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THEORY OF ECONOMIC MODELS FOR FORECASTING  
AND POLICY

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Part IPRELIMINARY SURVEY OF THE MACRO-ECONOMYChapter 1INTRODUCTION AND PURPOSE

The present study is an attempt to build a fairly detailed description of the complete economic system as a working, living organism. The ultimate goals toward which the work is pointed are prediction, policy and economic health and welfare. This study makes two steps toward that goal. It firstly attempts to describe how the total organism works, by studying its major functional organs one by one, and then by putting these together into an interrelated whole. This whole and its parts we may refer to as a complete 'model' of the macro-economy. The second step is to describe how this model can be used as an aid to economic forecasting, and the formulation of global economic policy. This requires a theory of the ultimate causes of economic change, of their prediction, and of the prediction of their impact and transmission through the economic system.

The term 'model' is used in the sense of being a simplification and an abstraction of the detailed complexity of the real world. The real system is thereby reduced to a scheme that can be worked with in a practical way. The reduction is accomplished partly by the aggregation of like micro-variables; and partly by abstracting

from the total economic system with its many facets, just the quantitative relations and interrelations.

While there is far more to the economic system than those aspects which can be counted and estimated numerically, it is these aspects which are of paramount importance in forecasting the movements of the system and in prescribing policy for it. Of course it must still be recognized that the other aspects of the economic system such as its human relations, institutions, value systems, exert a considerable influence on the outcome of the quantitative aspects. In fact these largely determine the structure, the parameters within which the numerical variables move and are mutually interdependent. It follows that the non-measurable aspects of the total system must not be lost sight of either in building the model or in using it. Structure itself is quantitative and can be estimated by econometric methods; but these non-measurables are causal with respect to structure and can cause it to change.

Working with a simplified model of the real economy will inevitably cause errors. But the alternative is such a maze of detail that no overall picture is possible; and then no progress can be made in the study and control of the macro-economy. All of the errors or divergences of a model must not be counted as losses, however. For the attempt is usually made to explain each deviation, and in this process new knowledge is often gained.

If a model is too aggregative the errors may be so large that the model is not usable. If, on the other hand, it becomes

too detailed, it may become practically as unwieldy as working with the real world itself. Hence a balance of some kind must be struck. In the present study the tendency will be to describe in more detail than has usually been customary in previous macro-models. This is done deliberately to bring the description and the model as close to the real world as possible ( and hence reduce the errors), within the limits of space and practicability. The purpose is to obtain a more complete view of the global economy and its anatomy; and at the same time to provide a firm base for a later reduction of the size of the model by further aggregation, for subsequent econometric research.

While it is hoped that the theoretical model developed will be interesting and useful in its own right, it is recognized that a model can become much more useful for practical forecasting and policy when its structure has been converted to quantitative form, - as in the case of an econometric model. Also some of the hypotheses that go into the design of the model cannot be tested until they are placed along side of the observed data of the real world. Many of the hypotheses can of course be tested by observing the behaviour of micro-units in the economy, and by introspection. But the whole set of hypotheses developed needs to be tested by the observed data of a total economic system before the model can be considered as fully usable for practical purposes.

For this reason the model developed will be aimed at econometric research, as well as at general description. But no econometric work will be attempted in the present study, for the field

to be covered in it is already sufficiently extensive.

While this work is aimed forward into prediction and policy, it is felt that its results could also assist in historical analysis. This is because one of the purposes of the model ( especially when its numerical form has been estimated) is to be able to explain the movements of the economy through analysing the separate influences on them of originating exogenous causes from outside and endogenous trends and cycles from within. This type of analysis, achieved through the cause-effect relations and interrelations depicted by the model, is no less necessary for a full comprehension of historical developments than it is for forecasting and policy.

In all such uses qualitative and descriptive models are helpful to understanding for many problems, but may become powerless against general equilibrium situations. They can be helpful for many types of partial analysis, where cause and effect are being studied within one section only. And they can be useful where the chains of cause and effect lead through many sectors but will always produce effects in the same direction. But when we are faced with a general equilibrium problem where all economic variables are affected, and where a final effect is a composition of both primary and induced causes working in opposite directions, qualitative models are often not helpful. For example, a raise in wage rates increases consumer demand and through it lifts the schedule of the demand for labour. But at the same time it moves the demand for labour backward along this schedule. What is the final effect on employment? And does it depend on the position from which the raise is made, and on the

magnitude of the raise? Qualitative models and algebra are largely powerless against problems of this sort, but arithmetic is not.

An adequate numerical model, can work through from initial cause to final effect, showing cause-effect linkages on the way if desired, and give a final definitive answer. The answer will be probabilistic, but this need not dismay us as long as the probability distribution of the answer is not excessively wide. There is in any event so much probabilism ( a distribution of possibilities, as distinct from certainty) in the affairs of men, that we have long ago learned to live with this problem and to allow for it.

There is nothing new in attempting to visualize an economy operating as a whole, as a complete organism. But it is only recently that attempts have been made to explain its movements through a system of interrelationships among its sectors and aggregative or macro-variables. The most important and revolutionary development in this direction was the "General Theory" of the late Lord Keynes [ 1.6 ]. His work must be counted as a 'great divide' in the field of economics. Disturbed by the needless suffering of the Great Depression he set to work in earnest to solve this problem by attempting to describe the inner behaviour of the total or macro-economy; and out of his labours some most revolutionary ( for the times ) conclusions emerged. For example: a system of interrelations among economic sectors and aggregative variables is implied; given the conditions of supply the level of employment is determined by the level of aggregate demand; there is no " hidden hand" guiding the economy constantly toward full employment and other desired economic condit-



ion, and it can remain far away from such an 'equilibrium' indefinitely.

If there is no "hidden hand" or 'governor' at work, then clearly a policy is needed to achieve the demands of an intelligent public opinion for a rational state of economic affairs. The logical executive of such a policy is the sovereign central government.

The possibility of successful government macro-policy, using mainly variations in taxation, government spending, and money supply to achieve certain desired goals hinges on the existence of macro relationships in the global economy. The hypothesis that such a system of relationships does exist and can be used is of course implicit in this study. The alternative or negative hypothesis would be that the systematic components in macro-economic behaviour are relatively small, so that the movements of the macro-economy are essentially random.

The positive hypothesis, to be used in the work to follow, is supported by general observation that the activities of human beings are usually more purposive than random; by general observation of the macro-economy and the effects of governmental policy on it; by the econometric work of men like Dr. Jan Tinbergen ( [1.18], [9.9] ) and Dr. Lawrence R. Klein ( [1.11], [9.5], [9.6] ); and by the ten years of practical experience which the present writer has had in an econometric program in the Canadian Government.

With these preliminaries the course is now set.

Our purpose is to attempt a fairly detailed description of the inner behaviour and functioning of the total economic organism,

of the aggregative cause-effect relationships which explain its movements. It is hoped that the model developed will be useful for subsequent econometric research and as an aid in historical research, economic forecasting and policy development.

The policy goals toward which it is directed are essentially over-all or global conditions such as full employment, price stability, and a satisfactory balance of payments. These are conditions of good health in the economy. And it is one of the desired functions of economics to be able to help the economy to preserve its good health, through detecting the onset of any illness, diagnosing its causes, and prescribing for its cure. The present writer has vivid memories of the Great Depression when the economy was very sick and human suffering was widespread. It is his hope that the models to be developed in what follows will help in a modest way toward preventing such disasters in the future, while helping to preserve a strong and healthy economic system.

Chapter 2.A BIRD'S EYE VIEW OF THE TOTAL ECONOMY CONSIDERED AS A VAST ORGANISM

Since our purpose is to study the total economy as a unity, it is helpful to attempt to get an overall, single comprehensive view of it. Since it consists of so many units ( individuals, households, firms and institutions), and covers such a wide geographical area, some preliminary simplification and aggregation scheme is necessary. For this purpose an analogy obtained by comparing the economy to a living animal organism consisting of many interrelated organs may be found to be useful. Each unit (individual etc.) in the economy now becomes a 'cell'. The cells are specialized to perform different functions. An aggregation of cells acting together to perform a particular function for the total organism becomes an organ. The organs acting in harmony together produce a unity - the complete animal or organism - which lives and grows. Let us consider briefly the further aspects of a total economic system which enable it to be compared roughly, but with some profit, as analagous to an organism.

The economy lives broadly based on its natural environment. This environment consists of the nation's share of the earth's surface, plus what is both under and above this surface. There are four main components to the natural environment -- the lithosphere (outer rock shell of the earth), hydrosphere, atmosphere, and the radiations received from the sun. The lithosphere contains all of the minerals and stored energy materials used by the organism at its particular stage of development. It also contains the vital topsoil out

of which, with the aid of sun, air and water, much of the organic food requirements of the organism flow. The hydrosphere provides fish, vegetable matter, and aided by lithosphere configurations and the sun, hydroelectric power. The atmosphere provides the organism with its vital oxygen for breathing and with other gases for industrial chemical processes. It also carries the carbon dioxide which is vital in the photosynthesis of the organic food supply. The sun has been the great and almost the sole supplier of energy to the environment and the organism. Some of this energy was stored up after photosynthesis in ages past, and appears now as coal, petroleum and natural gas. The remainder is the current, daily radiation supply which provides warmth, photosynthesis of food, and hydro power. Only recently has an additional source of energy other than the sun become significant. This is the nuclear energy existing in the atoms of the matter of the natural environment, and is of the same nature as the sun's energy itself.

The natural environment provides the 'food' supply of the organism, is: 'mother earth', and against it the mouth of the organism presses. The mouth is made up of the extractive or primary industries; agriculture and pastoral grazing, forestry, fishing and mining.

As the 'food' drawn up moves beyond the extractive industries or mouth, it passes into a preliminary digestive system which transforms the raw food into forms that the stomach desires (consumer goods), into reserve fat (inventory stocks), or into bone structure (capital goods). These transformative industries consist of manufacturing and construction, and are usually referred to as the

secondary industries.

The fat reserves are important as they enable the organism to maintain full activity throughout its system for a while, despite any temporary disruption in food intake or internal supply flows. The bone structure gives the organism greater sturdiness, and also multiplies the effectiveness with which it can use its 'energy foods' from the natural environment.

From the preliminary digestive organs the now transformed 'food' moves mainly to the stomach for current digestion and use in the household sector of the economy.

The organ which facilitates the passage of 'food' from the secondary industries to the stomach or consumption areas is made up of the retail trade and consumer services industries. We may call these the tertiary industries.

A further group of industries in the economic anatomy provide essential services to all of the primary (extractive), secondary (transformative) and tertiary (service) industries or organs already described. These may be called the energy and linking, or the cohesion and integrating industries. They help to convert the separate organs into a unified or integrated whole. They include those industries which carry energy, goods and information to all of the other industries and organs: the power, transportation and communications industries. These industries tend to reach all parts of the economy, with great trunk lines running between agglomerations of cells, or 'cluster' areas. Then at these cluster areas they 'filament' out to reach almost all individual units or cells in the area.

The communications industry is the transmission section of the nervous system of the organism. The transportation industry is its blood stream, carrying its essential 'foods' from the mouth, through the preliminary digestive organ to the stomach, the fat stores and the skeleton. The power industry is the heart and life force of the organism, keeping it 'alive' and keeping it supplied with energy.

Other industries in this 'energy and linking' group are wholesale trade, warehousing, and business services. Since the six main industries in this group service almost every organ in the organism, including themselves, they collectively provide excellent indicators of the general level of activity of the total organism.

Flowing through the anatomy or the body of the economy is a substance which acts partly like a 'glandular secretion' or 'hormone', and which is called money. Its primary function is to facilitate exchanges of 'food' between specialized cells. It achieves this through serving as "generalized purchasing power" to its holders, giving them a wide choice in the selection of their intakes, and also in the timing of their intakes. The quantity of this secretion appears to influence the rate of blood flow, and the general level of activity of the complete organism. It tends to run through parallel or the same channels as the blood flow, but in the opposite direction. It is secreted in and emanates from a major gland in the organism, the savings pool, within which cluster the banks and other financial institutions, all of which may be referred to as the finance industry.

As the money flows through the arteries of the productive system facilitating exchanges and serving as a counter and surrogate of real income, part of it flows back again as savings, into the savings pool. At the same time money flows out of the pool as investment spending, and thereby re-enters the productive system. A good deal of the health of the organism depends on a satisfactory balance between these inflows and outflows. If the pool fills up there is soon too little money and hence demand flowing through the productive industries. Unemployment speedily follows. If the pool runs dry it is a symptom either that the system does not have enough money, or that there is more demand and more money being used in the productive system than it can stand without suffering from a fever of inflation.

Another important regulatory or hormone like mechanism consists of the rates of exchange between goods and money which are formed in the many markets of the economy. These are 'stop' and 'go' signals which tell the units to produce or consume more of this and less of that. There may be some counterpart to these signals in a real organism, and it is perhaps the hormones and secretions of the ductless glands which are in some sense analagous to the price system.

This regulatory 'stop' and 'go' function is not handled exclusively by the price system, however. The latter works in close teamwork with the system of 'fat' stocks or inventories in a more complete performance of this important function. Prices in the modern economy are by no means completely flexible, and often excessive 'fat' accumulation in one product is the signal, rather than or along with

falling prices, to slow down on production and speed up consumption of this product. Similarly a shortage of inventory below a comfortable level acts as a stimulus to increase production and reduce consumption.

To turn next to the household sector of the organism, here we have a sector which embodies several different functional areas or organs. Firstly it consumes the 'food' which the organism has drawn up from the natural environment, prepared for digestion in the transformative industries, and passed on through the tertiary industries. This part of the households is the stomach organism. But the households also provide the labour force of the economy, the brains, muscles, and productive skills of the organism. The 'food' consumed in the stomach provides nourishment and strength to these brains and muscles, and it is these in turn which keep all of the other activities and organs within the system going.

The household sector also provides the personality of the organism. Following the lead of Dr. Sigmund Freud, and the psychiatric field of knowledge which has developed as a result of his work, we may separate personality into an ego which decides, manages and innovates; an id wherein lie the instinctive drives; and a super-ego which develops aspirational standards of what is just and good, and which tries to keep the whole personality from straying from these standards.

The ego function is essentially the government function, and it is performed by members of households. The government function is not restricted to the political government of a social organism, although it may reach its most important focus there. Decision,



management and innovation are also required in firms, institutions and households.

The super-ego function is likewise exercised in all of the individuals, households, firms and institutions of the social organism. But it also tends to reach its highest form and focus in certain institutions, which we may refer to as the social purpose and value building institutions. These include education, religion and the arts. It is the aggregate level of this super-ego function in any society, as fostered and enhanced by its institutional focus, that will have very much to do with the levels of justice, honesty, beauty, and happiness in the society.

The levels reached by the aggregate ego and super-ego of a society have a great bearing on the global productivity and other parts of the economic structure and development. In many ways in fact the global personality of a society filters through its economic system setting its pattern and development.

Our portrayal of the economic organism is not complete without mention of the foreign trade sector. Through its relationships with other countries of the world the domestic economy is able to obtain goods at all stages of production from abroad, which it either could not produce at all, or could only produce at excessive costs. For these goods it trades its own products which it is able to produce easily and cheaply. Through this international division of labour and specialization the real incomes of all countries are increased. In this trade domestic money flows out of the economy into an 'internat-

ional money exchange' for imports, and back into the domestic economy for exports.

The economic organism described above consists of a system of interrelated organs. Each organ performs a different function, necessary to the welfare of the complete organism. If the functional organs are all performing harmoniously with respect to each other and the total, the organism will be healthy. We may think of it as something alive with food flowing up from the natural environment, through the extractive industries and thence into the transformative industries. We see a continuous circulation of goods in one direction and money in the opposite direction through the arteries of the productive system and from industry to industry until the goods finally flow to the stomach, the bone structure, to fat storage and to foreign countries. We see money being paid to persons and firms for their productive efforts, and being partly respent within the system, but partly saved. The saved portions flow into the savings pool, adding to its level, but diminishing the flow of money and demand through the productive system. At the same time money flows out of the pool mainly to the industries producing investment goods, and thereby swelling the flow of money and demand in the productive system.

If one or more of the organs of this living system gets out of order or harmony, the total organism will become ill. Thus if the stomach demands excessive 'calorie food' or if this is all the system can produce the bone structure (capital equipment) may suffer. If too much fat (inventories) develops the organism must

slow down (with unemployment) until the fat is used up. If too little money is secreted the organism may become lethargic, lose its appetite, and develop unemployment. If too much money is secreted and pushes out of the savings pool, the organism's appetite may become voracious. It may then attempt to eat more 'food' than it can produce, with a resulting 'fever' of inflation.

This analogy of the economy as a living organism with its continuous activities and circulating flows, and with its possibilities of sickness and health, has been developed merely to facilitate the attainment of a bird's eye view of the total economy. In later sections the organs are studied in more detail, but mainly with respect to their quantitative behaviour. The analogy with a living organism is largely dropped in this analytic research. But when finally in Part IX the behaviour patterns and relations for the different sectors are all brought together in one large model, it may again be helpful to revert to this chapter to help get <sup>an</sup> overall view once again of the whole vast and interconnected system.



### Chapter 3.

## THE BASIC MOTIVATIONS AND GUIDES TO BEHAVIOUR WITHIN THE ECONOMIC ORGANISM

What keeps the whole complicated system in motion? What are the sources of energy or drive in it? What articulates and integrates the multitude of cells (households, firms and institutions), and the many organs (industries and sectors) into a smooth-running complete organism? What restraints and behaviour guides does the system provide to bring this about? These are some of the questions which this chapter will attempt to discuss briefly.

The economy is run by people assisted by the natural environment and various cultural products consisting basically of knowledge, materials, machines and structures. But without the people willing it, not an act or an iota of production would take place. Thus individuals move the system, and we must try to see what the various forces are that move them. We must try to get a comprehensive general view of the person, both as an individual and as a member of society. For each person plays a dual role - both as a separate personality, and as part of a social and cultural milieu.

The Freudian separation of the personality into three components again seems helpful. There is firstly the id representing the instinctive needs and urges associated with security, survival and procreation. The production of food, clothing, shelter and other consumer goods can be related to this part of the total personality. Next there is the ego, the part of the personality through which the

individual governs and manages himself, seeks creative activity, decides, innovates and organizes, and attempts to find his appropriate place in the social and productive hierarchy. Finally there is the super-ego which seeks for the personality a scale of values and aspirations, a code of ethics, and a purpose in life. This part of the personality tends to determine the individual's total preference system, within which his economic or material preference system lies.

Combining all parts of the individual and his personality we find that he wants material goods, but that he also wants much more than these. He wants to do, to create, to produce, to grow in performance to the full extent of his innate capacities, and to enjoy social fellowship in the process. He also wants to participate in social goals of social welfare. And finally he wants to feel that he is acting fairly and justly toward his fellow man and other living things in his environment; and that he is aiming toward the general purposes and values of the society and of his own personality.

Of course all individuals do not act with the full system of drives here described. Some have more, some have less, some personalities are not fully formed, some are ill. Only an average picture is here attempted, and as suggested before, it is the level of this average personality which has much to do with the cultural and developmental level of the whole society.

Given the pattern of goals and drives (the total preference system) within the individual, what must he do to attain and satisfy them? He must marshal his available resources and direct them in some kind of economizing, maximizing fashion toward his goals.

What are the resources or means available to the individual, that can be directed toward his want system? Stock-taking suggests personal economic wealth, personal abilities or capacities, personal energy and time.

Maximizing the level of a want system given a budget of various assets and flows is a familiar type of activity in the field of economic analysis. Here we carry the same type of analysis over into a wider area, encompassing the total man. Thus we assume that as each goal is invested in, the satisfaction from additional amounts of it will eventually begin to decline. At the same time the additional cost of pursuing it to the exclusion of other wants may begin to increase. He will not pursue any goal beyond the point where the cost of an additional unit becomes greater than the satisfaction from it. At the same time if a unit of resources produces an increment of goal A which yields more satisfaction than an increment of goal B also produced by a unit of resources, some resources will be switched from B to A. This type of shifting between goals at their marginal increments, will tend to continue until the marginal satisfaction from a unit of resources is equalized in all alternative goal and want uses.

This type of behaviour will gradually bring an individual to the greatest satisfaction or utility that he can derive from his resources. As means are applied to ends a multitude of psychophysiological reactions within the individual guide him at each instant into the paths which provide him the greatest net satisfaction. In the current weighing and balancing he will often of course have to estimate the present values of future satisfactions which present

actions are planned to bring about.

It remains to question whether an individual will in fact carry out any such maximizing procedure. On this point both general observation of living creatures and introspection lead one to suggest that it be taken as an axiom that at any instant an individual will select that one, of all of the opportunities apparent to him, which yields the highest net satisfaction to his total personality. Of course many individuals do not appear to follow such a maximizing behaviour. They give the impression of not wanting to be bothered with the effort of maximizing. But can this not be traced to their personality and goal preference system, which may set a high value on leisure and indolence? Given this value system they can still be assumed to maximize their satisfaction from given resources. Our axiom would still hold that any person will act in a way which pleases him most, and it can then be assumed that 'peculiar' behaviour follows from 'peculiar' preference systems, which differ from the customary norms.

From our axiom that every individual attempts to maximize his satisfactions, and hence that only preference systems, not maximizing behaviour, differ from person to person, certain principles of behaviour can be deduced. These principles will be found generally useful as aids in some of our subsequent model building, and they are helpful here in attempting to get a broad view of the general motives and guides which

motivate and channel the activities of the people, the movers, of the macro-economy. They are expressed here in terms of satisfactions or utilities, and disutilities, and as though these could be measured as amounts or quantities. There has been much recent controversy in the literature about whether utilities and disutilities can in fact be so treated. (As a guide to this literature the reader is referred to Professor J.R. Hicks [2.12], Professor Sir Dennis Robertson [1.19], ~~and~~ Professors J. Von Neuman and O. Morgenstern [3.26], and Dr. S. A. Ozga [1.17].) But here we are following the old-fashioned concept of utility or satisfaction as something which the individual feels in quantitative form, even though he does not explicitly measure it. How else could he compare the degree of increased satisfaction from an increased use of resources on element A of his total preference system with the cost of the resources? How else could he decide between different lines A and B, where different marginal quantities of resources are involved, and where many of the total resources concerned (e.g. time and energy) cannot be brought to any common objective measuring unit like money? Thus  $\Delta A$  is preferred to  $\Delta B$ , but  $\Delta A$  is felt to cost more than  $\Delta B$  in terms of resources. How can he choose whether to select  $\Delta A$  or  $\Delta B$  unless he can quantify in his own mind the net gain or loss in utility from  $\Delta A$  in comparison with  $\Delta B$ , taking into account the disutilities or costs connected with each?

Using the old-fashioned concepts and the axiom stated above, the following general principles or 'laws' of the total behaviour of an individual are worked out, and here summarized:

(1) If the total milieu of an individual remains fixed except for



the quantity variation of <sup>an</sup>element, line, or goal in his total goal or preference system, the extra or marginal satisfaction per additional unit of the goal will generally pass through three stages. In the early stage with only small quantities of the goal the marginal satisfaction will be increasing, at a medium stage it will become constant, and in a final stage for still larger quantities it will start to decline.

(2) In using a group of resources to produce a particular goal, if all resources but one are held constant and if this one is varied from a minimum to a maximum boundary, the marginal utility of the product (marginal product) for successive units of the varying input will often pass through three successive stages. These stages can be described as increasing, constant and decreasing marginal product or returns.

(3) An individual applies all his resources to the attainment of all of his goals, and allocates his resources in the following ways. Resources will be applied in appropriate groups toward any single goal in such a way that the marginal products of an equal valued unit of all resources are equalized. Until this all round equalization is reached a gain in utility can be had by using more of the resources whose marginal products are higher, or less of the resources whose marginal products are lower.

The different goals in his goal system will be pursued until the marginal utility of each goal per equal valued unit of resources used up is equalized for all goals. In reaching this final

position the individual will increase his pursuit of those goals with high marginal utilities and perhaps decrease his pursuit of those with low marginal utilities.

The resources of wealth, training, energy and time must be brought into some kind of common measure for this process. This must be done through the individual's subjective utility system, making use of economic costs, opportunity costs and disutilities in a complex of psycho-physiological reactions.

Certain side conditions are necessary in order for the individual to reach a stable maximum of his total satisfactions. In general marginal productivities and marginal utilities must have reached the third stage of diminishing marginal returns and diminishing marginal utilities, or the individual will try to increase his use of certain resources or his attainment of certain goals.

Since it was stated that all resources were applied toward goal attainment, there is no problem of deciding when to stop using resources. They are all used and allocated as outlined above.

(4) As a corollary to the above three basic principles we may consider the choice between work in the economic sphere and leisure outside of it. Leisure and recreation are of course part of the total goal system, but are only indirectly part of the economic system. Economic goals and resources can be defined as subsets of the total goals and resources, and such that they can be brought under the measuring rod of money.

(Figou [1.18]) They refer to the more materialistic aspects of the total range of personality and life.

The choice between work and leisure will be made by an individual in the following way. He will apply time, energy and wealth to the production of economic goods for present and future consumption until the marginal utility of these goods begins to fall, and the marginal cost of the resources used (opportunity costs and disutilities) begins to rise. The resources will continue to be applied until the marginal cost of resources becomes equal to the marginal utility of economic goods. ( Cf. Alfred Marshall's boy and berries [1.15] , p. 331).

It is suggested that the three principles and the corollary above underlie the social and economic behaviour of any society, primitive or modern. Societies may differ in their value and goal preference systems, and in their resources knowledge, and techniques of production. Also their value systems and productive techniques will change over time. But at any instant these will be given and will determine the motivations, drives and methods of the society. The behaviour laws outlined above will then guide the society automatically toward maximizing the satisfaction obtainable from its means in satisfying its total wants.

Any society can lift its current maximum of satisfaction to a higher level in the future, by lifting either or both of its scale of values, and its resources.

Chapter 4.SOME PRELIMINARY REMARKS ABOUT DEMAND AND SUPPLY

The goals of society consist of basic or survival needs, and culture inspired needs and wants. These goals provide a stimulus to action to satisfy them. The stimulus may be thought of as the essence underlying demand; the response to this stimulus is correspondingly the essence behind supply activities. If the desire for a goal is very great its total production in the society may be large, unless its cost in terms of wealth, effort and other goals foregone is also very high. Thus expressed demand is conditioned by supply conditions or costs. Expressed demand becomes a positive function of the urgency of desire and a negative function of cost. Production of a goal will begin as long as this urgency is felt to be greater than the cost, and will be expanded until the marginal satisfaction decreases to equality with the marginal cost.

In a primitive society demand (expressed demand) and supply are very closely related. The demand stimulus will tend to be followed by a supply response, and demand and supply will be continuously equated in this fashion. (An even closer relationship is provided by the fact that one of the needs of the individual is for purposive activity which gives expression to his capacities. The activity of supply satisfies this need up to a fatigue point.) A feeling of hunger for example may be immediately followed by an excursion to collect food. In such situations where demand tends to create an immediate and direct supply reaction we have an inverted

Say's Law. But in modern monetary-industrial societies while the ultimate relationships between demand and supply are not essentially different from these, the immediate relationships are completely different. Demand is expressed by means of money. Depending on the preference system, demand for any goal is still a function of the urgency of desire and cost. But cost is now a question of market price, opportunity cost (the prices of other goods), and the value of money to the individual. This latter value depends on the ease with which the individual can earn money income, and the extent of his existing wealth. Money becomes a 'generalized purchasing power' in which demand is expressed. It will not win all social and cultural goals. Some of these must still be won by direct activity response. But most economic goals and many social goals are attainable merely by the exchange of money for them.

The demand concept in modern societies can be summarized as an 'ex ante' willingness to spend money on a particular good, where the willingness is not a fixed amount, but is rather<sup>a</sup>/variable or function. It is an effect or function of several other causal factors, among which must be included the goal system or preference pattern the available quantity of money, and both money cost (price) and opportunity cost.

Supply activity is also significantly changed in the money-industry economy of the west. It is only seldom a direct response to an immediate felt want. It becomes a specialized activity in which a product is produced for sale on the markets in exchange for money. Since money has become the 'sine qua non' of demand, it

becomes an essential goal of supply. In general the greater the net receipts for supply the greater the supply output will be. The supply of any product tends accordingly to become, like demand, a variable or function, depending on certain causal factors. Among these we must include the techniques of production and the level of net receipts of money.

The system of specialization and the use of money for exchange has enormously increased productivity in modern economies. But it has separated the demand stimulus from the supply response to the point where their connection is only clearly recognizable in the longer run. They are very much separated by money in the short run. This causes one of the major complications of modern economies. In the primitive economy demand almost completely generates its own supply. But in the modern economy neither this nor its converse is true. Demand may be strong on the basis of all component causes but one - money holdings from income and wealth. It is thereby stifled. At the same time supply activity may be temporarily strong. But it can only continue to generate money incomes if it is all sold. But since demand is independent, and usually exercised by different people, supply may not all be sold. And even if it is all sold, the income generated may not necessarily be all spent, or spent in the same way, in the next round. For although spending surely creates income, income does not necessarily create spending. Thus a full employment supply can often not be sold, and piles up in inventories until supply activities are reduced and unemployment results. In

this way the divorcement of demand and supply through the money system in the modern economy can cause an illness to which the primitive society is not subject. In the primitive economy excess supply can occur, but it only leads to a temporary spell of increased leisure or ceremonial. In the complex modern economy it can degenerate through dynamic forces into widespread unemployment, and a form of social exile for the unfortunate unemployed.

In summary: In this and in the preceding chapter the attempt has been made to show that fundamental economic laws of nature are at work in all societies, primitive and modern. In any society the total personality and the social purpose and value building institutions set up a pattern of goals and a system of values. These become the basis of demand stimuli, which are then tempered by the difficulties of supply. In the primitive society demand in a sense creates or generates supply responses directly, and demand and supply almost never get out of balance. Demand is never inhibited by a money problem. But in the money society demand is expressed with money, and supply is produced for money. In the process demand becomes sufficiently removed from supply that the two can easily get out of step enough to produce severe hardships.

There is little in economic analysis that does not fall within the concepts of demand stimulus, supply response and rates of exchange of goods for money. A large portion of the subsequent analysis of the behaviour of the total economy is devoted to the study of the important aggregate demand, and aggregate supply flows, and of the causes behind their variation. Through this inquiry it is

hoped to be able to show how to discover the causes behind that lack of balance between demands and supply flows, which result in either inflation or unemployment. The results should then show what can consciously be done by the social ego (government) to bring about a proper balance.

The problem could be directly eliminated by rejecting specialization and money. But this would be tantamount to 'throwing out the baby with the wash water', an entirely unnecessary regression.



Chapter 5.THE PROBLEM OF AGGREGATION

In the analytical work which follows the general procedure will be to study the behaviour of the micro units or cells of the economy (individuals, households, firms and institutions) and then to aggregate the behaviour of these units into the total behaviour of its separate organs or sectors. In a real organism like a dog or a cat the individual cells do aggregate well and lose their individual identity in the overall identity of the dog or cat. In the case of an economic system however the individual cells do not lose their individual identity so completely, and this raises the question of how effective aggregation procedures can be in economics.

To obtain a clear picture of what aggregation for an economic system means, it will be helpful to consider symbolically all of the actual quantitative interrelations in such a system. Let the subscript  $i$  ( $i=1,2, \dots, G$ ) represent an economic activity or variable decided on and carried out by an economic unit. Let the different economic units in the complete system be designated by  $j$  ( $j=1, 2, \dots, N$ ). Then all of the micro-variables of the system can be designated by  $y_{ij}$ . Since any one decision and activity both influences others, and is influenced by others, there is a system of interrelations among the  $y_{ij}$ . But each micro decision and behaviour is also influenced by broad aggregate flows and trends in the economy. For example the buying decisions and behaviour of a firm are based on the rate at which its individual customers are buying from it, but

also on its view of the total level of activity in the economy. Likewise an individual consumer is influenced in his buying behaviour by the income which he receives from a firm or institution, but is also made more buoyant or less buoyant by his knowledge of the general economic climate, as this is reflected to him by various economic aggregates. In this way even the real micro-system of the economy embodies a use of aggregates - those which influence the behaviour of the micro-units. These aggregates are usually formed by simple summation or by simple or weighted arithmetic averaging. It is customary to aggregate only like or similar activities. A general system of aggregates which could be derived from all the  $y_{ij}$  might be designated as  $Y_k$  ( $k=1, \dots, H$ ).

To complete our generalized system we must mention the exogenous micro variables  $z_{rs}$  which enter the system from outside and influence it, but are not influenced by it. Let their relevant aggregates be  $Z_h$ . The total system of micro behaviour and interrelations can now be represented symbolically by

$$(1) \quad \left[ \begin{array}{l} y_{ij} = f_{ij}(y_{11}, \dots, y_{ij-1}, y_{ij+1}, \dots, Y_{GN}; z_{11}, \dots, z_{rs}, \dots; \\ \quad Y_1, \dots, Y_H; Z_1, \dots, Z_K) \\ (i=1, \dots, G; j=1, \dots, N) \\ Y_k = A_k(y_{k1}, \dots, y_{kN}); (k=1, \dots, H) \\ Z_h = B_h(z_{h1}, \dots, z_{hr}, \dots); (h=1, \dots, K), \text{ where the } A_k \text{ and the} \end{array} \right.$$

$B_h$  are the aggregation functions or procedures used by the micro units. There are  $GN + H + K$  equations in this total system with  $GN + H + K$  endogenous or mutually interdependent variables.

System (1) portrays the real interrelations of the economy. But it is too large and detailed to be used by the practical worker faced with macro-economic problems. It is necessary for him to get this real system down to a size that he can handle. His procedure in doing this is to arrange the micro-variables in (1) so that all similar causal variables are placed side by side horizontally in the  $f_{ij}$  functions, while at the same time all similar effect or decision variables are placed together in the vertical array of the  $Y_{ij}$  on the left hand side of the system of equations. Then he aggregates each set of similar variables into one aggregate variable. In doing this the system of interrelated variables reduces to

$$(2) Y_i = F_i ( Y_1, Y_2, \dots, Y_G ; Z_1, \dots, Z_K ).$$

$$(i = 1, \dots, G)$$

The system (2) is not a 'real' system. It is more like an 'average' of the system of micro-relations. (Cf. Tinbergen [9.9], Vol. I p. 14, where he says that the coefficients of the macro-equations "do not give any indications of the behaviour of individual entrepreneurs, consumers, etc., but only of the average reactions of many individuals"). The macro-system (2) can also be thought of as the shadow or outline which the real underlying micro-system creates, and reveals to an outside observer. It is further sometimes convenient to think of the macro-system (2) as an "analogy" to the micro-system (1). (Cf Theil [1.21]). Whichever term is used, what is clearly meant is as follows. In the true equation system we have

$$\begin{aligned}
 & y_{i1} \text{ influenced by } (y_{g1}, \dots, y_{gN}), (y_{h1}, \dots, y_{hN}), \dots \\
 & y_{i2} \text{ influenced by } (y_{g1}, \dots, y_{gN}), (y_{h1}, \dots, y_{hN}), \dots \\
 & \vdots \\
 (3) & \vdots \\
 & \vdots \\
 & y_{iN} \text{ influenced by } (y_{g1}, \dots, y_{gN}), (y_{h1}, \dots, y_{hN}), \dots
 \end{aligned}$$

where  $y_i$  is one kind of micro decision or behaviour and  $y_g$  and  $y_h$  are others. To simplify the system and make it manageable we replace the groups  $(y_{i1}, \dots, y_{iN}), (y_{g1}, \dots, y_{gN}), (y_{h1}, \dots, y_{hN}), \dots$  by their respective aggregates  $Y_i, Y_g, Y_h, \dots$ . Then we pretend that the aggregate  $Y_g$  influences the collectivity or aggregate  $Y_i$  in some average way.

Of course this does not in any sense imply that the aggregate system (2) which substitutes for (1) is as good as (1), or will give the 'true' results which might be obtained from (1). It is only an average or representative tendency of the multitude of micro relations. Hence it will likely contain similar weaknesses to those which apply to any average or index number, and is likely to produce errors. This has influenced some researchers to raise the question of whether some 'perfect' form of aggregation is possible, whereby the macro-system(2) would produce the identical results produced by (1). (Cf. Klein [1.8], [1.9], [1.11], [1.12]; Theil [1.21]; and the further references in these sources.)

While any such research is useful in throwing more light on a problem, it would seem that these searchers were asking for almost too much. They were seeking for as much information about the macro-system as could be obtained from a knowledge of the complete

micro-system; yet they were presumably attempting to avoid having to have this detailed knowledge of the micro-system. One of the main features they wanted of course for their system of aggregation was that it would give correct results even when the internal distribution of the  $y_{ij}$  within the  $Y_i$  changed. And both did achieve for limited circumstances a form of such 'perfect' aggregation. Klein worked on the case of industry aggregations with production functions assumed to be of the Cobb-Douglas type. In this case his 'perfect' aggregation method involves the use of geometric rather than arithmetic means. But this solution is not generally usable throughout the complete economic system. Theil on the other hand assumes linear systems and achieves 'perfect' aggregation at the expense it seems of a knowledge and use of the complete micro-system(1). It would be about as easy to use (1) in the first place, as to use Theil's method of aggregation. For each of his  $Y_k$  on the right hand side of (2) is a weighted arithmetic mean of the  $y_{kj}$ ; with weights  $b_{kj}$  which are the marginal rates of influence of  $y_{kj}$  on the  $y_{ij}$  on the left hand side of (1).

To obtain a preliminary view of the aggregation problem, a very simple example is now presented. Suppose that there are only two families 1 and 2 in a hypothetical society, and that their individual consumer demand equations are

$$(4) c_1 = a_1 + b_1 y_1$$

(5)  $c_2 = a_2 + b_2 y_2$ , where  $c$  represents consumer demand, and  $y$  represents household income.  $a_1$ ,  $a_2$ ,  $b_1$  and  $b_2$  are parameters, with  $b_1$

and  $b_2$  the "marginal propensities to consume". What is the average or analogous macro-relationship between aggregate consumer spending and household income? Let  $C$  and  $Y$  be the customary aggregates of consumer spending and household income, so that

$$C = \sum_{j=1}^N c_j = c_1 + c_2, \quad Y = \sum_{j=1}^N y_j = y_1 + y_2.$$

Then the desired macro-consumption function for the whole society is of the form

$$(6) \quad C = A + BY.$$

The important questions to be asked of (6) now are:

(a) Under what conditions will (6) give the same aggregate explanation of  $C$  as would be obtained by using both (4) and (5)? (b) How could  $Y$  be defined in terms of  $y_1$  and  $y_2$  so that (6) will always be equivalent to (4) and (5) for the aggregate explanation of  $C$ ?

To answer the first question we note that (6) will give the same result as (4) and (5) combined if

$$(7) \quad A = a_1 + a_2; \quad B = \frac{b_1 y_1 + b_2 y_2}{Y} = b_1 s_1 + b_2 s_2;$$

where  $s_1$  and  $s_2$  are the respective household shares in the income distribution. We assume the true micro-parameters  $a_1$ ,  $a_2$ ,  $b_1$ ,  $b_2$  to remain constant for the period under study. Then (6) will explain  $C$  correctly under a variety of income variations as long as the income distribution ( $s_1$ ,  $s_2$ ) does not change with changing incomes. Also there is one other condition under which (6) would explain  $C$  correctly. This is if the marginal propensity to consume for all families were equal. Thus if  $b_1 = b_2 = b$ , it follows from (7) that  $B = b$ . Under

this second condition (6) would explain C for any changes in income, and irrespective of the income distribution.

Faced with the limitations which the above two conditions impose on (6), Klein, Theil and others have set themselves a perfectionist problem. How could Y be redefined so that (6) would always give the true values of C without any such limitations? As mentioned above Theil tackles the linear problem, and comes out with the following aggregation formula for Y.

(8)  $Y = \frac{b_1 y_1 + b_2 y_2}{B}$  and is thus a weighted arithmetic mean of the individual  $y$ 's, with weights proportional to the corresponding micro-marginal propensities to consume.

If Y is aggregated by formula (8), it is clear that (6) will provide correct explanation of C irrespective of changes in the income distribution, and regardless of differences in the b's from household to household. But the cost of this aggregation is so high that almost nothing is gained. For the aggregation of Y and of other endogenous variables in a larger system requires full knowledge and use of the micro system. All that has really been done is a regrouping of the micro-system into the useful patterns suggested by (3), followed by the addition of like parts of the micro-system as it stands.

Theil's results incidentally apply only for a single macro-equation. He was not able to discover any 'perfect' aggregation for a system of macro-equations like (2). His methods do however apply to the reduced form of (2), found by solving for each Y in terms of the Z's,

(9)  $Y_i = H (Z_1, \dots, Z_k) ; ( i = 1, \dots, G )$ . For each equation of (9) can be treated as though it were a separate equation.

Theil's and Klein's researches are very useful in that they point to types of ideal solutions to the aggregation problem, and indicate the pitfalls of ordinary aggregation. But they are not of general applicability in applied work. In this connection our simple example above provides certain precepts for practical work. These are that we should try to select our boundaries of aggregation in any particular problem such that either (1) the distribution of the micro-variables which compose any aggregate is not likely to change much , or (2) the behaviour of the micro units involved with respect to the micro-variables is nearly the same within each aggregation boundary ( the case of  $b_1 = b_2 = b$  above). Aggregation of employment in a production function might be an example of aggregation which follows the first precept; while the division of total income into separate groups, such as wage, investment, and farm operator aggregates, is probably a good example of the second.

To sum up, it can be concluded that aggregation error will be one of the losses associated with the use of practical macro-models in economics for some time to come. Such models will usually be modeled on an arrangement of the real micro-system into groups of like behaviour, followed by the conversion of these groups into macro-variables with associated macro-parameters. Grouping should be carried out in keeping with the two precepts suggested above to reduce aggregation error as much as possible.



While the continued theoretical study of aggregation error in macro-models is undoubtedly important, the practical problem is almost solely a matter of the sizes of these errors. Research in the practical field on the development and use of macro-models, for example by Tinbergen [9.9], Klein [1.11], [9.5], [9.6], Brown [2.6], does not reveal that these errors are excessively large, or that they destroy the usability of the models. In fact, as Wold so well expresses it [1.23], the search for an exact relation among aggregates is rather academic in any event, since the underlying micro-relations are not assumed to be exact. They are, at least in practice, "disturbed by the influence of factors not accounted for in the theoretical analysis". In the macro-equation, aggregation error becomes simply another error to be added to the aggregate disturbances derived from the micro-disturbances. If the problem is approached this way the researcher is freed from the restrictions of attempting 'perfect' aggregation (not usually possible in any event for systems of simultaneous, linear, structural macro-equations), and can concentrate on the economic form of aggregate relations which might be most applicable to the particular problem at hand.

In the research which follows, economic form and content will be what is essentially sought for, and it will be understood that aggregation error will always be present. Only simple aggregates using weighted and unweighted arithmetic sums and means will be used conceptually for the macro-variables. Within this framework it is believed that the two rules or precepts developed above will help to minimize the aggregation error.

Chapter 6.THE METHOD TO BE FOLLOWED

It has been observed above that the quantitative aspects of a total economic system involve many relationships and interrelationships among all of the important economic variables. This pattern of interrelatedness could be represented by a network of interacting electric circuits, or by a network of interconnected tanks, pipes and valves with water flowing through the system. The same result can be achieved with a system of interrelated equations. In this system there is one separate equation to explain each individual economic variable. The equation explains the variable by making it equal to a function of all the other variables of the system which exert a causal influence on it. The variable being explained is in this context an effect or behaviour variable. But this same variable will usually in turn act as a causal variable in other parts or equations of the system, and it is in this way that the interrelatedness of the system is represented in the equation system. For example income will appear as a causal variable in the consumer demand equation in which consumer demand is the effect of behaviour variable being explained. But consumer demand will in turn appear as a causal variable in an equation in which income is the effect variable being explained.

In this type of arrangement there will be as many equations as there are variables which are explained by the system.

These are of course the endogenous variables. There is another class of variable in the system, however - the exogenous variables. These are causal factors or influences which act on the economic system from outside it, but which are not explained, or significantly influenced in any stable cause-effect relation, by the economic system. For example political, sociological, technological and physical forces are in the main exogenous to the economic system.

A further sub-class of variables in the economic model to be developed consists of those variables ( usually endogenous) which are lagged. They represent causal influences which do not produce effects in the current time period in which they are formed, but which instead have their effects delayed to some subsequent time period. Stock variables representing accumulations from previous time periods are a form of lagged variable. It is the lagged variables which tend to produce the cycles of economic life.

From the above description it can be seen that an equation system model of a complete economic system will require as many equations as there are endogenous variables explained or formed within the economic system. But the number of variables in the system or its model will exceed the number of equations by the number of exogenous and lagged variables which influence the system.

The purpose of the work which follows is both to describe the economic system and at the same time to develop a complete quantitative model of it, in theoretical algebraic form. What method will be generally used to arrive at each of the explanatory equations?

Each sector or 'organ' will be studied separately and one or more behaviour equations will be developed for it. The method of developing an equation will usually be to study the behaviour of the micro-unit or "cell" from first principles. From general observation and introspection, basic axioms and hypotheses are formed about the fundamental behaviour of the unit. The axioms and hypotheses are usually not subject to direct observation, but represent the very basic structure of the 'cell'. Deductive reasoning is then applied to the assumed basic structure, and is carried forward until a relationship between observable variables is reached. This first 'break-through' from unobservable and hypothetical structures like preference systems and maximization assumptions to a basic but observable relation we shall refer to as an "autonomous" micro-relation. (See also Haavelmo [1.4].) An autonomous relation is then observable, but fundamental, in that it cannot be reduced to any more basic or fundamental observable behaviour relation. Also it can be included in the class of "meaningful theorems" suggested by Samuelson [2.27].

The micro-autonomous relation is then aggregated in the manner outlined in the previous chapter, to provide us with an average, or shadow, or outline of the real behaviour of the multitude of cells. The global relation can be tested with aggregative economic statistics. If it simply does not fit the data, strong doubt is cast on the unobservable hypothetical structure used. But if it does fit the observed data well, it can be concluded that the original axioms and hypotheses of the basic structure at least have not been refuted.

The aggregate relation formed by aggregating autonomous micro-relations will also be referred to as an autonomous relation. Combinations of one or more autonomous relations into a single relation, with or without eliminating some of the original variables in the autonomous relations, will be referred to as "reduced forms". Although reduced forms may be useful in many cases, they have the defect of hiding basic behaviour. Also if reduced forms are fitted to statistical data the resulting equations may be difficult or impossible to alter when basic autonomous behaviour patterns change. For these reasons in the research that follows the aim will be to develop only autonomous relations for the model of the total economy.

A word should also be said at this point about the time units to be used in the economic model. Time could be treated as an instantaneous and continuous variable, in which case the model would be made up of differential and integral equations. The converse of the infinitesimally small time periods of continuous time analysis is the use of finite, consecutive, discrete time periods. The length of these could range from a day or a month in short-run analysis, to ten years for long-run studies.

The use of discrete time periods is a form of aggregation. Instantaneous stocks or rates of flow are averaged or aggregated over the finite time period to obtain the value of the variable for this period. If the time periods are long, much of the variation in a variable may be hidden within such an aggregate. For example a variable may pass crucial upper or lower turning points at some time

within the time period, and this important behaviour may be buried in the aggregate.

Of the two methods the infinitesimal and continuous time variable would represent the real world with the greater fidelity. But most economic statistics cannot be collected on this basis. It is usually only practical to collect them for finite periods like the week, month, etc. Since it is desired that the model be useful for practical work using practical statistics, the finite time variable is assumed in the subsequent development. But a small time period is assumed, in order to keep the model close to the short-run behaviour of the real world. The model developed can then be aggregated further with respect to time to make it a middle-run model or a long-run model, as the need arises. In general a month, or a quarter of a year, will be thought of as the time interval of the model, and the time period used will be referred to as a short time period. Middle-run time will be thought of as involving one and two year time periods, and long-run time will mean five and ten year time intervals.

The existence and detection of lagged influences will likely change considerably as we shift from short-period to long-period models. Few cause-effect relations are instantaneous, and hence most of the equations would involve lags when the instantaneous time variable is used. Many of them will still involve lags for short time periods. For example if a reaction from cause to effect takes one month to work out, the cause variable will be lagged by one time period in a model using time periods of one month. But the same relation

aggregated for time periods of one year would show the cause-effect reaction as 'instantaneous' or without observable or detectable time lag. Thus the appearance and structure of a model will change considerable as we aggregate from short to long time periods. Factors that may be important in the short-run may be of little relevance in long-run analysis, and conversely. But a middle and long-run model must still derive from the short-run model by aggregation.

As the model is developed sector by sector the final behaviour equations of the sector will tend to represent the summary or distillation of the quantitative behaviour of the sector. These final equations are transferred to Part IX, where the complete economic model is assembled in this summarized equation form. With it before one, one can attempt to get a grasp of the whole system.

Although mathematics is used from time to time through the work the non-mathematical reader need not be deterred for this reason. The main use of mathematics is the use of symbols to represent economic variables, and the placing of these symbols in equation form to show which variables are analysed to be causal with respect to the variable to be explained. In this way the symbols serve merely as a convenient, indeed a necessary, shorthand. Where mathematical methods are used to deduce the consequences of axioms and hypotheses, the non-mathematical reader can simply skip to the results. These will be stated both verbally and in the simple shorthand of the symbols.

All of the symbols used are listed alphabetically in Appendix B at the back with full definition and description. A shorter

description may then be used as each symbol is introduced in the text.

All bibliographical references are listed in Appendix A at the back, and are then referred to in the text by number.



Part IIANALYSIS OF BEHAVIOUR IN THE HOUSEHOLD SECTORChapter 7PRELIMINARY STUDY OF CONSUMER DEMAND USING COMPARATIVE STATICS.

We begin our detailed study of the total economy with the study of household demand for consumer goods. Here we have both the end and the beginning of economic activity. The bulk of the final goods of an economy flow into its households. And it is household needs and wants which play a major role in initiating economic activity. These needs and wants must inevitably represent a large part of the individual's total value and preference system, but as was suggested above they are still only a part.

It has been suggested in Chapter 4 that demand in a modern economy is an 'ex ante' willingness to exchange money for goods, and that this willingness is a variable function of certain causal factors. It is the purpose of the present section to attempt to isolate the major causal factors influencing the aggregate demand of households for economic goods and services. But only those influences which are revealed by a static or 'timeless' analysis will be studied in this chapter.

1. General Description of Household Behaviour.

The concept of "household" used in our model includes every individual disposing of his income on final goods for consumption

and wealth purposes, as distinct from production purposes. It is thus not limited to just family units. The consumer or household has a system of goals within which there is a pattern of preferences. Some of the goals will be economic, others non-economic. The economic goals may be loosely defined as those that can be bought or measured with money (Pigou [1.18]). The resources at the consumer's disposal for attaining his goals are his wealth, abilities, energy and time. His wealth consists of durable and semi-durable goods, and financial assets including money. We assume that he marshals his total resources toward the satisfaction of his total goals, in such a way that his inner feelings of satisfaction are maximized, even though he may not set out consciously and deliberately to reach a maximum. As a part of this process he allocates a certain portion of his time, energy, talents and wealth to economic production and income earning. Let this earn him a disposable money income of  $y$  in time period  $t$ . However his  $y$  is not independent of events within the consumer goods and other economic areas. For example price changes in consumer goods, or the rate of return on savings, will tend to influence his degree of participation in the labour force, and hence  $y$ . But these factors will be taken into account and permitted appropriate variation when we analyse the labour market and the determination of  $y$  from this market and the productive system. Here we take  $y$  as given from these areas and the consumer's preference system. Then we assume that as part of his overall maximization, he maximizes a sub-section -- the satisfactions obtainable by him from  $y$  in his purchase of economic goods.

Suppose that the consumer allocates his income among a vector of economic goals (1,2, ..., n), purchasing quantities  $(x_1, x_2, \dots, x_n) = (x)$  in such a way that the total utility derived from the selection is maximized. Thus,

(1)  $U(x_1, x_2, \dots, x_n) = \max.$ , where  $U(x_1, \dots, x_n)$  represents the sub-system of economic goals, and household preferences for them, in a functional form.  $U$  is a total utility or satisfaction index.

The quantities of individual economic goods purchased are constrained by their market prices  $(p_1, p_2, \dots, p_n) = (p)$  and by household income  $y$ . Thus

(2)  $y = p_1x_1 + \dots + p_nx_n = px'$  (vector product). All the goods but one represent purchases for current household use. This one good, say the  $n$ 'th, represents income saved for future needs. Thus  $p_nx_n = S_{nm}$  = household savings in money terms. The meaning of the real value of  $x_n$  and of the price  $p_n$  associated with it will be made clear later.

## 2. The Method of Comparative Statics for Deducing Household Demand Functions.

With goal systems, prices and incomes taken as given, we assume that the household will be at demand position  $(x)$  which maximizes  $U$  given  $(p)$  and  $y$ . As market conditions cause  $(p)$  and  $y$  to change, we assume that households will move to new positions which maximize  $U$  subject to the new constraints. Then a comparison of the changes in  $(x)$  with the corresponding changes in  $(p)$  and  $y$  gives us a basis of arriving at functional demand relations between the elements in  $(x)$  in terms of  $(p)$  and  $y$  as causal influences. The preference

system  $U(x_1, \dots, x_n)$  is assumed not to change. Given these assumptions a system of demand equations,

(3)  $(x)' = d'(p_1, \dots, p_n, y)$ , where  $(x)$  is the vector  $x = (x_1, \dots, x_n)$  could conceptually be arrived at. This could be done simply by observing the changing behaviour of a household under changing market conditions. Here however we attempt to deduce mathematically what properties are inherent in this system on the basis of our assumptions of a fixed preference system and the maximization of the consumer's position in it. The method of comparing one equilibrium position with another under changed conditions, without considering at all the time required or the time path taken in between the two equilibria, is known as the method of comparative statics.

### 3. Summary Mathematical Analysis of Consumer Demand Using Comparative Statics.

This summary draws heavily on the work of Professors Eugene Slutsky [2.28], R.G.D. Allen [2.2], J.R. Hicks [2.12], and P.A. Samuelson [2.27].

We begin by maximizing (1) above subject to the constraint (2). This could be accomplished by first substituting (2) into (1), but it is more simple to add (2) to (1) by the use of a Lagrange multiplier.

$$(4) \quad U(x_1, \dots, x_n) + \lambda (y - p_1 x_1 - \dots - p_n x_n) = U^*$$

Given  $(p)$  and  $y$ , what market basket or vector  $x$  will maximize  $U^*$ ? At a maximum  $dU^* = 0$ , and  $d^2U^* < 0$ .

Let

$$(5) \quad \frac{\partial U}{\partial x_i} = U_i, \text{ and } \frac{\partial^2 U}{\partial x_i \partial x_j} = U_{ij}. \text{ Then}$$

$$(6) \quad dU^* = (U_1 - \lambda p_1) dx_1 + \dots + (U_n - \lambda p_n) dx_n = 0$$

$$(7) \quad d^2 U^* = (dx) [U_{ij}] (dx)' < 0, \text{ assuming independent variations}$$

among the  $x$ 's; or

$$(7a) \quad d^2 U^* = d(xy) \begin{bmatrix} U_{ij} - p' & \\ -p & 0 \end{bmatrix} d \begin{bmatrix} x' \\ y \end{bmatrix} < 0,$$

taking the restraint (2) on the  $x$ 's into account.

From the first order conditions (6), we have the important result,

$$(8) \quad \frac{\partial U}{\partial x_1} / p_1 = \dots = \frac{\partial U}{\partial x_n} / p_n = \lambda.$$

Thus each good is purchased up to the point where its marginal utility is proportional to its price. Also if we select a small monetary unit such as a shilling or a ten cent piece, and consider the quantity of each type of good which can be purchased with one of these units we obtain the vector  $(\frac{1}{p_1}, \frac{1}{p_2}, \dots, \frac{1}{p_n})$ .

Equations (8) tell us that the marginal utility derived from the last shilling's or ten cent's worth of goods purchased in one line is equal to the last unit's worth in any other line. The ratios in (8) are clearly then the marginal utility of money for a consumer at maximum utility in equilibrium. Hence the Lagrange multiplier  $\lambda$  becomes equal to the marginal utility of money. Equations (8) suggest that the consumer portions out his money income according to the same princ-

iples with which he allocates his total resources among his total goals in life (Chapter 3).

Equilibrium equations (8) and the budget identity (2) provide us with  $n+1$  equations in  $x_1, \dots, x_n, \lambda$ . We are assuming the preference system  $U$  to remain unchanged, and we then solve the  $n+1$  equations for  $x_1, \dots, x_n, \lambda$  in terms of  $p_1, \dots, p_n, y$  and  $U$ .

$$(9) \begin{cases} x_1 = d^1(p_1, \dots, p_n, y) \\ \vdots \\ x_n = d^n(p_1, \dots, p_n, y) \\ \lambda = d^{n+1}(p_1, \dots, p_n, y) \end{cases}$$

The form and parameters of the  $U$  function are embedded in the  $d$  functions of (9). System (9) are the demand functions for which we are searching. They reveal the demand for individual consumer goods in terms of individual prices and income acting as causal factors. But they do not tell us qualitatively how demands will vary as prices and income vary, and this we would like to derive both as "meaningful hypotheses" (Samuelson [2.27]) and as "autonomous relations" (Haavelmo [1.4]) of consumer behaviour.

To move the analysis in this direction we assume small and independent changes in  $p$  and  $y$  in (2) and (8), permit the system to move to a new equilibrium, and then attempt to deduce how small changes in the  $x$  and  $\lambda$  arise as the effects of small changes in the  $p$  and  $y$ . We begin by taking the total differential of each equation in (8), and of (2). This gives us

$$(10) \begin{bmatrix} dx_1 \\ \vdots \\ dx_n \\ d\lambda \end{bmatrix} = \begin{bmatrix} U_{ij} & p \\ -p & 0 \end{bmatrix}^{-1} \begin{bmatrix} \lambda dp_1 \\ \vdots \\ \lambda dp_n \\ -dy + (dp)(x) \end{bmatrix}$$

From (10) it is possible to derive all of the desired rates of change of each  $x$  with respect to each causal variable  $p$  and  $y$ . Thus (10) permits the Jacobian matrix

$$(11) \left[ \frac{\partial(x_1, x_2, \dots, x_n, \lambda)}{\partial(p_1, p_2, \dots, p_n, y)} \right] \text{ to be computed.}$$

Note how the basic preference system  $U$  is embedded in the system(10) and hence in (11), albeit in rather complex form. Our problem now is to determine the qualitative nature of (11). Can we deduce how each demand  $x$ , and  $\lambda$ , will vary with changes in individual  $p$ 's, and  $y$ ?

From his pioneering analysis in this field Slutsky

[2.28] made the important observation that

$$(12) \frac{\partial x_i}{\partial p_j} + x_j \frac{\partial x_i}{\partial y} = \frac{\partial x_j}{\partial p_i} + x_i \frac{\partial x_j}{\partial y} \quad . \text{ This follows from}$$

the symmetry of the matrix in (10). For let

$$(10a) \begin{bmatrix} U_{ij} & -p \\ -p & 0 \end{bmatrix}^{-1} = R = [r_{st}], \text{ a symmetric } (n+1) \times (n+1) \text{ matrix.}$$

Then from (10) and (10a) we have

$$(10b) \frac{\partial x_i}{\partial p_j} = r_{ij} \lambda + r_{i,n+1} x_j, \text{ and}$$

$$(10c) \frac{\partial x_i}{\partial y} = -r_{i,n+1} \quad . \quad \text{Thus}$$

$$(10d) \frac{\partial x_i}{\partial p_j} + x_j \frac{\partial x_i}{\partial y} = r_{ij} \lambda = s_{ij},$$

with  $[s_{ij}] = S$ , an  $n \times n$  symmetric matrix. This result verifies (12).

But what do these expressions mean?

Assume an increase in  $p_j$  of  $dp_j$ . Then

(13)  $\frac{\partial x_i}{\partial p_j} dp_j$  is the total change in purchases of  $x_i$  induced

by  $dp_j$ .  $x_j dp_j$  on the other hand represents the loss of income arising from  $dp_j$  and hence

(14)  $x_j dp_j \frac{\partial x_i}{\partial y}$  represents an income effect on  $x_i$  as a result of

$dp_j$ . (13) will usually be a positive effect and (14) a negative one, for positive  $dp_j$ . But in (12) these two expressions are added. Hence,

(12a)  $\left[ \frac{\partial x_i}{\partial p_j} + x_j \frac{\partial x_i}{\partial y} \right] dp_j = s_{ij} dp_j$  is the amount of sub-

stitution in the market basket in favour of  $i$  as a result of the price increase  $dp_j$  and such that the income effect on  $x_i$  of  $dp_j$  is compensated for. Thus (12a) is the general substitution effect on demands for any compensated price changes; (14) is the income effect which would require compensation in order for (12a) to hold; and (13) is the combined income and substitution effect on demand when there is no compensation. These effects are all represented on an indifference curve diagram in Fig. 1.

AB represents the initial spending on  $i$  and  $j$  with  $p_{i1} / p_{j1} = OB / OA$ . The equilibrium position is 1. Now let the price of  $j$  increase relative to the price of  $i$ . We then have  $p_{i3} / p_{j3} = OC / OA$ . The consumer has reduced purchases of  $j$  and increased purchases of  $i$ , arriving at a new equilibrium position 3. The distance



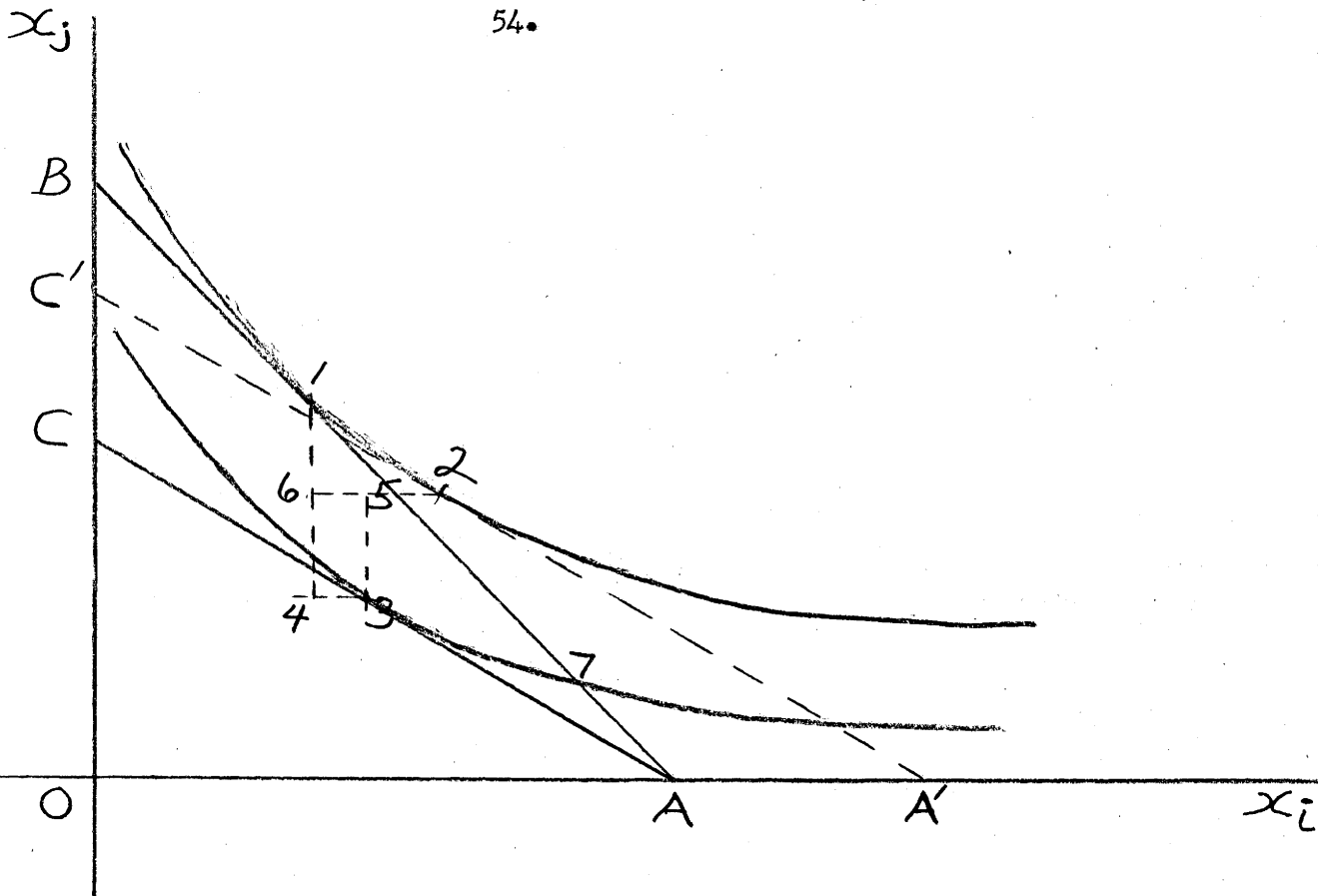


Fig. 1.

43 on the diagram represents the total effect on  $x_1$  and hence represents (13) above. But if the consumer could have been compensated for the income effect of  $dp_j$  he could have stayed on the same indifference curve and reached equilibrium position 2. Then 62 on the diagram shows the compensated substitution effect and hence represents (12a) above. This makes 25 on the diagram the negative income effect, corresponding to (14) above.

The total effect on demand  $x_1$  of a price increase  $dp_j$  is then the sum of these effects and is given by (12a) - (14) resulting in (13), or 43.

The analysis can be carried further using the indifference curve concepts, and taking differentials of (9) and (2), with all prices and income varying.

$$(15) \quad dx_i = \frac{\partial x_i}{\partial p_1} dp_1 + \dots + \frac{\partial x_i}{\partial p_n} dp_n + \frac{\partial x_i}{\partial y} dy$$

From (2) ,

$$(16) \quad dy = p_1 dx_1 + \dots + p_n dx_n + (dp_1)x_1 + \dots + (dp_n)x_n = pdx' + (dp)x'$$

In Fig. 1  $\sum p_1 x_1 < \sum p_1 x_2$  since position 1 represents minimum cost of a given level of satisfaction. Also  $\sum p_1 x_3 \leq \sum p_1 x_1$ . For 3 is below the income line AB. But 3 could be close to 7 which is on the income line AB. And  $\sum p_2 x_1 > \sum p_2 x_3$ , for 3 is on a lower income line than one passing through 1 and parallel to AC. This gives us (Samuelson [2.27]),

$$(17) \quad \sum p \Delta x \leq 0 \text{ and } \sum (p + \Delta p) \Delta x < 0. \text{ But 3 is not far from 7 (Fig. 1) where } \sum p \Delta x = 0. \text{ (See also (16) with } dy = 0, \text{ and } dp = 0.)$$

Hence in the limit,

$$(18) \quad \sum dp dx < 0.$$

Returning to (15) and (16) , and taking  $\sum pdx$  as = 0 in the limit, we obtain

$$(19) \quad dx_i = \left( \frac{\partial x_i}{\partial p} + x \cdot \frac{\partial x_i}{\partial y} \right) dp'$$

$$(20) \quad dx' = \left[ \frac{\partial x'}{\partial p} + \left( \frac{\partial x}{\partial y} \right)' x \right] dp' . \text{ Hence from (18),}$$

$$(21) \quad dp dx' = dp \left[ \frac{\partial x'}{\partial p} + \left( \frac{\partial x}{\partial y} \right)' x \right] dp' = dp [S] dp' < 0.$$

This result also follows from the second order maximum conditions (7a). For if the quadratic form in (7a) is negative definite, then a quadratic form with matrix the inverse of that in (7a) will also be negative definite, and a quadratic form with S as its matrix will be negative

semi-definite. (Samuelson [2.27] pp. 114, 375 - 9).

With the quadratic form in (21) negative definite, the principal minors of the matrix of the form, beginning with the first, alternate in sign from negative to positive. Hence if goods  $i$  and  $j$  are put in first and second positions we obtain

$$(22) \quad \frac{\partial x_i}{\partial p_i} + x_i \frac{\partial x_i}{\partial y} = s_{ii} < 0 ;$$

$$(23) \quad s_{ii}s_{jj} - (s_{ij})(s_{ji}) > 0 .$$

What general conclusions about consumer demand behaviour can we draw from all of the above analysis? We now attempt to pull these together.

(a) The substitution effect on demand for a good when its own price changes ( $s_{ii}$ ) will always be negative, and hence of opposite sign to the price change. (From (22)).

(b) The effect of an income change on demand  $x_i$  (see(10c) ) cannot be stated a priori. The indifference analysis (Fig. 1) suggests that it will usually be positive. The identity (2) tells us that at least some  $x_i$  must increase as  $y$  increases. But for some goods the consumer may already have all he wants, giving  $\frac{\partial x_k}{\partial y} = 0$  ; while others may

be inferior goods for him, so that  $\frac{\partial x_i}{\partial y} < 0$ . Thus  $\frac{\partial x_i}{\partial y} > 0$

usually, but may be equal to or less than 0 in some cases.

(c) The income effect of a price change  $-x_j \frac{\partial x_i}{\partial y}$  , or  $-x_i \frac{\partial x_j}{\partial y}$  ,

will usually be negative, but may be positive for an inferior good.

(d) The total effect on demand for a good when its price changes

$\left( \frac{\partial x_i}{\partial P_i} \right)$ , combining substitution and income effects, will usually be negative. The exceptions will be in the cases of inferior goods, where the income effect will be positive and may outweigh the substitution effect.

(e) The sign of the compensated substitution effect  $s_{ij}$  cannot be stated a priori. When it is positive we define the goods  $i$  and  $j$  to be substitute or competitive; when it is negative we define them to be complementary. When the goods are substitutes, the positive substitution effects  $s_{ij} = s_{ji}$  are less than the geometric mean of the direct and negative substitution effects  $s_{ii}$  and  $s_{jj}$ . (From (23)).

This seems reasonable since there may be several alternative substitute goods for the good whose price is changing.

(f) In the case of substitute goods  $i$  and  $j$  a price rise in  $j$  will produce a positive substitution effect and a negative income effect on  $i$ . Which one of these effects will outweigh the other depends partly upon the importance of  $j$  in the consumer's budget. If  $j$  is only a small part of the budget the substitution effect will probably be greater than the income effect, and conversely.

(g) If the two goods  $i$  and  $j$  are complementary and  $p_j$  increases, both substitution effects  $s_{jj}$  and  $s_{ij}$  will be negative. The income effect for both will usually be negative as well. Hence if the price of either one of a pair of complementary goods increases the quantity purchased of both will almost always decline.

These conclusions are by and large subject to confirmation by general observation and detailed budget studies. They do seem to be generally verified ( e.g. [2.23] ) and our underlying preference system hypothesis and axiom of maximization are not in general refuted, to the extent that original individualistic hypotheses are not refuted. We can accordingly now feel free to go ahead with aggregation of the micro-behaviour to an observable and hence testable macro-theory of consumer behaviour.

#### 4. An Aggregative Consumer Demand Equation under Static Assumptions.

What kind of aggregative consumer demand equation, which could be tested with aggregative economic statistics, can be derived from the above micro-theory? We begin by assuming that although different consumers have different preference patterns they will almost all follow the behaviour pattern suggested by the above theory. We also assume that the common culture of the many households in a given domestic economy will produce a reasonably large proportion of roughly similar preference patterns. Then if the distribution of income relative to preference patterns remains constant, we can draw some rough aggregative conclusions. Let  $\sum x_i$  over the  $N_h$  households of the economy be represented by  $X_1$ . Let  $y_{hkm}$  and  $Y_{hm} = \sum_{k=1}^k y_{nkm}$  ( $k=1, \dots, N_h$ ) be micro and macro disposable money income of households respectively. Let  $Y_h$  be the aggregate real income of all households in the economy. ( Cf. Note (1) Appendix B ). Since all household income is either spent on current consumer goods, or saved, we have

$$(24) \quad Y_{hm} = p_1 X_1 + \dots + p_n X_n, \quad \text{where } X_n \text{ represents aggregate savings}$$

in real terms. ( We shall discuss the interpretation of  $p_n$  and  $X_n$  more fully, later). Thus  $X$  is the vector of demand for all goods, including savings goods, while  $p$  represents their corresponding market prices.

To bring (24) into real terms the effect of a changing level as distinct from pattern of prices must be removed from both sides. To discuss the theory of how to do this would lead us into the whole field of index numbers and deflation. This area of research is most important to the present work, but it cannot be discussed here purely on grounds of lack of space. In this study we must take all index number and price deflation theory as given.

Deflating (24) to filter out price level movements and keep the variables at the constant price level of a base period essentially involves the reweighting of the weight or valuation vector  $p$  in the vector product  $pX$ . This is done with a price index  $P_C$ . With it we arrive at the real value flow of household income during time period  $t$ , given by

$$(24a) \quad Y_{ht} = \frac{p_{1t}X_{1t} + \dots + p_{n-1,t}X_{n-1,t} + X_{nt}}{P_{Ct}}$$

In most of the theory which follows time period subscripts  $0, 1, \dots, t-1, t, t+1, \dots$  will be omitted, but they are almost always understood to be present in concept. Equation (24a) can be further aggregated to

$$(24b) \quad Y_{ht} = C_t + S_{ht}; \text{ where } C_t \text{ is the aggregate real flow into households during time period } t, \text{ of all consumer goods valued in fixed base period valuation units (e.g. constant dollars or pounds); and } S_{ht} = X_{nt} \text{ is household real income saved for future use during period } t.$$

The basic choice involved in the fully aggregative consumer demand equation is the choice between consumption spending and saving. In the micro-theory above we saw how price effects and income effects from price changes altered the pattern of allocation of total income among available goods and savings, through income and substitution effects. We also reasoned that pure income effects apart from relative price changes would increase spending on most goods, except for inferior goods. Hence our first deduction about aggregate consumer spending is that an increase in both money and real income will cause money and real consumption and savings to increase. Our next and much more difficult deduction must be concerned with broad aggregate income and substitution effects as between consumption and savings goods as the relative price levels of these two aggregates change.

What do we mean by "savings", and what is its 'price level'? Saving is done basically to provide for future consumption. This future consumption is an important element in the overall goal system and preference pattern of the individual. It includes plans for the future purchase of some durable good such as a house or car, for the education of children, for future sickness and accident, and for old age. As a first approximation we can say that income can be saved by holding it in the form of money, by purchasing securities, or by buying residential real estate. A more complete definition will be developed later.

Savings flows gradually accumulate from time period to time period into a stock of household wealth,  $W_{ht}$  (as measured

at end of time period  $t$ ). It is out of this stock that the goals for which saving is undertaken are purchased

But what 'price level' concept can be attached to savings goods or wealth. If the price of an ordinary good increases a fixed amount of money will buy less of it. If the price level of goods in the economy goes up, the value or purchasing power of money in general goes down. Such price level movements affect our appraisal of the value of money saved equally<sup>1</sup> with the value of money for current use, assuming for the present no dynamic upward trend in price expectations. Under this assumption a rise in  $P_C$  will not usually cause a substitution effect away from  $C$  and in favour of  $S_h$ , unless the goods for which  $S_h$  is planned do not rise in price as much as  $P_C$ . But apart from such a case  $P_C$  changes can only exert income effects on both  $C$  and  $S_h$ .

There is however a 'wealth effect' involved which does not show up in the micro-theory. The micro-theory could be broadened to include this, but here we shall make the adjustment by less rigorous methods. The real or purchasing power value of all money and securities (apart from variation in security prices) is reduced by the increase in  $P_C$ , presumably below their relative position in the desired real wealth pattern in the individual's total preference system. The individual may attempt to redress this balance by a substitution away from

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1. I am indebted to Professor T.W. Swan for a very helpful discussion around this point, and for suggestions which lead to the development of formula (27).



current consumer goods in favour of real liquidity, and hence savings. (Cf. Patinkin [7.16]). To the extent that there is "money illusion" in the economy, however, such a shift would not occur.

Paralleling  $P_C$  changes in the choice between  $C$  and  $S_h$ , what can happen in the realm of savings goods to influence substitution between  $C$  and  $S_h$ ? Assume for the moment no price changes anywhere in the economic system, but that the yield on savings goods (based on interest, dividends, rents, profits, but defined more precisely later, for example in Parts III and VII) goes up. This means that the amount of future goods purchasable by a unit of money today has gone up. By analogy we can say then that the 'price' of savings goods has declined relative to the price level of consumption goods. This will tend to produce a substitution effect in favour of saving, and a positive income effect increasing both current consumption and saving. Let  $i_h$  represent the average or aggregate yield on the savings goods held by households. Then  $1/i_h$  tends to represent a price variable to accompany  $X_h = S_h$  in our demand system.

So far we have assumed that  $P_C$  remained constant while we discussed the influence of yield on saving. But if  $P_C$  should rise by the same proportion as  $i_h$  rises, the real yield of future goods from a dollar or pound saved today is unchanged. Hence it is something like the ratio  $P_C/i_h$  which is the 'price' of savings goods. Now an increase in  $P_C$ ,  $i_h$  constant, will reduce both  $C$  and  $S_h$  by an income effect. This loss in real income will likely cause a shift away from saving (a decline in the saving ratio), for saving tends to be a luxury good which is postponable.

Our static and preliminary aggregate demand equation for consumer goods can now be represented by

(25)  $C^d = f_1 \left\{ \frac{Y_{hm}}{\Phi(P_C, i_h)}, \frac{i_h}{P_C} \right\}$  . The first term shows the effect of changes in income earned, and the income effects of changes in  $P_C$  and  $i_h$ . The second term shows substitution effects as between the prices of consumer goods and the 'price' or yield on savings goods. (The superscript "d" will often be used to designate a demand function, while superscript "s" will be used for supply functions.)

It may be found in practice that  $C^d$  is relatively inelastic with respect to variation in  $i_h$ . However, our theory suggests that for sufficiently large variation of  $i_h$ ,  $C^d$  will be influenced. The reason that both  $C^d$  and  $S_h^d$  are likely to be inelastic with respect to  $i_h$  at low values of  $i_h$  is that  $S_h^d$  would occur in substantial volume even if  $i_h$  were zero. Saving depends mainly on the high value placed on future consumption needs by modern societies. The preference pattern makes the basic division between C and  $S_h$ , and we merely suggest that at this margin of separation some influence can be played by  $i_h$  on the ratio of separation, for sufficiently large values of  $i_h$ .

Since (25) shows the choice between C and  $S_h$  a separate equation for  $S_h^d$  would seem to be redundant. For  $Y_h$  is here taken as given, and then (24b) is sufficient to determine  $S_h$ , along with (25). This is the procedure to be followed in the present model, with  $S_h^d$  not a residual demand, since  $C^d$  is a choice between C and  $S_h$ . An alternative procedure would be to set up three equations in the two unknowns, and solve these with one degree of freedom by a

method similar to that used for least squares parameter solutions.

We must now consider more deeply the meaning to be attached to  $S_h$  in (24b), where all variables are in real terms.  $S_{hm}$  represents present money income foregone currently, and its opportunity cost is  $\frac{S_{hm}}{P_C}$ . But in addition to providing this real value at some time in the future it can also provide earnings in the interim periods. These earnings bring the savings up to a value  $(S_{hm}/P_C)(1 + i_h)^n$  by the time they are needed for future consumption, where  $n$  is some average number of time periods for which savings are held. Another way of looking at this problem, combining the concepts of both the future purchasing power of the savings and of its yield, is to ask the question: How much money was required to be invested in the period before the base period to accumulate to one constant pound or dollar of value in the base period; and how does this compare with the current period? Let these amounts be  $P_{Sho}$  and  $P_{Sht}$ . The index of these 'prices' will be  $P_{Sh} = P_{Shot} = P_{Sht}/P_{Sho}$ .

$$(26) \quad P_{Sho} (1 + i_{ho}) = 1 \text{ base period purchasing power or value unit of one pound or dollar.}$$

$$(26a) \quad P_{Sht}(1 + i_{ht}) / P_{Cot} = 1 \text{ base period value unit.}$$

$$(27) \quad P_{Shot} = P_{Cot} \left( \frac{1 + i_{ho}}{1 + i_{ht}} \right) = P_{Cot} / P_{ihot}$$

We can now define  $S_h$  rather precisely in meaning as,

$$(28) \quad S_h = S_{hm} / P_{Sh} = S_{hm} \times \frac{P_{ih}}{P_C} .$$

We can now place a more precise meaning on the deflation of  $Y_{hm}$  (money value) to  $Y_h$  (real purchasing power value). The

appropriate deflating index  $P_h$  is derived as follows:

$$(29) \quad Y_{hm} = C_m + S_{hm} \quad \text{and}$$

$$(29a) \quad Y_{hm} / P_h = C_m / P_C + S_{hm} / P_{sh}, \text{ giving}$$

$$(30) \quad P_h = \frac{P_C (C_m + S_{hm})}{(C_m + S_{hm} P_{ih})} .$$

It is now nearly possible, with a little alteration to (25), to complete our consumer demand equation. But there is one respect in which this equation violates one of our rules or precepts for aggregation. Total income  $Y_h$  ranges through such a wide range of income sizes and sources that we cannot construe a close similarity in the parameters associated with  $y_h$  in the micro-equations. This means that changes in the income distribution could cause considerable aggregation error in an equation like (25). A partial solution to this problem is to separate  $Y_h$  into functional income shares, following the footsteps of Dr. L.R. Klein [1.11], [9.6]. (See also Brown [2.6]).

Let  $Y_{wm}$  = disposable wage-salary income,

$Y_{\pi nam}$  = disposable nonwage or property and enterprise income in the non-agricultural sector,

$Y_{\pi am}$  = disposable nonwage income in the farm sector. Then the

static consumer demand equation suggested by our analysis, and expressed this time as a linear function, is given by

$$(31) \quad C^d = a_0 + a_1 \frac{Y_{wm}}{P_{hw}} + a_2 \frac{Y_{\pi nam}}{P_h \pi na} + a_3 \frac{Y_{\pi am}}{P_h \pi a} + a_4 \frac{P_{sh}}{P_C} .$$

This equation allows for basic income changes in the economy, real income effects arising from price changes, and a substitution effect between consuming and saving, induced by relative 'price' changes.

It also eliminates aggregation error arising from shifts in the functional income distribution, which can be rather large over the phases of the business cycle.

Chapter 8.AGGREGATE CONSUMER DEMAND UNDER DYNAMIC INFLUENCES.

It has been found that an aggregate consumer demand equation like (31) in Chapter 7 does not fit the observed data particularly well. This turns out to be mainly because it is essentially timeless or static. In one particular, it assumes implicitly that as the environment of prices and incomes changes, households shift 'immediately', or within one time period, from the old to a new equilibrium position. Further it ignores the fact that households accumulate stocks of goods, which are the accumulated results of past behaviour, and have a significant influence on present and future demand. Finally, it makes no allowance for expectations whereby the developments of the past and present are projected into the future, and then react back to influence the present. But these time aspects of consumer behaviour, where the dating of certain variables may be important, do come into much of human behaviour, and are of particular relevance when we are studying behaviour in the short-run. Long-range behaviour on the other hand may come much closer to the static equation of Chapter 7.

In the present chapter we shall attempt to complete the analysis of consumer demand by adding the dynamic influences of reaction times, stocks of wealth and durable goods, and expectations to the theory. Before we commence the dynamic study however a classification of consumer demands on the basis of degree of need may be

found useful. This is germane to the general study of consumer behaviour, but seems to be especially relevant in dynamic theory.

### 1. Classification of Consumer Demand By Degree of Need.

(a) Basic Needs. These are required for survival. They involve sufficient food and water, and sufficient protection of body warmth from the elements, to sustain biological life. They are met by certain levels of food, clothing and shelter, and must be satisfied by even the most primitive society. The demand for basic needs will be inelastic with respect to both price and income. The level of these needs will change but little through time.

(b) Culture Needs. These have gradually become a part of the social culture as it evolves. The culture tends to impel the individual to use the particular goods involved, by law or by social pressure. They are complementary to certain standards of cleanliness, dress, housing, schooling of children, and so on. Failure to consume these goods tends to cause the individual psychic, as distinct from physiological, pain.

The demand for culture needs will be quite inelastic with respect to both price and income, but levels of these will go down somewhat during depressed times. The drop in real incomes forces a decline, social pressure is relaxed because of this, and there are sufficient other households involved that an individual household finds itself in the company of others as it drops its standards.

The level of culture needs has undergone a remarkable upward trend as the economic productivity of civilization has grown

through historical time.

(c) Culture Wants. Our increasing productivity and facility at producing goods has enabled us to develop a third source of final demand which we may classify as wants. Wants may be defined as those goods which we do not need ("need" in the sense defined above), but for which we are willing to expend our energy and resources. These wants are very much influenced by the cultural climate and milieu, by the pattern and level of values reached by the culture and its social purpose and value building institutions. They are also under continuous stimulation by the advertising industry. Some of the influences are 'Veblenesque' [2.29], and the goods are desired so that their "conspicuous consumption" can demonstrate the 'superiority' of the possessor. Other goods in the category of the wants have a higher motivation, in that they make life truly more abundant. They add comfort or health or beauty to life, or save time and energy from necessary household tasks. This saving can then be transferred to other activities such as sports, hobbies, social activities, education, the arts, social service, and so on.

Many of the consumer durable goods of modern industrial societies could be classed as culture wants, and we might mention the automobile, refrigerator, automatic washing machine, radio, record player, and television set. But some of the semi-durables (clothing and house furnishings), and many of the services (e.g. entertainment, recreation, vacations, travel) also fall into this category. Since they are not needs, the demand for them will be elastic with respect



to income and price. Because of this, and because they contain so many durables, they add to or accelerate any economic instability. Also, like the culture needs, they evidence a very dynamic upward trend. This seems to accompany the western culture's toleration of innovation and encouragement of science and invention. Culture wants thus tend to be continuously expanding, and as this goes on many of them cross the line and become culture needs.

The totality of economic needs and wants in a culture constitutes the "standard of living" to which it aspires and is willing to give exertion (cf. Davis [2.8]). This concept will be found useful in our final breakdown of consumer spending into  $C_s$  (services),  $C_p$  (perishables),  $C_{sd}$  (semi-durables),  $C_{hd}$  (household durables),  $C_a$  (automobiles), and  $C_{d+1}$  (dwellings plus land). Some of these fall more into the needs class, others more into the wants. Otherwise this classification is based on durability, and statistical availability, and represents a workable compromise.

Our method in the present chapter will be to study the various dynamic influences on consumer demand, without attempting to assemble specific equations until the very end. We begin with an investigation of the influence of time duration and time path between consumer equilibria.

## 2. Habit Persistence in Consumer Behaviour.

In the analysis of consumer reactions to changes in prices and income by the method of comparative statics, it was assumed

either that consumers reacted instantaneously to such changes, or that whatever time it might take for consumers to move from one equilibrium position to the next was irrelevant to the static problem. But if in fact it does require time for consumers to adapt to the new circumstances so that they move only slowly from one equilibrium position to another, the aggregate demand equation suggested by the static or 'timeless' methods would not fit the observed data, at least where the observations are of consecutive short time periods. Such an equation would be unsuitable for practical work in forecasting and policy in the real world. Hence it is of great importance to know whether or not consumers do require an appreciable length of time to adapt to price and income changes in the environment.

There is considerable opinion and some evidence that human beings form consumption habits which are sufficiently strong to slow down their reactions to changes in the economic environment. Lord Keynes suggested in the "General Theory" [1.6 p. 97] that such habits would exist and would retard household reactions in the short-run. Dr. Ruth P. Mack [2.17] p. 42 concluded that much consumer buying is strongly habitual and that the "organism seeks relief from the strain of making subtle decisions". Dr. Ruby T. Norris [2.20] emphasized that the consumer is conditioned by the culture, is a creature of habits, and requires time to break old habits and adopt new ones. The present writer in an econometric study of aggregate consumer demand [2.6] found a definite tendency for changes in real consumer spending to lag behind changes in real income. The empirical work of Drs.

Franco Modigliani [2.19] and J.S. Duesenberry [2.9] similarly reveals a lag of consumer spending behind income changes.

There is thus sufficient evidence to suggest a theory that the act of consuming a particular pattern and level of goods forms a psycho-physiological habit pattern within the individual. These habits then display a certain hysteresis (like magnetism in iron) or persistence, such that they are strong when the stimulus has just occurred, but gradually die away as time passes.

It is clear then that when we are working with time periods of say a year or less we must adapt the static theory to take account of this dynamic feature of human behaviour. How can this be done? Suppose for example that we are working with the static equation

$$(1) \quad C = a_0 + a_1 \frac{Y_m}{P} + a_2 \frac{P_C}{P_S} .$$

Suppose that the consumer sector is in equilibrium at time 0 with values  $C_0$ ,  $Y_{m0}$ ,  $P_0$ ,  $P_{C0}$  and  $P_{S0}$ , and then is disturbed by a change in both money income and prices to new values of  $Y_{m1}$ ,  $P_1$ ,  $P_{C1}$  and  $P_{S1}$ . Assume that the new stimuli last from period 0 to n and beyond, but that the consumer sector reaches a 'static' equilibrium again in period n. The new static equilibrium will be given by

$$(2) \quad C_n = a_0 + a_1 \frac{Y_{m1}}{P_1} + a_2 \frac{P_{C1}}{P_{S1}} .$$

But if consumer spending in the short-run is continuously influenced by previous consumption habits, then the time path of consumer demand from 0 to n must contain  $C_{t-1}$  as a causal variable. If this is the only further influence on C, the addition of this variable to the static equation produces a dynamic relation, which will describe the time-path of C in moving from

$C_0$  to  $C_n$ .

To study this time path suppose we simplify the static equation to

(2a)  $C = a_0 + a_1 Y$ . The dynamic equation will then be

(3)  $C_t = b_0 + b_1 Y_1 + b_2 C_{t-1}$ . Here  $b_1$  and  $b_2$  will be  $< 1$  and  $b_1 < a_1$  (cf. [2.6]). The values reached by  $C$  in successive time periods are given by,

$$(4) C_0 = a_0 + a_1 Y_0$$

$$C_1 = b_0 + b_1 Y_1 + b_2 C_0$$

$$C_2 = b_0 + b_1 Y_1 + b_2 (b_0 + b_1 Y_1 + b_2 C_0)$$

$$= b_0 (1 + b_2) + b_1 (1 + b_2) Y_1 + b_2^2 C_0$$

$$\vdots$$

$$C_t = b_0 (1 + b_2 + \dots + b_2^{t-1}) + b_1 (1 + b_2 + \dots + b_2^{t-1}) Y_1 + b_2^t C_0$$

$$\vdots$$

Consider now the variation in  $C$  and the influences on it as  $t$  varies from time period 0 to  $n$ .

When  $t$  is still near 0  $C_0$  will exert a strong influence on  $C_t$ , while  $Y_1$  will have much less influence. Thus  $C_t$  will still be close to  $C_0$ . But as  $t$  becomes large and comes closer to  $n$  the influence of  $C_0$  wanes and finally disappears, while the influence of  $Y_1$  waxes and becomes dominant. The number  $n$  must be large before the influence of  $C_0$  can vanish, which tells us that the theory of habit persistence or hysteresis is especially relevant when the time periods are short - that is in short-run analysis.

When time period  $n$  is reached  $C_t$  will have climbed up

to its 'static' equilibrium value

$$(5) \quad C = \frac{b_0}{1 - b_2} + \left( \frac{b_1}{1 - b_2} \right) Y_1, \text{ which is a stationary}$$

equilibrium in dynamic terms. This equilibrium is stable since  $n$  is large. Hence it will continue at the value  $C_n$  as long as  $Y$  is undisturbed from its value  $Y_1$ , and the behaviour structure is not changed.

The relationship between the 'static' or long-run equation (2a) and the dynamical short-run equation (3) is found by comparing (5) and (2a). We obtain

$$(6) \quad a_0 = b_0 / (1 - b_2) ; a_1 = b_1 / (1 - b_2).$$

Thus  $a_0 > b_0$  and  $a_1 > b_1$ . It follows that both the marginal propensity to consume from real income and its related multiplier are small in the short run (Brown [2.6]) but grow with time, reaching much higher asymptotic values in the long-run.

How shall we incorporate the habit persistence theory and the dynamic analysis into our model of the economy? Since our main concern is short-run prediction and policy problems we can simply use an equation of form (3) to take us from one time period to the next. For any long-run problems equations like (2) and (2a) will serve, while for intermediate cases (4), or difference equation solutions, can be used.

### 3. Special Factors Related to Consumer Demands for Durable Goods.

#### (a) The Dynamic Influence of Stocks in the Demands for Durable Goods.

Many of the durable goods fall into the category of culture wants. The desire for them will be very strong, but the demand for them will be fairly elastic relative to real income. This feature, combined with their bulk and the volume of employment and materials that go into them, causes them to accelerate or magnify demand and income changes which may have originated in other parts of the economy.

Another characteristic of durable goods which also makes the demand for them undergo wide variation is their durability. Perishable goods and most services are used up rather soon after purchase, and the demand for them is accordingly often recurrent at short intervals. But once a household has made its purchase say of an automobile, its demand for a new automobile tends to be over for the duration of the useful life of this durable. The desire for the automobile still continues to be strong but it is now satisfied by the existing car, and no new market demand is expressed. In this way consumer stocks of durables (and of semi-durables) exert a negative influence on current demand.

This means that the current market demand for a particular type of durable must be expressed by those households who aspire to own it but who have not as yet purchased it. A family will aspire to a particular durable if it feels that its income prospects make the aspiration reasonable likely to be achieved, and if the durable is a part of the "standard of living" which it urgently desires and is willing to strive for (Cf. J.S. Davis [2.8]). Let  $A_a$  represent the aspirational level of automobiles desired by the

consumers of a society. (Cf. C.F. Roos and V.S. von Szeliski [2.25], [2.26]).  $A_a$  will be based on the number of income earners, the income distribution, the general outlook of consumers regarding their future earnings and employment possibilities, and finally the aspirations and standards which the total culture has fostered. There will tend to be a relationship between  $Y_h$  and  $A_a$ , but for the present study we shall treat  $A_a$  as exogenous, determined largely by demographic and sociological forces, but capable of estimation with the help of sample survey methods.

Standing opposite this aspirational level of ownership of automobiles is the actual existing real stock of cars owned by households. Let it be designated  $K_a$ .  $K_a$  is explained by the dynamic equation

(7)  $K_{at} = K_{at-1} + C_{at} - D_{at}$ , where  $C_{at}$  represents consumer purchases of new automobiles during period  $t$ , and  $D_{at}$  is the decline in the real value of the consumer stock of cars during period  $t$  due to normal wearing and deterioration, obsolescence, and destruction by accident or by scrapping.  $D_{at}$  is referred to as 'depreciation' for short.  $K_{at}$  is the resultant of all past purchases and depreciation of cars, at least as far back as the life of the longest lived car.

How do these stock variables affect the demand for automobiles? Clearly if  $A_a$  and  $K_a$  were equal both at micro and macro levels, the demand for cars would be zero, and the market could be described as saturated. It is then the shortfall of  $K_a$  below  $A_a$  that is both a necessary condition for any demand to exist, and that is a

causal influence on the current demand. Thus  $(A_a - K_a)$  becomes an important causal variable in the demand equation for cars, and it is in this more precise sense that the existing stocks of a durable exert a negative influence on the demand for it.

(b) Second Hand Markets.

As a durable ages it may fall below the standard or aspirational level of the household. The household will then want to sell this durable and purchase a new or at least newer one. Many aspirational levels are not for a new durable, but only for one of a particular age and quality. Because of this there is a continuous flow of supply and demand for second hand or used durables. Second hand markets, operated both by organized dealers and through the classified advertisements of newspapers, expedite these desired sales and purchases.

But the economic model which we are attempting to construct is essentially concerned with the demand for and supply of newly produced goods. This is because full employment is one of its major policy objectives. Hence our demand equations will all be cast in terms of purchases of newly produced goods. Yet we cannot ignore the second hand markets; for they are an important part of the total economy, and they represent an important source of goods which are competitive with new production for satisfying the demand for durables. Hence each demand equation for newly produced durable goods must treat the corresponding second hand durables as competitive or substitute goods. This requires the addition of new causal variables to the



demand equations. In the case of automobiles the two terms  $\frac{P_{as}}{P_a}$  and  $K_{as}$  are suggested, although one of these terms may be found to be redundant.  $P_{as}$  is a price index of second hand cars, and  $K_{as}$  is the real stock of second hand cars which are on the market for sale.

(c) The Availability and Terms of Consumer Credit.

It is customary to buy all of the smaller items of consumption for cash out of current income, but to borrow money for a bulky item like a house. It would take the average household a good many years to save up the full price of a house, but the family can live in the house for all of those years if it is able to borrow the money and is willing to pay interest on the loan. It all depends on the marginal rate of substitution between a house one year hence and a house today ( the rate of time preference for a house), in comparison with the interest rate. The same principle applies to household durables and automobiles, although here a smaller proportion of people borrow than is the case with housing since the amount of money involved is not so great.

There is however an increasing tendency for consumers to find their time preference for durables greater than interest costs as levels of real incomes have increased and interest rates have declined. Hence consumer credit has become increasingly important in financing these other durables, as well as housing.

The availability of credit for this purpose, relative to the demand for it, is indicated by the effective rate of interest

charged ( $i_{sh}$ ), the proportion ( $d$ ) of the purchase price which suppliers are willing to lend, and the time ( $t$ ) within which the loan must be completely repaid. The overall availability of money to lend in this area, relative to demand can be indicated by an index number of the three terms of borrowing  $i, d, t$ , which can be represented by  $B_s(i, d, t)$  for short-term loans, and  $B_m(i, d, t)$  for long term house mortgage loans. If we let  $D_{eshf}$  represent the short-term debt owed by households to firms (who hold corresponding securities  $S_{eshf}$ ), then the rate of growth of this debt is some indication of the ease of credit  $B_s(i, d, t)$ . Similarly for mortgage debt of households (to other households as well as firms) the rate of growth of  $D_{emh} (= S_{emh})$  is indicative of the availability of credit as well as of other economic conditions.

There is some question about the importance of the interest rate, particularly in the case of  $B_s$ . The interest rate is the price for obtaining the use of liquidity for one year. Like any other price it presumably has some influence on demand. But it appears that when the loan is short-term, and when incomes are high, this demand is rather inelastic with respect to the interest rate. This is because the income effect of  $i$  changes is small in this case. But the interest rate carries much more influence in the case of long-term mortgage loans, for now there is a large income effect. This is because the loan is usually for a large amount, and is outstanding for a long time. (Cf. J.R. Hicks [2.12], Chap. XVIII).

(d) Consumer Debt.

These obligations tend to force the households who have incurred them to save more than they might otherwise do as they make the necessary periodic repayments. It follows that either  $D_{eshf}$  and  $D_{emh}$  separately, or as a sum  $D_{eh}$ , may exert a negative influence on some consumer demands. It would be mainly culture wants - some services and some of the durables - which would be so affected. Since however mortgage payments tend to substitute largely for rent payments, probably only  $D_{eshf}$  exerts a true drag on some consumer spending. Whether the drag of existing debt  $D_{eshf}$  will ever be serious with respect to current spending depends on the size to which it has grown relative to income  $Y_h$ . Should the principle and interest payments per time period become large relative to  $Y_h$ , the drag on current demand from this cause could be serious, with serious employment effects.

An additional variable (or degree of freedom) could be saved in the relevant equations by combining  $L_h^1$  with  $D_{eshf}$  to arrive at net liquidity  $NL_h = L_h - D_{eshf}$ . But the micro-holders of large liquid assets will usually be different from those who owe  $S_{eshf}$ , partly because of the heavy interest charges on  $S_{eshf}$ . Hence the separate liquidity and debt variables will probably provide a sharper analysis where degrees of freedom are adequate.

(e) The Rate of Family Formation.

Families have a greater demand for durable goods than do single individuals, since some durables (especially houses and the

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1. The influence of liquid asset holdings  $L_h$  is discussed on page 87.

larger household durables) are usually only used by families. The rate of family formation ( $\Delta N_f$ ) varies cyclically parallel to income, but is still probably a relevant variable in the  $C_{d+1}^d$  (dwellings plus land) and  $C_{hd}^d$  (household durables) equations.

(f) Complementarity.

The chief complementarity in our breakdown of consumer spending is between  $C_{d+1}$  and  $C_{hd}$ , with a lesser complementarity between  $C_{d+1}$  and  $C_{sd}$  (semi-durables). In each case  $C_{d+1}$  is the dominant variable, and will appear as a causal variable at least in the  $C_{hd}^d$  equation.

4. Special Aspects of the Demand for Housing.

Housing combines all the features of durable goods discussed above, but is the most long-lived consumer durable as well as the most bulky in cost. In addition there are three separate markets from which this demand can be satisfied - the new house, existing house, and the house rental markets. Only the demand for new houses affects employment. But the demand for new houses is very much influenced by conditions and prices in the other two competitive or substitute markets. Substitution among these three markets does not depend solely on price, because the preference pattern of the individual usually favours one of the three markets apart from price. But substitution will still take place at the margin as the three relative prices and conditions change.

Let  $P_{d+1}$  represent the price level in the new house market, and  $B_m(i,d,t)$  its terms of credit. Then  $P_{(d+1)s}$  and  $B_{ms}(i,d,t)$

are the corresponding variables in the existing house or second-hand market. We take these as given or 'exogenous' in the present model.

It remains for us to consider the house rental, or more broadly the dwelling unit rental market, including here all apartments as well as houses.  $K_{d+1}$  includes all such dwelling units, whether new or used, and whether owner occupied, for sale or for rent. In the short -run it represents the total supply of housing. But from this total supply we must deduct owner-occupied and those for sale in the new or used markets. This gives us rental supply  $K_{(d+1)r}^S$ . The rental market is not one of competitive equilibrium, but is rather of the type where prices move only slowly toward an equilibrium (See Part VIII). The rent price index will accordingly be explained by a dynamic equation of the form

$$(15) \quad P_r = f \left\{ P_{r,-1}, (K_{(d+1)r}^S - K_{(d+1)r}^d) \right\}.$$

Excess supply in this market is readily observable, as long as it is positive. It is the number of empty houses for rent. Let it be represented by  $E_{dr}$ , and treated in this analysis as given or 'exogenous'.

Our analysis of the demand for new dwelling units omits detailed analysis of the second hand and rental markets to enable us to concentrate on building a model which aims at explaining employment, the price level, and the balance of payments. It does however consider roughly the value of all dwelling services, in conjunction with estimating  $A_{d+1}$  and  $K_{d+1}$ . These are priced at  $P_r$  levels, and are aggregated into  $C_s$ , aggregated into  $C_d$ , as is usually done in national income accounting.

5. The Influence of Household Ownership of Wealth on Consumer Demand.

Up to this point it has been assumed that consumer spending has been made out of current money income only. But this would be the same as to assume that a firm could only make sales out of its current production. Actually a firm can make sales out of its inventories or stocks, and similarly a household can make purchases out of its store of wealth. Both inventories and stores of purchasing power are analagous to 'fat' in the organism.

Household wealth is the accumulated stock of net household savings. It consists of savings in both financial and real forms. Financial wealth consists of the money holdings of households,  $M_h$ , and securities,  $S_{eh}$ <sup>1</sup>. It thus consists of stored purchasing power, promises to pay stored purchasing power and equity in the assets and profits of corporations. Real wealth consists of household stocks of semi-durables  $K_{sd}$ , household durables  $K_{hd}$ , automobiles  $K_a$ , dwellings and associated land  $K_{d+1}$ , and the net worth of unincorporated business  $K_{ub}$ . The total wealth holdings of households is then defined by the identity

$$(8) \quad W_h = \frac{M_h}{P_h} + \frac{S_{eh}}{P_h} + K_{sd} + K_{hd} + K_a + K_{d+1} + K_{ub}.$$

For the economy as a whole financial items cannot be counted as wealth. But for the household sector alone, money and the promises to pay of other sectors (business and government) represent purchasing power, and hence a generalized store of value.

1. As a simplification we here assume insurance companies and pension funds to be a part of the household sector.

An offsetting factor in the household wealth structure is household debt, both short-term ( consumer credit), and long-term (mortgages). The short-term and mortgage debt owned by households to firms is  $D_{eshf}$  and  $D_{emhf}$ . The corresponding securities held as assets by the firms are  $S_{eshf}$  and  $S_{emhf}$ . Short term debt owed by one household to another is excluded from  $S_{eh}$ , and hence need not be included in the debt side. But mortgages held by household or personal mortgagees ( $S_{emhh}$ ) are of more importance, and are a part of  $S_{eh}$ . We can now define the net wealth of households as

$$(9) \quad NW_h = W_h - D_{eshf} - D_{emh}, \quad \text{where } D_{emh} = S_{emhh} + S_{emhf}.$$

How does wealth influence consumer behaviour and household demands for goods? It is accumulated by saving out of income. It is desired partly for the current utility of the durable goods included in it, and partly because it represents a store of future purchasing power. It causes the system to be dynamic, since it is a stock, and is explained by the equation

$$(10) \quad NW_t = NW_{t-1} + S_t. \quad \text{It enables a household to spend more than its income for a number of time periods, and hence permits the marginal propensity to consume to be, for a time, greater than unity.}$$

The micro-analysis of the preceding chapter falls short of reality by omitting wealth holdings, both from the utility function to be maximized, and from the economic resources to be applied to this purpose. A more correct representation would be,

$$(11) \quad U ( X_1 , \dots , X_{n-1}, S_h \mid NW_{h,-1} ) \text{ is to be maximized, subject to}$$

the budget constraint

$$(12) \quad Y_{hm} + NW_{hm,-1} = p_1 X_1 + \dots + p_{n-1} X_{n-1} + P_{Sh} S_h + NW_{hm,-1} \\ = C_m + NW_{hm}.$$

A micro analysis followed by aggregation would now show us that consumer demand for  $(X_1, \dots, X_{n-1}, S_h)$  will be influenced by income effects of both income and price changes (mainly price changes in consumption goods), wealth effects of both wealth and price changes (mainly price changes in net wealth goods), and substitution effects of price changes assumed compensated for income and wealth effects.

Space considerations forbid the development of this analysis in detail, but it would follow the pattern of Chapter 7, but with equations like (10), (11), (12) above at the start of the analysis. The full detailed analysis would show the re-arrangement of holdings of components within  $NW_t$  during  $t$ , on the basis of changes in yields ('prices') among the components, as well as the choices among  $X_1, \dots, X_{n-1}$  based on price and income changes, and finally the important choice between  $C_m$  and  $S_{hm}$  or  $\Delta NW_{hm}$ . When all of the re-allocation of wealth items and the choice of consumer goods is complete, the final identity of household economic behaviour will be of the form

$$(13) \quad Y_h + NW_{h,-1} = C_p + C_s + D_{sd} + D_{hd} + D_a + D_{d+1} + M_{hr} + S_{ehr} + K_{sd} \\ + K_{hd} + K_a + K_{d+1} + K_{ub}.$$

Because wealth is accumulated for future consumption, it follows that some wealth is always being consumed. For this reason as well as from the above resource and utility analysis it appears



that wealth is likely to act as a causal variable on aggregate consumption in a way somewhat parallel to the way income acts. That it influences consumer purchases in this way is confirmed by recent theoretical and empirical work by Professor L.G. Melville [2.18] and Dr. W. Hamburger [2.10], [2.11]. But it has this peculiarity in that it acts as both a consumer good (eqn. 11) and as a source of spending (eqns. (12) and (13)). As a consumer good it will eventually produce diminishing marginal utility as it grows, and hence will ultimately at certain levels induce the household to consume more out of current income. This influence reinforces the current using up of wealth by those households who are using it for the purposes for which it was accumulated -- accident, travel, education, old age, and so on.

While these assumptions may be quite valid for moderate levels of income and wealth, there may be considerable non-linearity in the consumer demand equations when income and wealth become very large. Consumption may then reach satiation (Cf. Dr. Ruth Mack [2.17]), and begin to level off. Then we would find both  $\frac{\partial C}{\partial Y}$  and  $\frac{\partial C}{\partial NW} \rightarrow 0$ . This would cause wealth to accumulate at a faster rate (eqn. (10)), provided that income was maintained by adequate investment. It follows that a shift to greater <sup>in</sup> equality in the wealth and income distributions could be accompanied by negative  $\frac{\partial C}{\partial Y}$  and  $\frac{\partial C}{\partial NW}$ , in some cases.

## 6. The Influence of Liquid Wealth.

One component of wealth holdings may have a more than proportionate influence on current consumer demand. This is the portion of wealth that is liquid, and which hence represents immediately or almost immediately available generalized purchasing power. Any definition of liquidity must make an arbitrary 'cut-off' on the 'liquidity scale', and it is here suggested that we cut off to include only money and central government securities. (Actually the precise dividing line must be found by statistical and econometric research.) We shall temporarily define for this study the liquid asset holdings of households as

$$(14) \quad L_h = M_h + S_{egh}.$$

If  $L_h$  is an especially sensitive wealth variable then it should be included in the demand equations separate and distinct from total wealth. Assuming no "money illusion" it should be deflated in the demand equations by the household price index  $P_h$  to give a causal term  $L_h/P_h$ . We thereby incorporate in the demand equations the "Pigou Effect" [2.22]. It is probable that  $L_h$  will be most important in demand equations involving a combination of culture wants and bulky costs, that is in some of the services, and in the durable goods where current cash income in excess of basic and culture needs may be inadequate for making the bulky purchase.

## 7. Population

Growth of population will tend to have an important expansive influence on consumer demand. But it is believed that population is already adequately represented in the causal variables.

Population changes will affect the labour force and through this  $Y_w$ . Also population and net new family formation must be included in the estimation of the aspirational variables, A.

### 8. Expectations.

Each economic unit must base its plans for the future, and hence many of its current actions, on its forecast of the future. It must in fact forecast and act accordingly in order to survive. Living only in the present might mean short life. Its forecasts of the short-run and the longer run future are its expectations. The forecasts will be based mainly on current endogenous trends, but will include some exogenous (e.g. political, international) trends and foreknowledge.

Where expectations are formed from current economic trends, the variables involved can be brought into the equations through their lagged values. Thus,

(16)  $E(Y) = f(Y_{-2}, Y_{-1}, Y) = f(\vec{Y})$ . But where the expectation is of a particular event, some quantitative estimate of the event must be brought into the appropriate parts of the model.

Economic expectations can affect current demand in ways like the following:

(a) If consumers expect the price of a good to go up between now and  $t$ , many of them will buy now. (Substitution through time to lower priced good).

(b) If prices are declining consumers will tend to wait until it

appears that the trend is reversing before buying.

(c) If consumers expect supplies to cease to exist by some future date they will rush to buy now.

(d) If real incomes are expected to increase in the future they will tend to purchase now on the strength of the present value of this future gain.

(e) If real incomes are expected to decline, this loss will also be discounted as a reduction of future wealth and some purchases will be foregone.

Political and international expectations can influence consumers in ways like the following:

(a) Expectations about future government indirect taxes on goods can cause price expectations about these goods to be up or down, with demand reactions as indicated above.

(b) If some national or international disturbance creates strong feelings of uncertainty and insecurity about the future, households may refrain from the purchase of durables. This may explain the low sales of durables in the United States in 1939-41. The Korean War on the other hand did not create such deep feelings, and there was a rush in 1950, on the expectations of future shortages, to purchase durables.

An expectations variable  $E_h$  should be included in each demand equation having a significant proportion of culture wants in it. This variable can be built from economic and political-international trends. For the latter it is suggested that the values 1, simple

fractions of 1, and 0 be used, and that these values be selected on the basis of a historical chronology which can accompany all economic statistical time series.

#### 8. Summary of Consumer Demand Equations under Dynamic Conditions.

It is now possible to pull together the results of our static and dynamic analysis of consumer behaviour. Since the dynamic analysis shows that the underlying forces behind consumer demand are different for different categories of goods, we need a separate demand equation for each separate category of goods. In particular it will be recalled that behaviour was deduced to vary with the degree of need and want ( basic needs, culture needs, culture wants) and with the 'size' and degree of durability of the goods involved.

Using the shorthand of our symbols we can summarize all of the above theory in a set of equations, one for each consumer good category. These categories are selected on the basis of a compromise between theoretical behaviour differences and availability of statistical data. In each equation the variable on the left hand side is the demand variable being explained, while the variables on the right hand side are the variables which cause or influence the variable being explained.

The meaning of the symbols was given in the text above, but have also been listed alphabetically in Appendix B for the complete model. All of the equations developed for the complete model

are brought forward in a grand summary to Part IX.

The equation summary for consumer or household demands now follows.

(a) Demands for Consumer Goods.

$$(17) C_p^d = f_1 \left\{ \frac{Y_{wm}}{P_h}, \frac{Y \Pi nam}{P_h}, \frac{Y \Pi am}{P_h}, \frac{P_p}{P_h}, C_{p,-1}, \frac{NW_{hm}}{P_h} \right\}$$

$$(18) C_s^d = f_2 \left\{ \frac{Y_{wm}}{P_h}, \frac{Y \Pi nam}{P_h}, \frac{Y \Pi am}{P_h}, \frac{P_s}{P_h}, C_{s,-1}, \frac{L_h}{P_h}, \frac{NW_{hm}}{P_h}, E_h \right\}$$

$$(19) C_{sd}^d = f_3 \left\{ \frac{Y_{wm}}{P_h}, \frac{Y \Pi nam}{P_h}, \frac{Y \Pi am}{P_h}, \frac{P_{sd}}{P_h}, \frac{L_h}{P_h}, \frac{NW_{hm}}{P_h}, (A_{sd} - K_{sd}), E_h \right\}$$

$$(20) C_{hd}^d = f_4 \left\{ \frac{Y_{wm}}{P_h}, \frac{Y \Pi nam}{P_h}, \frac{Y \Pi am}{P_h}, \frac{P_{hd}}{P_h}, \frac{L_h}{P_h}, \frac{D_{eshf}}{P_h}, \frac{NW_{hm}}{P_h}, (A_{hd} - K_{hd}), B_s(i,d,t), K_{hds}, \frac{P_{hds}}{P_{hd}}, C_{d+1}, \Delta N_f, E_h \right\}$$

$$(21) C_a^d = f_5 \left\{ \frac{Y_{wm}}{P_h}, \frac{Y \Pi nam}{P_h}, \frac{Y \Pi am}{P_h}, \frac{P_a}{P_h}, \frac{L_h}{P_h}, \frac{D_{eshf}}{P_h}, \frac{NW_{hm}}{P_h}, (A_a - K_a), B_s(i,d,t), K_{as}, \frac{P_{as}}{P_a}, E_h \right\}$$

$$(22) C_{d+1}^d = f_6 \left\{ \frac{Y_{wm}}{P_h}, \frac{Y \Pi nam}{P_h}, \frac{Y \Pi am}{P_h}, \frac{P_{d+1}}{P_h}, \frac{P_r}{P_{d+1}}, \frac{P_{(d+1)s}}{P_{d+1}}, B_m(i, d,t), B_{ms}(i,d,t), \Delta N_f, \frac{L_h}{P_h}, \frac{NW_{hm}}{P_h}, (A_{d+1} - K_{d+1}), \vec{P}_h, E_h \right\}$$

$$(23) P_r = f_7 (P_{r,-1}, E_{dr})$$

$$(24) C_l = f_8 \{ C_{d+1}, P_l \} \text{ (land acquired by the household sector from the frontier, the government or the business sector).}$$

(b) Household Budget Identities.

$$(25) Y_h = Y_w + Y_{\pi} - \prod_{cu} \quad (\text{undistributed profits of corporations})$$

$$(26) K_{sd} = K_{sd,-1} + C_{sd} - D_{sd} \quad (\text{Similarly for hd,a,d})$$

$$(27) D_{sd} = C_{sd} - \Delta K_{sd}$$

$$(28) Y_h + NW_{h,-1} = C_s + C_p + C_{sd} + C_{hd} + C_a + C_{d+1} - \Delta K_{sd} - \Delta K_{hd} -$$

$$\Delta K_a - \Delta K_{d+1} + NW_h \cdot \text{Conventional savings out of income}$$

Conventional savings out of income:

$$(29) S_h = Y_h - C_s - C_p - C_{sd} - C_{hd} - C_a = Y_h - C \quad \text{True savings out of income}$$

True savings out of income:

$$(30) S_h = \Delta NW_h = Y_h - C_s - C_p - C_{sd} - C_{hd} - C_a - C_{d+1} + \Delta K_{sd} + \Delta K_{hd} \\ + \Delta K_a + \Delta K_{d+1}$$

$$(31) \Delta NW_h = \Delta ( M_{hr} + S_{ehr} + K_{sd} + K_{hd} + K_a + K_{d+1} + K_{ub} - S_{eshfr} - S_{emhr} )$$

Total household economic activity during a time period is revealed by combining (28) and (31), to obtain

$$(32) Y_h = C_s + C_p + C_{sd} + C_{hd} + C_a + C_{d+1} + \Delta M_{hr} + \Delta S_{ehr} + \Delta K_{ub} \\ - \Delta S_{eshfr} - \Delta S_{emhr} \cdot$$

Equation (32) reflects household demand for goods (ex post), plus household re-arrangement of all of its money and financial assets and liabilities. Since it includes the net wealth components it permits consumer spending in any period to exceed consumer income by running down assets or by borrowing. Equations (17) to (22), and (32) represent a demand system of seven equations in eleven unknowns.

The unknowns are the economic variables which appear in (32), other than  $Y_h$  which is explained elsewhere. It is clear that if we wish to fully portray household demand activity, we must develop four or five more equations. Four will make our system just determinate, with one variable such as  $M_h$  determined as a residual by the budget constraint (32). But five behaviour equations could be developed for each of the five financial variables, thereby overdetermining the system and giving it one degree of freedom. Here however we shall make our equations represent choices between assets like money and securities, and then four further equations are quite adequate. But these further household behaviour equations will be deferred to Part VII, The Money System, where they can be analysed better.

Note that in place of a single consumption function for the household sector we shall have developed eleven behaviour equations through attempting to aggregate goods demanded into groups that are fairly homogeneous both in themselves and with respect to consumer behaviour. Note also that our model shows both true consumption (eqn. (30)), and consumer demand for goods. The former is necessary for wealth and welfare analyses, the latter for employment and price level explanation.

This concludes our study of the household and its demands for final goods and wealth. We now shift our attention to the firm, and its production and supply of these goods.



Part IIITHE FIRM, THE PRODUCTIVE SYSTEM AND SUPPLYChapter 9PRELIMINARY STUDY OF THE FIRM WITH COMPARATIVE STATICS1. Introductory.

Production involves the obtainment of materials and energy from the natural environment or from foreign trade, followed by the transformation of these step by step into final goods. At each step or process as the goods move through the system value is added to them. Final goods are those on which the domestic economy will perform no further processes or add no further value, and consist of consumer goods, government purchases, producers' capital goods, and exports.

Value is added to initial goods by the application of labour, capital equipment, and organization or entrepreneurship. Total domestic production is equal to total value added by factors of production located within the geographic boundaries of the domestic economy [ 3.10 p. 15 ]. Thus Gross Domestic Production (GD P) during a time period is equal to the aggregate of sales of all final goods (GDS) plus the value added to pipeline goods still within the production process, but not as yet sold as final goods, less imports from abroad. Thus  $GDP = GDS + \Delta H - F_1$ .

The basic unit or 'cell' of the productive system is the individual activity, process or operation. But the basic autonomous and decision making unit is the individual firm, a small aggregation



of basic activities. The function of the firm is to bring together and coordinate factors of production or resources in a sequence of activities, with the purpose of creating a product of higher economic value than the value of the resources used up. The firm is thus the basic unit of responsibility, finance, and decision making in the productive system. The test of the firm's success in adding value to the resources it uses is that the market is willing to pay a price for its product which is greater than the value of the resources it has used up.

Profit can be ascribed to two factors of production. Part of it represents the marginal productivity of the capital invested in the firm. The remainder is attributable to the entrepreneurial or enterprising function of organizing production and of risking capital in conditions of uncertainty (Knight [1.13]). The more efficiently these functions are performed the greater the residual profit will be to the enterpriser.

High profits are not always a measure of efficiency in serving social needs and wants, however. They can also occur as a result of monopoly and restrictive practices which keep marginal utility and price high for goods with inelastic demand where need or want is great. Such practices need to be kept under public scrutiny, policy and regulation.

It has often been assumed in economic theory that the firm's behaviour is guided by a sole aim - to maximize its profit. This is analagous to assuming that the consumer maximizes a utility

function which includes only economic goods. In the early days of industrial capitalism the assumption of profit maximizing as the sole guide to the behaviour of the firm was probably close to reality. But in the modern world the firm must justify its acts before the bar of a much stronger and better informed public opinion. Hence while it is highly concerned about profit earning, it can no longer take short-run steps which are detrimental to human and social values without serious opposition. This alters the preference system of the firm in modern society. Maximum profit will still be an important part of it, but is no longer of overriding dominance. Other important parts of its preference system include: survival, growth, economic and political power, good public relations, wealth and power of top executives, goodwill of customers, service to society, provision of a pleasant milieu for the productive and creative needs of its personnel. A firm in the modern world cannot pursue maximum profits in the short-run and achieve these goals. But if it pursues many of these goals it is likely to achieve high profits in the long run.

Thus a very important distinction arises between a firm's behaviour in the short run and in the long run. In the short run we assume that the modern firm attempts to maximize a utility function of various goals, among which we include high profits. But the firm will not take advantage of all of its profit opportunities in the short run ( for example: increasing prices to all that the traffic will bear; short-run layoffs of employees to suit company convenience) for fear of destroying good relations with its customers, its employees

and with the general public. But many of the opportunities open to it can be obtained if they are taken slowly, and in accompaniment with long range planning and development.

It follows that profit maximization can be taken as a good first approximation to the underlying motivation of the firm, as long as we expect many reactions to environmental changes to take place slowly, and hence to <sup>be</sup> applicable only to the longer run. Comparative statics can once again be a fruitful way to begin our analysis.

## 2. Study of the Behaviour of the Firm Using Comparative Statics.

Here we are concerned with the firm as a producer and supplier of goods, and as a user or demander of factors of production. Let the output of an individual firm during a time period  $t$  be  $q_t$ , which may be a vector of different products. Let the price at which this output is sold be  $p$ , so that the firm's total revenue is given by

(1)  $TR = p q$  ( or  $p q'$  for a vector product).

We omit the time subscript, but during the same time period the firm uses factors of production 1,2, ---,  $n$  in quantities  $x_1, x_2, \dots, x_n = x$ . Given the technology available to the firm, the quality of its entrepreneurship and management, the skills and attitudes of its labour force there is a physical technological relationship between the quantities of its inputs and the level of its outputs,

(2)  $q = f(x_1, \dots, x_n)$ . This is the production function of the firm.

If  $v_1, \dots, v_n$  are the market prices of the factors of

production, and if  $D$  represents certain fixed costs which occur in the time period independently of the level of output, we can state the total value of resources used up by the firm in a time period, or its total cost, as

$$(3) \quad TC = v_1 x_1 + \dots + v_n x_n + D.$$

The profit earned by the firm during a time period is the value added by its capital and enterprise,

$$(4) \quad \Pi = p q - (v_1 x_1 + \dots + v_n x_n) - D.$$

In the process of maximizing  $\Pi$  the firm must make two different kinds of adjustment. One is internal and refers to producing with the greatest efficiency or least cost. The other is external and has to do with deciding on the scale of output and the price to charge, given the demand curve confronting the firm. The first of these may be referred to as an engineering-management type of adjustment, the second as a sales-management adaptation. These two forms of adjustment to the environment cannot be carried on in completely separate compartments however, but must be integrated by the overall management. But for our analysis of the firm's behaviour we shall follow a similar 'division of labour', analysing first the internal or engineering-management adaptations, then the external or sales-management adaptations, and finally the overall integration of these. The mathematical phase of parts of this analysis draws heavily on the kind of approach initiated by Slutsky [2.28] and developed so usefully by Allen [2.2], Hicks [2.12] and Samuelson [2.27].

(a) Minimum Cost Adaptations.

Assume that the engineer-management must take as given the factor prices  $v$ , fixed costs  $D$ , the scale of output  $q$ , product price  $p$  and the form and parameters of their production function  $f$ . Their job is to select  $x$ 's which minimize  $TC$ , given by (3), but with the  $x$ 's constrained by (2). Hence they must minimize

$$(5) \quad TC = v_1 x_1 + \dots + v_n x_n + D - \lambda \left[ f(x_1, \dots, x_n) - q \right]$$

Then,

$$(6) \quad dTC = 0, \quad d^2TC > 0.$$

$$\text{Let } \frac{\partial f}{\partial x_i} = f_i, \text{ and } \frac{\partial^2 f}{\partial x_i \partial x_j} = f_{ij}.$$

From (5) and (6) we have

$$(7) \quad \frac{\partial TC}{\partial x_i} = 0; \quad v_i - \lambda f_i = 0. \quad (i = 1, 2, \dots, n)$$

$$(7a) \quad \frac{f_i}{v_i} = \dots = \frac{f_n}{v_n} = \frac{1}{\lambda}. \quad \text{Thus the marginal product of an extra}$$

pound or dollar spent on any one factor must be no greater or no less than that obtained from an extra unit of money spent on any other

factor. Note that at equilibrium one additional unit of money spent

on any one factor or split up among all the factors produces  $1/\lambda$

of additional product. Then one additional unit of product costs  $\lambda$

monetary units.  $\lambda$  is accordingly equal to marginal cost,  $\frac{dTC}{dq}$ , at

least cost equilibrium.

Consider next second order conditions.

$$(8) \quad (dx) \left[ f_{ij} \right] (dx)' (-\lambda) > 0, \text{ subject to the linear constraint}$$

(9)  $dq = f_1 dx_1 + \dots = f_n dx_n = 0$ . Let  $(f_1, f_2, \dots, f_n) = (f_i)$ .

Then the principal minors of

$$(10) \begin{bmatrix} 0 & (f_i)' \\ (f_i) & f_{ij} \end{bmatrix} \text{ must be alternately positive and negative. Let}$$

$$\begin{bmatrix} f_{ij} & (f_i)' \\ (f_i) & 0 \end{bmatrix} = F.$$

A total cost function or surface can be found by solving (7a) and (2) for  $x_1, \dots, x_n, \lambda$  in terms of  $v_1, \dots, v_n, q$ , and then substituting the  $x$ 's in (3).

(11)  $TC = c(v_1, \dots, v_n, q)$ . Each point on this cost surface is a minimum cost equilibrium for the assumed  $v$  and  $q$  values.

Next we shall see what can be deduced by displacing the equilibrium of (7a) and (11), using the method of comparative statics.

From (7a)

$$(12) \quad dv_i = d(f_i \lambda) = \lambda df_i + f_i d\lambda.$$

$$(13) \quad df_i = f_{i1} dx_1 + \dots + f_{in} dx_n.$$

From (2),

$$(14) \quad dq = f_1 dx_1 + \dots + f_n dx_n.$$

From (12), (13) and (14),

$$(15) \quad \begin{bmatrix} f_{ij} & \frac{(f_i)'}{\lambda} \\ (f_i) & 0 \end{bmatrix} (dx, d\lambda)' = \left( \frac{dv}{\lambda}, dq \right)' = [g_{km}] (dx, d\lambda)'$$

$$(16) \quad (dx, d\lambda)' \begin{bmatrix} \frac{G_{11}}{G_0} & \dots & \frac{G_{n+1,1}}{G_0} \\ \vdots & & \vdots \\ \frac{G_{1,n+1}}{G_0} & \dots & \frac{G_{n+1,n+1}}{G_0} \end{bmatrix} \left( \frac{dv}{\lambda}, dq \right)',$$

Where  $G_0 = \det. G_0$ .

$$(17) \quad dx_k = \frac{G_{1k} dv_1}{G \cdot \lambda} + \dots + \frac{G_{nk} dv_n}{G \cdot \lambda} + \frac{G_{n+1,k}}{G} dq$$

$$(18) \quad d\lambda = \frac{G_{1,n+1}}{G} \cdot \frac{dv_1}{\lambda} + \dots + \frac{G_{n,n+1}}{G} \frac{dv_n}{\lambda} + \frac{G_{n+1,n+1} dq}{G}$$

Comparison of matrices F and G reveals the following,

$$(19) \quad \det G = \det F / \lambda ; \quad G_{ij} = F_{ij} / \lambda = F_{ji} / \lambda = G_{j(i,j \neq n+1)};$$

$$G_{i,n+1} = F_{i,n+1} ; \quad G_{n+1,j} = F_{n+1,j} / \lambda ; \quad G_{i,n+1} = \lambda G_{n+1,i}$$

$$G_{n+1,n+1} = F_{n+1,n+1} \cdot$$

From (17) and (19)

$$(20) \quad \frac{\partial x_k}{\partial v_k} = \frac{F_{kk}}{\lambda F} ; \quad \frac{\partial x_k}{\partial v_j} = \frac{F_{jk}}{\lambda F} ; \quad \frac{\partial x_k}{\partial q} = \frac{F_{n+1,k}}{F}$$

$$(21) \quad \frac{\partial \lambda}{\partial v_k} = \frac{F_{k,n+1}}{F} ; \quad \frac{\partial \lambda}{\partial q} = \lambda \frac{F_{n+1,n+1}}{F}$$

Equations (20) and (21) provide us with comparative static equilibria. Once  $x$  and  $\lambda$  are in equilibrium with respect to given  $v$  and  $q$ , how will these equilibrium values change as the given environment of  $v$  and  $q$  change? To answer this question we must attempt to evaluate the signs of (20) and (21).

(i) Because of (10), and the fact that  $f_{kk}$  could be brought into the south east corner in (10),  $F_{kk}$  and  $F$  are of opposite sign. Also  $\lambda =$  marginal cost will always be positive. Therefore,

$$(22) \quad \frac{\partial x_k}{\partial v_k} < 0, \text{ and } \underline{\text{an increase in the price of any factor will}}$$

always result in less of it being used. (Since we shall be aggregating, problems of indivisibilities can be ignored.)

(ii) Nothing definite can be said about the sign of  $\frac{\partial x_k}{\partial v_j}$ .



It could be either positive or negative. If  $k$  and  $j$  were complementary inputs  $\frac{\partial x_k}{\partial v_j}$  would be negative. But if  $k$  and  $j$  were substitutes or

competitive  $\frac{\partial x_k}{\partial v_j}$  would be positive.

(iii) The firm behaves symmetrically with respect to any pair of factors  $j$  and  $k$ , as either one of their prices change, while output remains

constant for  $\frac{\partial x_k}{\partial v_j} = \frac{\partial x_j}{\partial v_k}$ . If  $j$  and  $k$  are complementary, both

reactions will be equally positive.

(iv) We cannot say anything 'a priori' about the sign of  $F_{n+1,k}$

relative to that of  $F$ . But in almost every case we know that  $\frac{\partial x_k}{\partial q}$

will be positive. The only case where this wouldn't be so would be where production methods were being changed with increasing  $q$  such that  $k$  was being partially or totally replaced.

(v) As long as  $\frac{\partial x_k}{\partial q} > 0$ ,  $\frac{\partial \lambda}{\partial v_k} > 0$ , for surprisingly,  $\frac{\partial x_k}{\partial q} =$

$\frac{\partial \lambda}{\partial v_k}$ . But one would expect that marginal cost would increase with the price of  $k$ , again except in the case where  $k$  was being partially or totally replaced in the production process.

(vi)  $\frac{\partial \lambda}{\partial q}$  is the slope of the marginal cost curve of the firm, derived from (11), and can also be expressed as  $\frac{\partial^2 TC}{\partial q^2}$ . Note that

it consists of the Hessian determinant of the production function  $f$  divided by  $F$ ., and multiplied by marginal cost  $\lambda$ . The generalized mathematical analysis can tell us no more than this, and cannot define

the sign of  $\frac{\partial \lambda}{\partial q}$ . This is quite as it should be, for the slope of the marginal cost curve will change from negative to zero to pos-

itive as the firm passes through increasing, constant and finally diminishing returns, as the scale of  $q$  is increased from zero upward. As these changes occur the  $f_{ii}$  and  $f_{ij}$  will tend to be successively positive, zero and negative.

(b) Adaptations to External Changes in Market Demand.

Given the internal minimization of cost by engineering-management resulting in a total cost surface (11), sales-management must determine the most profitable price and output for the firm.

From (4) and (11) we have

$$(24) \quad \Pi = p q - c(v, q). \quad \text{Assuming that price is given by the market,}$$

we have,

$$(25) \quad \frac{\partial \Pi}{\partial q} = p - \frac{\partial c}{\partial q} = 0 \text{ at max.}$$

$$(26) \quad \frac{\partial^2 \Pi}{\partial q^2} = - \frac{\partial^2 c}{\partial q^2} < 0 \text{ at max.}$$

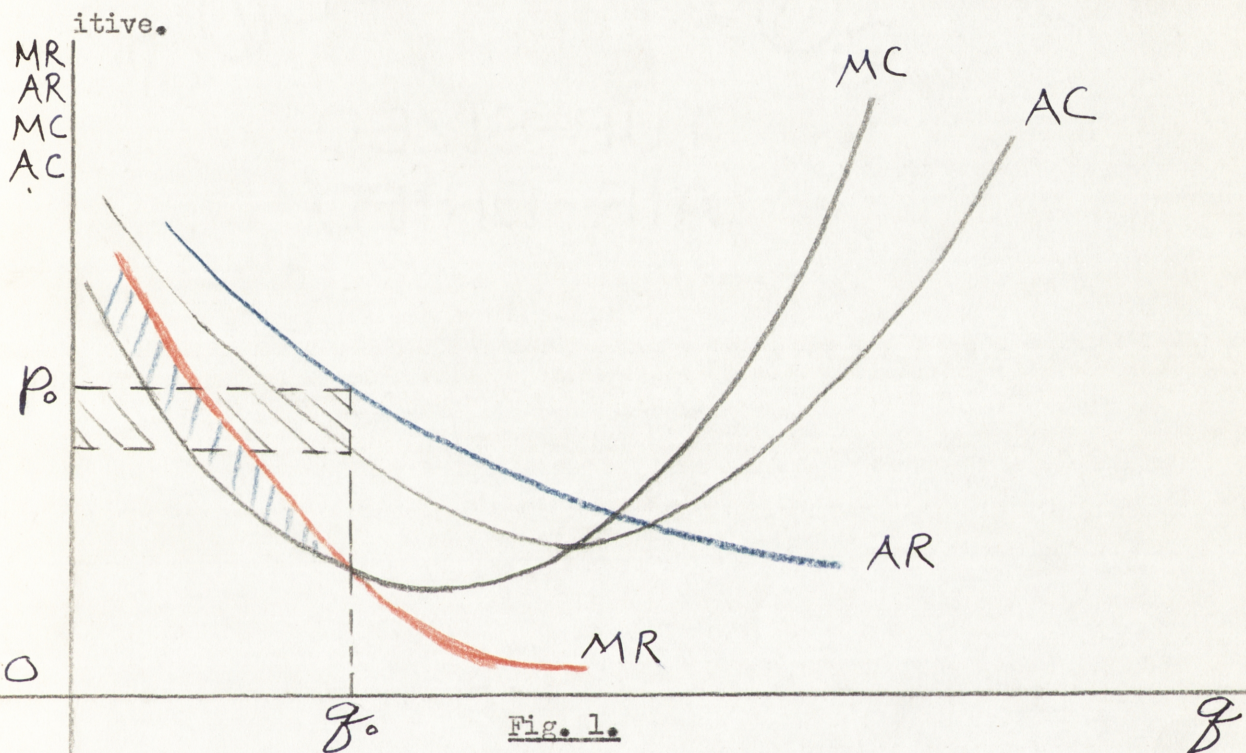
Thus output must be expanded until marginal cost equals price, and marginal cost must be rising. Hence under pure competitive conditions the given market price must be greater than or equal to minimum average cost.

If the firm is not in a pure competitive situation, and this applies to the majority of firms, it faces a downward sloping demand curve.

$$(27) \quad \frac{d \Pi}{d q} = p + q \frac{dp}{dq} - \frac{dc}{dq} = 0. \quad (v \text{ constant}).$$

$$(28) \quad \frac{d^2 \Pi}{dq^2} = 2 \frac{dp}{dq} + q \frac{d^2 p}{dq^2} - \frac{d^2 c}{dq^2} < 0$$

Thus output will be expanded until marginal cost (MC) equals marginal revenue (MR), and such that the marginal cost curve cuts the marginal revenue curve from underneath. A further equilibrium condition for (long-run) survival is a positive  $\Pi$ . But this condition borders on dynamic analysis. It means that for any lengthy time period average revenue (AR) must be greater than average cost (AC). Thus in Fig. 1 both of the shaded sections represent  $\max. \Pi$ , and these must be positive.



On the assumptions of Fig. 1 certain conclusions can readily be drawn about the reaction of the firm to an increase in demand, that is a shift of AR and MR upward or to the right. The firm will always increase output  $q$ . Also the firm will usually increase its price  $p$ . There can be occasions however when for moderate increases in demand price may decline.

(c) Composite Adaptations to Environmental Changes.

The external adaptations of the sales-management are brought into alignment with the internal adjustments of the engineering-management through the principle that MR and MC must be brought into equality in order to maximize profits. Thus from (7a) and (27) we have,

$$(29) \quad \frac{v_1}{f_1} = \dots = \frac{v_n}{f_n} = \lambda = p + q \frac{dp}{dq} \quad . \text{ Thus an increase in any } v_1$$

tends to cause  $\lambda$  to increase, despite the substitutions that this increase will call forth. The right hand side of (29) must then be raised by decreasing output. Also if the engineers are able to increase any marginal productivity  $f_k$  then  $\lambda$  tends to decrease, and the sales-management can then decrease  $p + q \frac{dp}{dq}$  by increasing output.

These results can be made slightly more general by assuming that both product demands and factor supplies have finite elasticities,  $e_p$  and  $e_i$ . Assume that the firm is at internal and external equilibrium, when an increase in demand for its product occurs.

$$(30) \quad MR = d(pq) = pdq + qdp = p \left( 1 + \frac{1}{e_p} \right) dq.$$

To achieve  $dq$  the firm must increase some or all of  $x$ . The additional cost of  $dx_i$  is

$$(31) \quad d(v_i x_i) = v_i \left( 1 + \frac{1}{e_i} \right) dx_i = MC_i$$

The marginal revenue attributable to hiring  $dx_i$  is

$$(32) \quad MR_i = p \left( 1 + \frac{1}{e_p} \right) f_i dx_i$$

$e_p$  will usually be negative,  $e_i$  positive.

Combining now both internal and external equilibrium, the firm will expand each  $x_i$  until each  $MC_i = MR_i$ . At this composite equilibrium,

$$(33) \quad \frac{f_1}{v_1 \left(1 + \frac{1}{e_1}\right)} = \dots = \frac{f_n}{v_n \left(1 + \frac{1}{e_n}\right)} = \frac{1}{p \left(1 + \frac{1}{e_p}\right)} .$$

As in (7a) the last dollar or pound spent on each factor can be related to an equal quantity of product;  $MC$  or  $\lambda$  in combined internal and external equilibrium comes into equality with  $p \left(1 + \frac{1}{e_p}\right) = MR$ ; and the marginal money cost of each factor equals the marginal productivity of the factor times marginal revenue from an additional unit of product.

To Sum up:

- (a) If product demand increases either  $p$  or  $q$  ( usually both will increase). If  $q$  increases most  $x_i^d$  will increase ( Sec. 2a(iv), (v) ). If  $p$  increases, and  $v_i$ ,  $e_i$  and  $e_p$  remain constant, all  $f_i$  must be reduced to restore external equilibrium and hence the  $x_i$  will generally be increased.
- (b) If the price of factor  $i$  decreases  $x_i^d$  will increase (22), other things equal.
- (c) If the price of any other factor  $j$  decreases there will be a "substitution effect" ( Hicks [2.12] Chap. VII ) away from  $x_i^d$  in favour of  $x_j^d$ , unless  $i$  and  $j$  are complementary. The substitution effects restore internal equilibrium.
- (d) If  $v_j$  decreases as in (c) the firm will likely be out of external equilibrium as a result of the decreased cost of its operation. All

of the ratios in (33) will now have increased. The firm will expand production to restore this external equilibrium. This has been called a "production effect" ( [2.12] Chap VIII) or an "output effect" ( Samuelson [1.20] pp. 538-9).

### 3. A Digression on the Firm's Method of Approach to a Maximum Profit Position.

The firm cannot move directly to a maximum profit position as indicated by Fig. 1, and by equations (27) - (33). This is because it usually does not know with any degree of accuracy the demand and MR curves confronting it. It does know part of its AC curve, and its breakdown of AC as between direct labour, raw materials and overhead cost. Hence the firm may follow something like the following procedure in deciding on either the price to charge, or the level of output. First a rough price and quantity combination will be estimated for the firm's market on the basis of past experience. Then the cost department will estimate the variable prime costs ( direct labour and materials) and the fixed overhead, for this level of output. This gives TC and AC. A percentage markup will be added to average prime cost to cover both fixed overhead and a desired margin of profit, or to AC to cover the desired margin of profit. This will give a trial selling price. The market is then tested. Market testing may show that a greater output can be sold at the test price, or that a higher markup can be added to the test level of output. By testing the level of profit associated with both of these variations a local profit max-



imum can easily be reached. With growing experience, and trial and error, the firm will tend to keep moving toward its highest maximum. It will perform most of its calculations using some type of markup on costs, but there is no reason why this markup percentage has to remain fixed and sacrosanct. Nor does it in practice, (Cf. Klein [1.11], p. 39, Samuelson [1.20], pp. 518-20, Gabor and Pearce [8.6], Pearce [8.12], [8.13] (with Ramey)).

#### 4. Summary: The Static Demand of the Firm for a Factor of Production.

The analysis of Sec. 2 above reveals a substitution effect away from factors whose prices have increased and toward factors whose prices have decreased, in a relative sense. This is required to reach internal equilibrium. It also reveals a production effect, from which a decline in any factor price causes all factors to be demanded in greater quantity to restore external equilibrium. In addition we found a general expansion effect whenever either  $p$  or  $q$  increased due to an increase in product demand. These three effects can be fitted into a preliminary demand equation for factor  $i$  as follows:

$$(34) \quad x_i^d = f \left\{ \frac{v_i}{P_v}, \frac{p}{v_i}, q \right\}, \text{ or}$$

$$(34 a) \quad \frac{x_i^d v_i}{p} = a_0 + a_1 q + a_2 \frac{v_i}{P_v}, \text{ where } P_v \text{ is a price index of } v_1, \dots, v_n.$$

These equations serve as prototypes for aggregation, and for the analysis of the demands of firms for labour, inventories and fixed capital under the dynamic conditions of the real world.

Chapter 10.AGGREGATE DEMANDS FOR FACTORS OF PRODUCTION UNDER DYNAMIC CONDITIONS.

The firm uses three broad classes of factors of production in its operations. These are labour, materials, and fixed capital equipment. These factors differ by being human vs. material, storable vs. non-storable, durable vs. transient. We shall begin with the factor that is human and non-storable, man-hours of productive employment. It is of course the demand for this factor which is the most important variable in the model being developed. For this model is intended mainly as an aid to full employment policy.

1. The Demand for Labour.

- Let:  $N_p$  = average number of paid workers employed by firms during a time period;
- $h$  = average hours worked per employed paid worker during the time period;
- $w_{ph}$  = average hourly earnings of paid workers hired by firms;
- $P_w$  = price index of hourly wage rates;
- $P_{GDP} = P$  = price level of gross domestic product;
- $GDP$  = real volume of flow of gross domestic product during the time period;
- $P_m$  = price index of industrial materials;
- $P_K$  = price index of new fixed capital;
- $P_{mK}$  = combined price index of industrial materials and producers fixed capital;



The demand for labour by firms aggregated from the last chapter, and translated into the new symbols becomes,

$$(1) \frac{N_p^d h w_{ph}}{P} = a_0 + a_1 \text{GDP} + a_2 \frac{P_w}{P_{mK}} .$$

We now move into the dynamic and other influences that may further affect this demand relationship in the real world.

(a) Expectations.

It takes time to produce output and hence what is finished product for the market today must have been set into production at some time in the past. This means that <sup>much</sup> ~~a~~ output must be planned for a future market on the basis of today's expectations or forecasts about that future market. Today's expectations are based largely on today's level of activity and its trends. The trend of production is represented by  $\overrightarrow{\text{GDP}} = f(\text{GDP}_{-2}, \text{GDP}_{-1}, \text{GDP})$ . The number of lags it would be necessary to include in  $\overrightarrow{\text{GDP}}$  will depend on the length of the time period being used.

There are other types of expectations of a technological, political and international nature which will also influence the plans of business men, and these can be included by adding a term  $E_f$  to the demand equation.

(b) Wealth.

The total assets of a firm consist of money ( $M_f$ ), securities (including accounts receivable and bills of exchange) ( $S_{ef}$ ), inventories (H), and fixed capital made up of plant, construction works, machinery and equipment ( $K_{PCM}$ ) and land ( $K_{lf}$ ). The firm owns all of this wealth, but it has offsetting debts to others which coun-

terbalance part of it. The value actually owned by the firm or its owners is its net worth ( $NW_f$ ). But, from a managerial point of view, the wealth of the firm may still be thought of as the total assets under control. That is,

$$(2) W_f = M_{fr} + S_{efr} + H + K_{PCML}.$$

The net worth of firms is equal to their total assets or wealth holdings less their issues of fixed interest (non equity) securities held by other firms, households and government,

$$S_{eif(hfg)} = S_{eifo}.$$

(3)  $NW_f = W_f - S_{eifor}$ . Note that the net worth of firms should be approximately equal to the market value of equity securities (share stock) issued by firms plus the net worth of unincorporated business

$$(4) NW_f = S_{eior} + K_{ub}.$$

Unlike the consumer, the firm does not need to prepare for old age. However because it puts a high value on survival and growth, the size of its wealth and its net worth are of great importance to it. But the greater these become the less it needs to worry about survival and growth, and the more attention, courage and resources it can devote to current production, and to research on new lines of production. Thus growth may beget growth, and there may be a wealth effect on the current level of demand for factors of production.

### (c) Liquid Wealth.

That part of a firm's total wealth which is liquid may exert a disproportionate influence on its current activity. We might define its liquid wealth as its stock of money and securities, less

mortgages, which are usually not considered as saleable for short run cash needs.

Thus

$$(5) \quad L_f = M_f + S_{ef} - S_{emf}$$

The actual money available to a firm for financing current needs is reduced by its current liabilities,  $S_{esf}(hfg) = D_{esf}$ , consisting mainly of accounts payable and bills of exchange payable. Thus net liquidity,

(6)  $NL_f = L_f - D_{esf}$ , is probably a more meaningful causal variable demonstrating the separate influence of liquid wealth on a firm's activities.

In its cycle of production a firm uses money to purchase factors which create products which the firm sells for more money. Thus money to the firm is in a sense a factor of production with which it garners more money. The firm has certain needs for money based on its level of transactions and its precautions against emergencies. Once these transactions and precautionary needs ( Keynes [1.6] ) are met, the firm is likely to think about putting any surplus funds it has into further productive uses or into securities with good yields. This may often mean increasing its demands for factors of production. It may apply these toward expanding output in its present product lines, or it may add new product lines.

In addition to possibly encouraging it to expand, a firm's level of liquidity may also be permissive, a necessary though

not a sufficient condition for expansion.

(d) Undesired Inventories of Finished Goods,  $u_{fg}$ .

The meaning and importance of this variable as an influence on output is explained below.

(e) Summary.

$$(7) \frac{N_p^d h^d w_{ph}}{P} = f \left\{ \overrightarrow{GDP}, GDP, \frac{P_w}{P_{mK}}, \frac{NL_f}{P_{wmK}}, W_f, u_{fg,-1}, E_f \right\}$$

(f) Demand for Hours of Work.

When  $GDP_f$  (gross domestic production in firms) is at a full capacity value  $\overline{GDP_f}$  we assume that hours of work are at a normal or standard value  $h_s$ . When firms attempt to produce at over capacity they usually demand overtime work at overtime rates of pay. This is because at full capacity there is not adequate room for more employees, and there is likely to be full employment in the economy in any event. But when output falls to below capacity, man-hours can be reduced by reducing both employment and hours. The employer will often prefer to reduce hours in the case of key personnel rather than lay them off, as a means of retaining the part of their services still required during the slack period. The following equations are suggested for hours demanded by employers.

$$(7a) h^d = h_s + a (GDP_f^d - \overline{GDP_f})$$

(7b)  $h^d = h_s + a (GDP_f^d - GDP_f^s)$ . In these equations  $h$  rises above  $h_s$  when there is over capacity production, or excess demand, and conversely.

## 2. The Demand for Inventory Stocks.

It is necessary for inventories to be spread rather evenly throughout the productive system to ensure smooth and continuous operation. The build up and running down of these stocks act as control mechanisms along with the price system, and exert particularly dynamic influences on the level of productive activity and employment. The inventories held by a firm tend to fall into three classes - raw materials purchased from other firms  $H_{rm}$ , goods in process in the firm  $H_{gp}$ , and finished goods of the firm ready for sale  $H_{fg}$ . These inventories of materials are of obvious importance in the productive process, for it is on them that the labour and capital equipment perform their productive operations. Since <sup>the</sup> three classes of inventories perform different functions in the productive process, we analyse the demand for each class separately.

### (a) Demand for Raw Materials.

The cost of holding raw materials is a combination of their price  $P_m$  and the costs of storage which include  $P_{pc}$  and  $P_w$ . Let this composite cost index be  $P_{rm}$ .

Since inventories must be planned for well in advance of needs the expectational component in the demand for them is strong. Also they bring a positive or negative speculative yield if their price trend is upward or downward. Because of the time required for ordering and shipping, and the time required for expectations about current trends to become firm, ex post  $H_{rm}$  will tend to lag behind GDP.

Abramovitz [3.2] estimates this lag at two to three months for the U.S.

$$(8) H_{rm}^d = f \left\{ \frac{P_{rm}}{P}, \frac{P_{rm}}{P_{wK}}, \overrightarrow{GDP}, GDP, \overrightarrow{P}, E_f, \frac{NL_f}{P_{wmK}}, W_f, u_{fg, -1} \right\}.$$

(b) Demand for Goods in Process.

These are the materials actually undergoing transformation in the productive system. Their quantity will be very complementary with the amount of labour employed, although there will be room for a little substitution at the margin.

$$(9) H_{gp}^d = f \left\{ \frac{P_{gp}}{P}, \frac{P_{gp}}{P_{wK}}, \overrightarrow{GDP}, GDP, \overrightarrow{P}, E_f, \frac{NL_f}{P_{wmK}}, W_f, u_{fg, -1} \right\}$$

$H_{gp}$  should coincide rather closely with the cycle of GDP, though with a minimum lag of one production period. [3.2].

(c) Demand for Inventory Stocks of Finished Goods.

From the point of view of the firm a large stock of finished goods insures against loss of sales in a rising market, but is costly in terms of warehousing and the use of money. It is also a good speculative investment if prices are expected to rise. From the overall point of view of the overall productive system they facilitate the smooth flow of goods from one stage to the next. The demand for these stocks will accordingly be similar to that for the other factors.

$$(10) H_{fg}^d = f \left\{ \frac{P_{fg}}{P}, \frac{P_{fg}}{P_{wK}}, \overrightarrow{GDP}, GDP, \overrightarrow{P}, E_f, \frac{NL_f}{P_{wmK}}, W_f \right\}$$

As suggested above these stocks play a further important role in the economic system. Since the system lacks price flexibility (CF. Part VIII) some control mechanism complementary to the price system is required. Inventories, especially of finished goods

provide this mechanism. If stocks pile up above what is desired under falling demand for product, they flash a signal to slow down production. Conversely should they fall below what is desired, as demand for product rises, a signal is flashed to speed up production. Thus undesired inventories  $u_{fg}$  become a feed-back mechanism ( similar to a thermostat controlling the temperature and heat source of a room) which guides the economy in a dynamic adjustment process. The process is one which aims to reduce  $u_{fg}$  to zero.

Let  $H_{fg}^d$  be the demand for finished goods inventories, and  $H_{fg}^d$  the ex post solution value of this variable in a complete model.  $H_{fgt}$  is the corresponding observed value of the stocks at the end of period  $t$ . Then we define,

(11)  $H_{fgt} - H_{fgt}^d = u_{fgt}$ . This is the total disturbance or residual for the variable  $H_{fg}$  in a complete model ( Cf. Brown [2.6] p. 365-6), and is the same in concept as could be obtained by surveying the actual and desired inventories of all firms at the end of a time period.

Because of the time required for  $u_{fg}$  to correct  $H_{fg}$ , this inventory stock will tend to lag, like  $H_{rm}$ , behind the GDP cycle.

The accelerative effect of inventory demand on the GDP cycle has important consequences for employment and price level theory. The inventory cycle follows the GDP cycle causally, but with a lag. This lag is short enough relative to GDP cycle periods to produce 'resonance' and serious widening of GDP amplitudes. It is of course  $\Delta H$  not  $H$  ( $= H_{rm} + H_{gp} + H_{fg}$ ) which affects GDP and employment, and its dynamic time path (which leads that of  $H$ ) must be followed to derive

its impact in any particular time period. Because of lags (2-4 months [3.2]) it may in some periods oppose the cycle, though it will usually reinforce it.

### 3. The Demand for Fixed Investment Goods.

It is through the purchase of new capital goods that the money savings of the economy flow back into the productive system, and the unemployment that saving would otherwise produce is thereby prevented. Capital goods are desired because of the great productivity which they add to productive operations when combined with the other factors of production. Their yield in terms of the value they create over their useful life, in excess of the value of resources used up to produce them, is so great that they become the main highway to economic development and higher standards of living.

The scientist and the engineer play leading roles in the development of modern machines and equipment, part of a historic process which began with the Industrial Revolution and still continues. Investment in tools and equipment had taken place from earliest times, but now the energy of fossil fuels was harnessed in machines and engines to run the tools. Mighty inanimate power had replaced animate muscle power. This process has gained momentum as it passed through the steam, electric and petroleum periods, and as it now enters the atomic era.

Coming back to economic principles, the decision to invest in fixed capital is based on the same economic forces as the decision to hire more labour or to purchase more materials. Labour and



materials will be purchased up to the point where their marginal revenue productivity becomes equal to their marginal cost. The marginal cost of a new piece of fixed capital can be estimated fairly readily. But what is its marginal revenue productivity?

Let the present cost of a piece of new fixed capital be  $P_{GI} \cdot GI_0$ . The engineers estimate installation cost plus additional overhead costs associated with the use of this capital to have a present value of  $A_0$ . They also estimate that the marginal physical output attributable to this capital in future time periods 1, 2, ---, n will be  $q_1, q_2, \dots, q_n$ , and that the scrap value will then be  $S_n$ . The management estimates the marginal net revenue per unit of these future outputs to be  $v_1, v_2, \dots, v_n$ . The stream of future income associated with the investment is then  $v_1 q_1, \dots, v_n q_n, S_n$ . This is the estimated marginal revenue productivity of the new capital. The  $v$  vector is based on the firm's forecasts and expectations regarding future economic conditions, and the future prospects of its particular industry.

But in order for the marginal revenue productivity to be compared with marginal cost  $P_{GI} \cdot GI_0 + A_0$ , the stream of future returns must be brought to the same time base as the marginal cost. This can be done through the time preference rate of the firm. Suppose that the firm can be reasonably certain of earning at least  $i_{lf}$  on its money, where  $i_{lf}$  is the long-run yield on corporation bonds. Then the present value of the expected stream of income from the new capital is

$$(12) \quad MR_0 = \frac{v_1 q_1}{(1+i_{1f})^1} + \dots + \frac{v_n q_n}{(1+i_{1f})^n} + \frac{S_n}{(1+i_{1f})^n} \cdot$$

As long as  $MR_0$  is greater than the marginal cost the firm is likely to decide in favour of the new investment.

What is the actual expected yield from this investment?

It could be defined as a rate  $r$  such that if the initial cost of the investment is accumulated at this rate over the life of the investment, it would be equal to the stream of income derived from the investment in successive time periods and accumulated at the same rate over the same lifetime. This gives an equation

$$(13) \quad P_{GI} \cdot GI_0 + A_0 = \frac{v_1 q_1}{(1+r)} + \dots + \frac{v_n q_n}{(1+r)^n} + \frac{S_n}{(1+r)^n}$$

from which an appropriate solution of  $r$  can be obtained. " $r$ " is of course the "marginal efficiency of capital" of J.M. Keynes [1.6] p. 135.]

As Keynes observed, the basic decision to invest depends on the value of the "marginal efficiency" of the investment,  $r$  in relation to the cost of borrowing or the opportunity cost of using money for investment,  $i_{1f}$ . The average firm will want this yield or marginal efficiency to be well above  $i_{1f}$ , sufficiently above to cover the added risks and uncertainties associated with production as compared to investment in 'guilt-edged' bonds. One way of assuring this is to interpret  $r$  as the expected value of a normal probability distribution, and insist that  $r$  be at least two estimated standard deviations above  $i_{1f}$ . Or it might insist that  $r$  be computed from the estimates of the returns for the first five years only of the investment,

and that this  $r$  be at least one standard deviation higher than  $i_{lf}$ . Through some such rules as these the firm can attempt to protect itself against the inevitable uncertainties involved in its expectations.

The factors which influence the level of the marginal efficiency of an investment can be listed from an inspection of (13). It will be greater the higher is the marginal physical productivity vector  $q$ ; the more this output vector is concentrated in the earlier years ( Hicks [2.12] Chap 14); the higher is the expected price vector  $v$ ; the more that higher prices are concentrated in earlier than later years; the higher the scrap value; the longer the useful life  $n$  of the equipment; the less the resources required in the original equipment,  $GI_0$ ; the less the price level of the original equipment  $P_{GI}$ ; the less the installation and additional overhead costs  $A_0$ .

On the basis of the theory of factor demand for firms and the marginal yield of capital we can set up a preliminary demand function for new capital goods.

$$(14) \quad GI^d = f \left\{ \frac{P_{GI}}{P}, \overrightarrow{GDP}, \overrightarrow{P}, \frac{P_{GI}}{P_{wm}}, i_{lf}, \frac{NL_f}{P_{wmK}}, W_f, u_{fg,-1}, E_f \right\}$$

One defect of this equation may be that it does not bring marginal efficiency or yield explicitly into the equation. This may be more important in the case of a long lived asset than in the case of factors that are used up in the short run. In any event we have already shown that capital is more closely related to profit or yield, since it along with enterprise is the claimant of this residual yield after current labour and material costs have been paid.

In considering refinements to this equation we shall plan to compare the marginal efficiency of capital and  $i_{1f}$  as Keynes suggested. In this process we shall also attempt to consider some other major special influences which affect the demand for capital goods, and which arise from their large cost, and long life.

(a) The Stock of Durable Capital Goods.

Because of their durability, these goods accumulate into stocks, as indicated by the dynamic equation

(15)  $K_{PCML} = K_{PCML,-1} + GI_{PCML} - D_{PCM} = K_f$ . The subscripts P,C, M,l refer to plant, construction, machinery and equipment, and land, respectively. Once a capital good has been purchased its continued existence inhibits further demand for a new good like it over its economic lifetime. Its economic life time can be stretched if economic conditions are poor, or shortened if economic conditions are good, so that the demand for these goods can add a tremendous 'resonance' effect to the amplitude of a GDP cycle, however the cycle once got started. The existing stocks contribute further to cyclical behaviour in the economy in that as large parts of them tend to be built up at the same time, due to innovations or prosperous times, these parts will tend to be due for replacement at the same time in the future. In this way 'long waves' of durable goods demand tend to be inherent in the system. (Schumpeter [3.33]). No such waves would occur for short-lived goods (Burns [3.3]). We can conclude that the existing stock of capital by itself exerts a negative influence in any time

period on the demand for further capital, and that the changes in this stock contribute very much to the tendency of economic activity to be cyclical.

(b) A Saturation Level of Capital Goods.

Closely related to the size of the stock of capital is the concept of its marginal yield  $r$ . Despite the inhibiting influence of the existing stock, presumably capital will be desired as long as its yield is greater than the interest rate  $i_{1f}$ . It also seems likely that at a given state of the arts and of population,  $K_f$  will eventually reach diminishing marginal productivity, so that as  $K_f$  grows  $r$  will tend to fall to  $i_{1f}$ . Let the  $K_f$  when this point is reached be a saturation level  $K_s$ . Have we any way of estimating  $K_s$ ?

Let the aggregate production function of firms be represented by

(16)  $GDP_f = F(N, H, K_f)$ . Assume this function to be roughly homogeneous of the first degree. Then using Euler's Theorem, the property and enterprise income originating in firms is roughly given by

$$(17) \quad \Pi_f = \frac{\partial F}{\partial K'} (H + K_f), \quad (K' = H + K_f), \text{ whence}$$

$$(18) \quad r = g\left(\frac{\partial f}{\partial K'}\right) = g\left(\frac{\Pi_f}{K'}\right); \quad \frac{\Pi_f}{H+K_f} = r' \doteq r.$$

$K_f$  can be profitably increased until

$$(19) \quad \left( \frac{\Pi_f}{H + K_f} - i_{1f} \right) \text{ reaches some minimum value, at which } K_f$$

becomes  $K_s$ . If (19) is inserted as an explanatory term in the investment demand equation, it will by itself give effect to four causal influences. These are the negative influence of  $K_f$ , the positive in-

fluence of the gap ( $K_s - K_f$ ), the positive influence of the productivity of capital and enterprise  $\prod_f$ , and the negative influence of money costs  $i_{1f}$ , based on money market conditions.

(c) The Acceleration Principle.

The  $K_s$  concept was based partly on the assumption of a stationary economy, with given population and arts. Let us assume now that growth takes place as a result of population increase or rising standards of living, with no change in the arts of production. Assuming that  $K_f$  had reached  $K_s$  prior to this change, and that there was full employment of existing factors, we should now find some rise in prices and hence in  $r'$ , as well as a positive excess demand ( $GDP^d - GDP^s$ ). The latter is a broader concept than  $u_{fg}$ , but is closely related to it, and will be defined in Part VIII.

On both of these scores  $\prod_f$  can be increased by expanding capacity and hence  $K_f$ .  $GI_{PCM}^d$  can once again become greater than  $D_{PCM}$ .  $K_f$  will resume growth by a duplication of existing types of capital equipment, and will increase in quantity with no change in quality. Under the assumptions of this section  $K_s / GDP_{max}$ , where  $GDP_{max}$  is full employment GDP, might tend to be roughly some constant  $a$ , so that  $K_s$  and  $GI$  becomes related to output in a simple formula

(20)  $\Delta K_s = GI - D = a(GDP - GDP_{-1})$ . "a" has been called the accelerator, and (20) the "acceleration principle". (Cf. J.M. Clark [3.8], J.R.Hicks [3.15], and A.F. Burns [3.3]). But (20) can only be of limited use for our purposes, since it applies only to movements of  $K_s$  relative to movements in  $GDP_{max}$ , and assumes a linear or 'plane'

and homogeneous production function. For our purposes the terms (19) and  $(GDP^d - GDP^s)$  include the acceleration principle cases, but are more widely applicable, to any time periods and any K and GDP. It may however still be advisable to test  $(K_f / GDP_f)$  in our demand equation, since it may provide a separate representation of the acceleration principle.

(d) Innovations and Real Investment: Schumpeter.

Since the Industrial Revolution it is doubtful if any western economy has reached a technological saturation with capital for any extended period, although many economies have reached an 'economic saturation' under conditions of depression and unemployment. The reason for this is that the technological saturation level keeps receding under the influence of new discoveries in the industrial and managerial arts, in technology and in science. It was Professor Joseph A. Schumpeter [3.33] who emphasized the importance of such new discoveries and their influence on investment. Schumpeter went on to observe that discovery is not enough. It requires men who have the vision and the courage to put the new discoveries into operation, usually against the social pressure of the environment which favours the repetition of tried and true methods. He called the new discoveries when put into operation innovations, and the men who push them through the opposition of the environment, innovators. An entrepreneur as such is essentially an innovator, and true profit on Schumpeter's definition is the reward to the entrepreneur for innovation.

In almost every case an innovation, whether it involves production of a new product, or the production of old products in new ways, requires a change in both the quality and the quantity of capital goods. Once an innovation proves to be successful a swarm of imitators enter the field and the demand for the new kind of capital expands enormously. The Industrial Revolution began with a major innovation, the steam engine fuelled with coal. Subsequent major innovations have been: the railways, the use of electric power, and the automobile. We now appear to be on the threshold of a new major innovation - the use of atomic power.

How can the effect of innovations on the demand for new capital be brought into our demand function? The essence of innovation from the economic point of view is that it raises the productivity of the factors of production. Usually it involves the use of real capital in a new form, and increased productivity.

In the short-run the innovator or enterpriser, and the new capital, can claim the profit arising from the increased productivity. This lifts the value of  $r'$  and will usually establish a new saturation level of capital,  $K_S$ . With  $K_S$  now greater than  $K_F$ , imitators [3.33] will soon swarm into the new product or production method.  $GI^d$  will now expand considerably, and  $K_F$  will grow until it once again reaches  $K_S$ . During this process  $r'$  (or  $r$ ) will first be lifted to new heights by the innovation. Subsequently it will gradually fall as  $K_F$  is expanded, until once again it approaches  $i_{1F}$ .



With regard to an appropriate variable to reflect these developments on  $GI^d$ , it would seem that (19) will respond to most of them. The actual invention or discovery behind the innovation is largely exogenous, but once the innovation has got under way (19) will carry the dynamic process along.

(e) Lags in Investment Demand.

Because real investment so often involves large scale construction projects it must be planned considerably in advance of actual production. Plans must be based on the conditions and trends observable at the time they are made. Hence many of the variables which influence current production of  $GI^d$  must be lagged to the time periods when plans were completed. Current values of causal variables should also be of some importance however as plans for smaller projects can be advanced, altered, postponed or abandoned as current conditions change. If short time periods are being used the carry-over of large projects may be quite large and then  $GI_{-1}$  may become a further important causal influence.

(f) The Finance of Investment - The Securities Market.

Investment capital goods usually require large amounts of money for their purchase. The savings of firms are usually not sufficient in the aggregate to finance all these needs. The practice is for them to obtain such money requirements from the 'savings pool', partly from their own retained earnings, partly from the sale of new securities, and sometimes from the banking system. If an abundance of money is flowing into the securities markets and the demand for sec-

urities is high relative to their supply, it will be easy for firms to float off a new issue of stocks or bonds for capital purposes. In the case of bonds and debentures these circumstances produce high values of  $S_{eifo}$ , and consequently low values of  $i_{lf}$ . This encourages investment and is already included in the demand equation via (19).

The influence of conditions in the equity share stock market is quite similar to this. High prices in this market indicate some or all of an abundant flow of money into the market, confident expectations of good future yields from equities, and a willingness to accept lower yields currently. It follows that  $P_e$ , the price level of equity securities, should be added to the demand equation as indicative of ease of financing. This variable should be lagged to the period when investment decisions are being made.

The influence of financial conditions can be further indicated by the actual net drawing of money from the savings pool by firms for investment purposes. This operation will follow the decision to invest, but will tend to precede actual construction. The net flow  $\Delta (S_{ee(hfg)} + S_{eif(hfg)}) = \Delta S_{elfo}$ , or  $(\Delta S_{elfo})_{-1}$ , represents the main external finance drawn in by firms. The rest of their needs must be drawn from internal sources, essentially the accumulation of past savings in the form of undistributed profits and depreciation reserves, to the extent that these are included in  $L_f$  and  $NL_f$ .

(g) Net Capital Inflow from Abroad.

Capital inflow from abroad whether for portfolio or

direct investment adds to the cash reserves of the banking system and enables the money supply to expand. This will influence the demand for investment indirectly through the liquidity variable  $L_f$ . The foreign demand for portfolio (and some direct) investment will also influence domestic investment indirectly through its influence on  $P_e$  in the stock market and on  $i_{1f}$  in the bond market. But direct investment  $\Delta K_{df}$ , as its name implies, is often meant for specific expansion projects. These are likely to follow some time after this capital import. It is accordingly likely to have a separate and direct influence on  $GI^d$ .

(h) Summary Demand Equation for New Investment Goods.

$$(21) \quad GI^d = f \left\{ \frac{P_{GI}}{P}, GDP, \vec{GDP}, \vec{P}, \frac{P_{GI}}{P_{wm}}, \frac{L_f}{P_{wmK}}, \frac{NL_f}{P_{wmK}}, W_f, E_f, \right. \\ \left. (r' - i_{1f}), (r' - i_{1f})_{-1}, \cdot (GDP^d - GDP^s)_{-1}, P_{e,*1}, (\Delta S_{elfo})_{-1}, \right. \\ \left. (\Delta K_{df})_{-1}, GI_{-1} \right\}$$

This overall demand can be separated into  $GI_{PC}$  and  $GI_M$  with quite similar equations, except that there is now some complementarity, and hence  $GI_{PC}$  might appear as a causal variable in  $GI_M$ .

Chapter 11THE PRODUCTION FUNCTION, PRODUCTIVITY, AND SUPPLY.1. The Production Function.

This function is the technological relationship between inputs of factors of production and outputs of desired product. The global relationship is an aggregation of all of the micro-relationships, one for each separate process or activity in each firm; it can be represented by

(1)  $GDP_f = F \left\{ (N_p + N_{enp})h, H, K_f \right\}$ , where  $N_{enp}$  is the number of entrepreneurs and unpaid family workers engaged in production in firms.

It is assumed that the quantities of factors which go into (1) have been carefully selected on the basis of their relative costs and marginal productivities, in accordance with the theory of the preceding two chapters. It is also assumed that the complete hypersurface (1) is shaped like a 'hill' so that any cross-section parallel to a co-ordinate 'plane' will display in succession increasing, constant, decreasing, and finally negative returns, as any one factor is varied from zero to infinity, with all other factors held constant (Chapter 3).

For any particular economy in a particular time period each of the factors will have a boundary or maximum value, representing the maximum availability of the factor at that time. The complete surface is accordingly truncated or bounded in this way, with the maximum points on its outer boundary representing a short-run production

"ceiling". The ultimate limitations on production, apart from boundary values of the factors, are the size and quality of the natural environment, the availability of leadership and enterprise, and problems of administration. It is these and perhaps other limitations that cause (1) to eventually turn down, producing the far side of the production 'hill'.

One further characteristic of the global production function is that, for a modern industrial economy, it does not start at the origin and is zero or near zero near the coordinate axes. This means that the factors of production are complementary in the large, and are only substitutable or competitive at the margin and in the small.

## 2. Productivity.

Productivity relates to the level of output obtained from given resources, and hence to the efficiency with which resources are used. It is thus a concept which is very close to the production function itself. In its most general form it can be defined as the average level of the production function within any useful boundary of factor values. Let this be a definition of total productivity.

Productivity improvements are of vital importance to any economy and can be achieved by two general and distinctly different types of action. The first is to alter factor quantities and proportions, to bring the economy to a more favourable area of a given production surface, relative to factor costs. The second is to discover new and improved ways of using factors in production so that

a new and higher production surface is reached, within the relevant factor region or boundaries. The second approach is accordingly to find and to sponsor innovations. Recall that Professor Schumpeter defined innovation as a change to a new production function, or as a change in the form of the production function ( [ 3.33 ] Vol. I p. 87 ).

Many lesser innovations involve only improvements in the organization of production, with no change in the quantity or the quality (form) of capital equipment. But most major innovations involve great changes in both the quantity and the form of capital.

A more particular and commonly used productivity concept is the ratio

(2)  $p = \text{GDP}_f / (N_p + N_{\text{enp}}) h$ , known as real output per man-hour or productivity per man-hour. This partial measure has important welfare implications, for it considers man in a dual role, as both a factor and a consumer of production. It represents the average return to society of an hour of work, and hence reveals the current opportunity cost of goods in terms of leisure, or of leisure in terms of goods.

The productivity concept is at the heart of economic development. Development can proceed by the two roads mentioned above - reaching a more favourable point on a given production function; lifting the global function by innovations. The first approach often involves expansion of H and K to a more favourable proportionality with population and the natural environment . (The natural environment is constant in the short run and hence is included in the parameters and form of (1) ). A combination of (1) and (2) shows that p will usually

be greater in a society with proportionately more of such capital.

In this connection, productivity is likely to be highest at full use of capacity, for factor proportions are then optimum from the point of view of indirect labour and engineering design. It follows that productivity may fall during depression and rise during prosperity. Full employment and economic development are related in this sense as well as in others.

Both development processes are going on continuously in progressive economies. Can we allow for them in our dynamic model? Equations (1) and (2) can interpret productivity within a short time period, and can allow for changes in factor proportions, and expansions of H and K. But how can innovations, and the gradual growth in knowledge and skills, even changing human attitudes, be incorporated into our equation? One way would be to date the function F of (1), making it  $F_t$ . Another method, closer to econometric needs, would be to add a factor  $q_t$  to (1) which would shift it up and down from time period to time period, as innovation takes place, and as the qualities of the factors of production change.  $q_t$  is not something that can be readily observed, but its accumulated effect is embedded in  $GDP_{f,-1}$ . By casting (1) into the dynamic form

$$(1a) \quad GDP_f = F \left\{ GDP_{f,-1}^0, (N_p + N_{enp})h, H, K_f \right\}, \quad \text{where } GDP_{f,-1}^0$$

is the highest previous level reached by  $GDP_f$ , changes in  $q_t$ , and hence  $q_t$  itself can be estimated by econometric means. Through this technique total productivity changes become revealed by past levels of total output.

### 3. An Equation of Supply.

Equation (1a) shows the formation of the supply of gross domestic product originating in firms, but it is not a supply equation in the usual sense. A supply equation is usually considered to be a relationship showing quantity which would be offered for sale at varying product prices. This function would then shift forward or backward with decreases or increases in factor costs. Such an equation can be formed from (1a) if we substitute in it ex ante demand functions for the factors of production, rather than ex post values of these. Thus,

$$(1b) \quad GDP_f^S = F \left\{ GDP_{f,-1}^O, (N_p^d + N_{emp})h^d, H^d, K_f^d \right\}.$$

Supply then becomes a system of some nine equations - (1b) from this chapter and (7), (7b), (8), (9), (10), (11), (15) and (21) from Chapter 10. These equations could be combined into a reduced form (3)  $GDP_f^S = F (w_{ph}, P_m, P_{GI}, P)$ , which would then resemble the conventional concept of a supply equation. But (3) is much less autonomous (Chapter 6) than the nine equations from which it is derived. Hence it can offer much less explanation about underlying behaviour and structure than they can, and it is accordingly not the sort of equation deemed adequate for the present model. For our purposes then supply will be accounted for in the model by (1b) and the eight equations which most directly lead up to it.

### 4. Production and Supply Included in Domestic Output but not Produced in Firms.

The Gross Domestic Product of a total economic system



includes the output of government and of various other institutions, to which social forces rather than market profitability provide the necessary impetus and control. Social welfare bodies, and most of the social purpose and value building institutions fall within these groups. The production of these institutions must be valued by the cost of the factors they use up, since no market price is available. Also it is usually found to be necessary to include in the national accounts, various imputed items like the rent earned by owner-occupiers of dwellings. These institutional and imputed productions of value must be added to the product of firms to arrive at an estimate of total GDP. It is best to treat these items as exogenous, with demand always equal to supply.

Let the factor costs in the institutional and imputed items be as follows. Let the wage bills of government be  $w_g N_g$  for its civilian employees, and  $w_m N_m$  for the military personnel or armed forces. Let the wage bill of all other non-commercial institutions be  $w_i N_i$ . Let the interest on debt associated with government fixed capital be  $\Pi_{go}$ , and the interest on the debt of the other non-commercial institutions be  $\Pi_{io}$ . Let all imputed nonwage income in the national accounts be  $\Pi_{ri}$ . Then the production of or value added by government can be represented by

$$(4) \quad G_1 = (w_g N_g) / P_h + (w_m N_m) / P_h + \Pi_{go}.$$

Similarly the value added by institutions can be represented by

$$(5) \quad I = (w_i N_i) / P_h + \Pi_{io}.$$

Total gross domestic production or value added can now be summed up as

$$(6) \quad \text{GDP} = \text{GDP}_f + G_1 + I + \Pi_{ri}.$$

With the aid of (1b) equation (6)

can be readily converted to a global supply equation for the domestic economy.

5. Some Important Identities Related to The Firm and The Productive System.

(a) Total Sales in domestic economy.

$$(7) \text{ GDS} = D + \text{GI}_d + \text{GI} + G + F_2.$$

G = total government spending on goods and services.

$F_2$  = total exports of goods and services, excluding  $\Pi_{fd}$ , interest and dividends received from abroad plus undistributed profit in direct investment owned abroad.

(b) Total Flow of New Goods through domestic economy

$$(7a) \text{ GDF} = \text{GDS} + \Delta H = \text{gross supply flow to domestic markets.}$$

(c) Total Production or Value Added in domestic economy.

$$(8) \text{ GDP} = \text{GDS} + \Delta H - F_1$$

$F_1$  = imports of goods and services, excluding  $\Pi_{di}$ , payments of interest and dividends to foreigners.

$$(9) \text{ GNP} = \text{GDP} + \Pi_{id} - \Pi_{di} = \text{Gross National Product.}$$

(d) Total Payments to Factors of Production in domestic economy.

$$(10) \text{ GDP} = (w_{ph}N_{ph} + w_gN_g + w_mN_m + w_iN_i)_r + \Pi_f - J + \Pi_{go} + \Pi_{io} \\ + \Pi_{ri} + T_{i-s} + D_{f1} + D_{f2}.$$

J = capital gains or losses on inventories due to price changes.

$T_{i-s}$  = indirect taxes less subsidies.

$D_{f1}$  = transfers by firms to reserves for depreciation,

$D_{f2}$  = bad debt losses of firms.

Equation (10) defines  $\Pi_f - J$  as the residual claimant to GDP after wage contractual and institutional obligations have been met.

(e) Gross and Net Saving of Firms.

$$(11) S_f = \Pi_f - J + D_{f1} + D_{f2} - \Pi_{ifo} - \Pi_{rfo} - \Pi_{dfo} - \Pi_{dub} = \text{gross saving of firms}$$

$\Pi_{ifo}$  = interest payments by firms, to other sectors o.

$\Pi_{rfo}$  = rent payments by firms,

$\Pi_{dfo}$  = dividend payments by firms.

$\Pi_{dub}$  = net drawings by owners of unincorporated business from current profits.

$$(12) NS_f = S_f - D_{f1} - D_{f2} = \text{net saving of firms.}$$

$$(13) NS'_f = S_f - D_{PCM} = \text{true saving of firms.}$$

(f) Change in Net Worth of Firms and Reallocation of Wealth.

$$(14) NW_f = NW_{f,-1} + NS'_f + \Delta_1 K_{ub} + \Delta_1 S_{eeor} .$$

$\Delta_1 K_{ub}$  = net investment or withdrawal of capital from unincorporated business.

$\Delta_1 S_{eeor}$  = net issue and sale of new equity securities by corporations.

$$(15) NS'_f = \Delta M_{fr} + \Delta S_{efr} + \Delta H + \Delta K_f - \Delta S_{eifor} - \Delta_1 K_{ub} - \Delta_1 S_{eeor} .$$

(g) The Finance of Investment.

$$(16) \Delta H + GI = S_f - \Delta M_{fr} - \Delta S_{efr} + \Delta S_{eifor} + \Delta_1 K_{ub} + \Delta_1 S_{eeor} .$$

Part IVTHE LABOUR MARKETChapter 12.THE SUPPLY OF LABOUR1. General Concepts.

The supply of labour is not easy to define, because it has many dimensions. To arrive at the number of 'units' of labour offered on labour markets throughout the economy during any time period we should combine at least the following factors: the number of persons  $N_1$  who offer their services; the hours of work  $h$  offered; the average degree of ability skill and training of the workers; attitudes and intensity of effort. In our model we shall only be able to include  $N_1$  and  $h$  explicitly, as components of labour supply. But the other two are included implicitly in the parameters and shifts of the production function, and will be reflected in productivity movements. The motivations and forces behind the supply of people and of hours are somewhat different, and hence we study them separately.

2. The Number of Workers in the Labour Force,  $N_1^S$ .

Population  $N$  is the basic pool out of which the labour supply or labour force is drawn. This supply consists of those who are at work, who have jobs but are temporarily not at work, who are without jobs and seeking for work, and finally those who would have been seeking for work, but believed there were no jobs available Cf. [4.5]).

We can add a further fringe group who do not consider themselves eligible for employment, normally. They would only be able to work part time, or do not feel that they have adequate training, or are in some way infirm; but all are capable of performing some useful work, and would take jobs for economic and psychological reasons if the demand for employment became so high that it became easy for them to become employed. Finally there is a group who could do useful work but are under no economic or psychological pressure to do so, and who would come into the labour force in times of national emergency.

What are the motives which normally impel people to enter the labour force? The most basic motive is of course the need to consume. But there are other needs in the total preference system which also influence the total man. Some of these are: the need for purposive and constructive activity; the need to exercise native ability; the need to participate in social goals related to the general welfare. In addition to these influences there are sociological forces which influence the ages of participation, the participation of women, the extent to which education influences participation, and so on.

The economic motives can be treated as variables in the supply function, but the sociological and other forces will be embedded in the parameters of the function, and will follow exogenous trends. Let "a" be a vector of the age-sex distribution of  $N$ , and "b" a corresponding vector of participation rates. Then

(1)  $\sum a = N$  ;  $ba' = N_1$  . It is the rates  $b$  which must now be related to economic influences.

The decision to participate represents a choice between economic needs and wants on the one hand, and the non-economic goals in the total preference system on the other. If the real wage ( average hourly earnings in our model)  $w_{ph}/P_h$  goes up there will be a tendency for participation to increase in the short run (cf. sec. 3 below). As average levels of household wealth  $W_h/N$  go up, there will be a longer term tendency for participation to decrease, for now economic needs are not so pressing. An increase in the standard of possessions desired by households  $A_h$ , relative to existing stocks  $K_h$ , is likely to expand participation because of the bulky cost of many of the durables.

Institutional and cultural patterns would tend to add persistence to past rates of participation, which suggests adding these to the supply function. Gradual changes in these patterns would normally cause the function to shift through time, but the influence of lagged rates builds these shifts into the function.

The labour supply or labour force equation is accordingly suggested to be

$$(2) \quad N_1^S = b^S a' ; b^S = f_1 \left\{ b_{-1}, w_{ph}/P_h, W_h/N, (A_h - K_h) \right\}.$$

### 3. The Supply of Hours of Work per Day and per Week.

The individual has 24 hours of 'time income' each day to spend. On the average, roughly eight hours must be devoted to sleep, and two hours to meals and personal care. This leaves a balance of 14 hours to be spent on the economic and non-economic goals in the indiv-



individual's total preference system. The choice is in a sense based on relative 'prices' and income and substitution effects ( Cf. Samuelson [1.20] p. 592 ). Let the apportionment of time income be  $h_d$  (average hours of work per day) to economic, and  $h_l$  (average hours of leisure per day) to non-economic goals, where  $h_d + h_l = 14$ . The opportunities open to a society for converting their time into economic goods depend on its aggregate productivity. These opportunities and a corresponding preference system are sketched in Fig. 1.

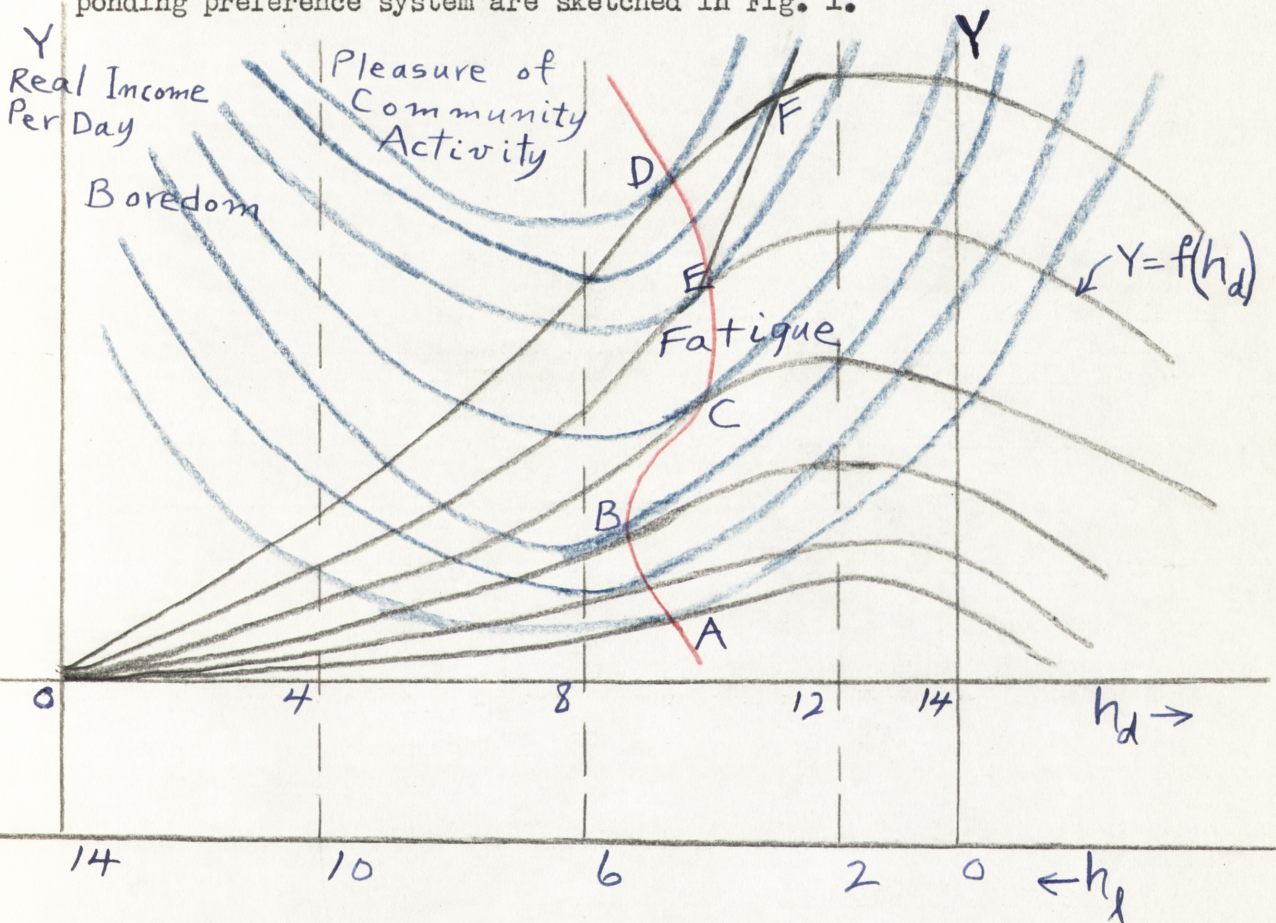


Fig. 1

Each production function in Fig. 1 is an 'opportunity line' showing the social possibilities as between economic goods and leisure. Different

production functions represent different states of the productive arts. At tangential points between the production functions and their total preference systems the individuals reach their highest attainable levels of satisfaction. At these points the marginal rates of substitution between goods and leisure in the two systems become equalized.

Average hourly earnings in real terms,  $w_{ph}/P_h = w_{phr}$ , are represented in Fig. 1 by  $f(h_d)/h_d$ . The 'time price' of goods is  $1/w_{phr}$  hours, the time required to earn one constant dollar or pound's worth of goods. As  $w_{phr}$  increases the time price of goods drops. This is what happens when the production function (productivity) rotates to a higher level. How does this affect the equilibrium choice between income and leisure?

Given a production and preference system as assumed in Fig. 1 we find by comparing static equilibria at B and C that, at relatively low incomes, an increase in productivity results in a choice of more goods and less leisure. This result can be construed to be a net change, composed of a substitution effect in favour of goods and away from leisure, plus an income effect from which both more goods and leisure are chosen. As we move to still higher levels of real income from C to E to D. the increase in productivity produces an income effect which swamps the substitution effect, and the net choice now is for both more leisure and more goods.

The equilibrium line ABCD represents a social average. Any individual can be tempted off this norm to a higher preference level at increased hours of work by the offer of higher marginal pay



rates, as indicated by the movement E F.

Factors other than real income which can affect the supply of hours are the standard of goods aspired to in relation to attainment, and the average level of wealth. These affect the preference system. Should the increasing availability of durable household goods give society a materialistic bent, the preference system might rock to the right. This would shift BCED to the right, increasing the supply of hours. A growth in wealth on the other hand might cause the system to rock to the left, and the supply of hours to decrease.

Let  $h^S$  be the average supply of hours of labour per worker for the time period of the model, and  $h_s$  the "standard hours" of work at full capacity and full employment.

$$(3) \quad h^S = f_2 \left\{ h_s, -1, \frac{w_{ph}}{P_h}, (A_h - K_h), \frac{W_h}{N} \right\}.$$

Chapter 13.SHORT-RUN MARKET ADJUSTMENT AND LABOUR SUPPLY1. Introductory.

The main market theory in our study comes in Part VIII further on. But here some part of this analysis must be anticipated. A market consists of demand forces and supply forces, which interact to produce an outcome of price and quantity rate of flow. The markets of the economy separate broadly into goods markets and labour markets. But supply and demand on the goods markets depend on the outcome of the labour market, for labour is both the major factor of production and a major demander of goods. For this reason it seems appropriate to complete the analysis of the labour market here.

The demand forces in the labour market are summarized by equations (7) and (7b) of Chap. 10, while supply forces are found in (2) and (3) of Chap. 12. The supply equations assume no "money illusion" on the part of workers, for all causal variables in them are in real terms. They are accordingly "classical". But the outcome of the day-to-day bargaining of the market is a money wage rate, and a level of employment. Hence there may be some money illusion in the market in the short-run, as Keynes suggested ([1.6] pp 10-11). Our purpose now is to attempt to analyse what does happen in the labour market in the short-run, and to relate this to longer-run behaviour.

2. Institutional Aspects of the Labour Market.

A special feature of the labour market is that the

product or service for sale is tied like a siamese twin to the supplier. For this reason labour supply is more closely related to the needs of man and to all his emotions, and in fact to his total personality and preference system, than is any other product or factor of production. In the beginnings of modern industrial capitalism ( say 1750 - 1850 ) labour was forced into selling itself competitively, while employers often combined in their purchase of labour. Human values were degraded in the interests of wealth production ( Cf. Heaton [4.7], Lester [4.11], Marx [4.14] ). Labour slowly and painfully developed methods of defence, with the major approach the formation of trade unions and other protective organizations. At a later stage political activities within the democratic framework were added to the program, to enlist part of the power of the state in this defensive process.

In the modern world labour organizations and political successes have gradually added to labour's power, until at some times it almost seems as though society now needs defence from some of the labour organizations. For there is an increasing tendency for these, as they grow in size and strength, to lose their democratic nature, to ignore the welfare of individual workers, and to ignore the welfare of the total society in the interests of their particular group.

The evils against which the early trade unions fought were : poverty, insecurity, unsafe and unhealthy working conditions, long hours, technological unemployment, cyclical unemployment. We cannot go into all the details of this courageous struggle here. Suff-

ice it to say that in some countries of the western world the battle has been largely won - partly by trade union activities, partly by the state under the political influence of labour unions and their sympathizers. But a harmonious division of labour between these two areas of action has not yet been fully worked out. Some union defensive tactics still hark back to past needs, and are out of step with a modern central government full employment, high productivity, high income economy.

It is in the struggle against poverty and for high wage rates that the greatest anachronism occurs. This struggle has always been waged on two related fronts. One is the restriction of the supply of labour, at least in the protected areas. The other is the direct pressure for higher wages in good times, under the threat of the strike, coupled with holding the wage line as much as possible during depressed times.

The methods of reducing supply in the trade unions, and in the professions, have included the restriction of entry by high dues and qualifications, long training and apprentice periods, jurisdictional rules, and the reduction of productivity. The latter was achieved by various "featherbedding" devices and "go slow" rules which restricted intensity of effort.

The indirect (supply) and the direct approach to high wages were found to reach substantially the same goal (Cf. Hicks [4.8], Samuelson [1.17] pp 598-600, Bronfenbrenner [4.2]). But while direct restriction of supply in skilled trades and professions caused an un-

employment of skills, with lower wage rates in unprotected areas, indirect control of the number employed by the wage rate method alone, if applied to the whole labour market, could cause general unemployment. This could keep all of the marginal workers, and many others as well, from appearing on the labour market and actively seeking for work.

It is natural for workers to resist a fall in money wage rates since they associate this with declining levels of living, a difficult process when habits have been formed. The unions have strong support from the workers here, even when prices are falling and there is some short-run money illusion. This downward rigidity to money wages is abetted by employers as well, for they hesitate to cut wages for fear of incurring the ill will of their employees and the general public. Since however no such opprobrium attaches to lay-offs during slack times they find this course the easier one to follow.

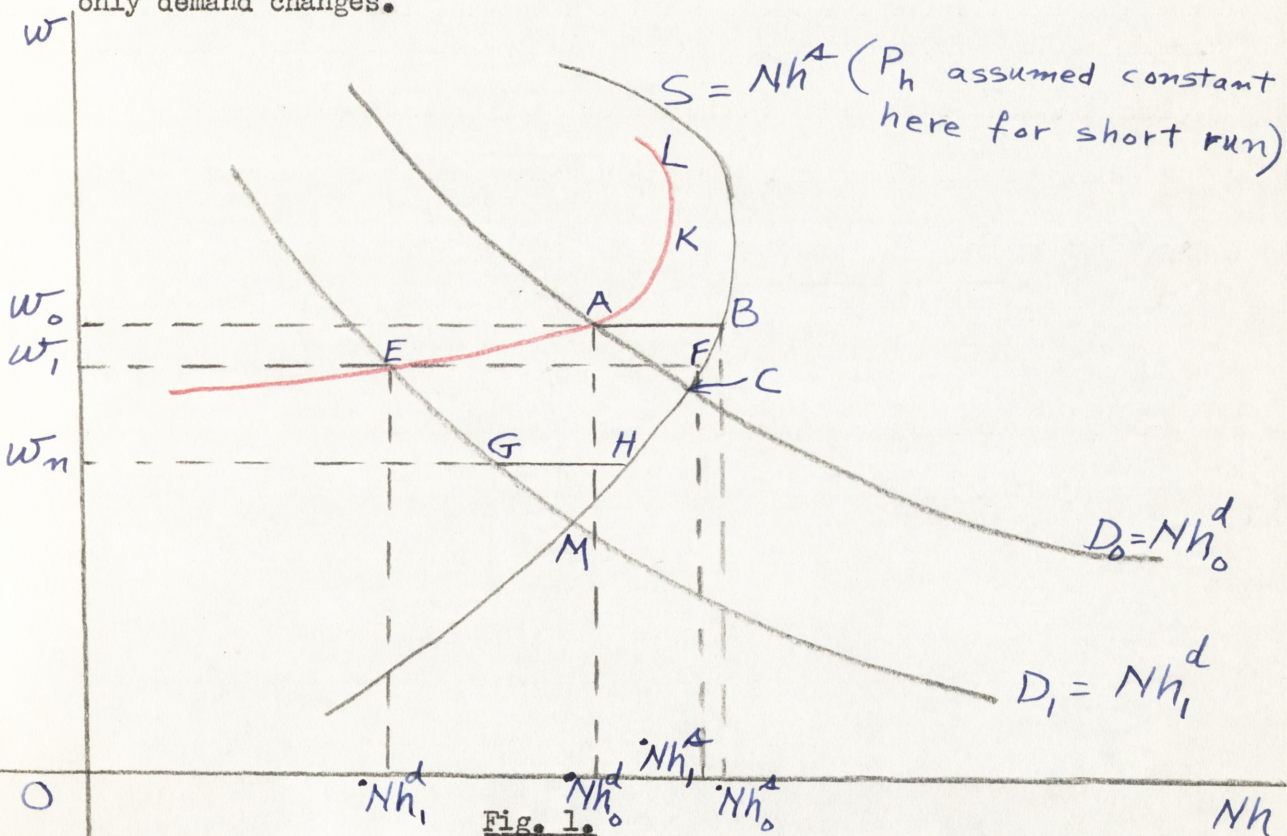
Certain key personnel must be retained, but eventually only a fraction of their services is needed. The solution to this problem is to reduce their hours of work rather than their wage rates, and it then may be necessary to shorten the hours of other employees to a suitable complementarity.

From this very brief survey of the institutional aspects of the labour market some conclusions can be drawn. Money wage rates will tend to be rigid downward. There will be no such corresponding upward rigidity, but labour supply will tend to be almost infinitely elastic at the going wage level, until nearly full employment is reached. Full employment occurs when the total labour supply

at the going wage rate is employed, except for the frictional unemployment required for labour mobility to more suitable jobs. Should demand press beyond full employment it is assumed that wage rates will rise rapidly.

3. A Model of Short-Run Behaviour in the Labour Market.

A model or schematic simplification of the above institutional picture is initially represented by Fig. 1. Assume that full employment was reached at average hourly earning  $w_0$  and man-hours of  $Nh_0^d$ . AB is frictional unemployment. Let us now study the short-run positions the market might move to from A should economic conditions change. We assume S to be stable in the short run, so that only demand changes.





Suppose first that  $D$  drops from  $D_0$  to  $D_1$ . Because of the downward rigidity of money wage rates the market will not move along  $S$  from  $ABC$  to a new full employment position  $GHM$ . Instead it will move along a nearly horizontal line like  $AE$ . Unemployment will grow rapidly from  $AB$  to  $EF$ . Only in the long-run might wage levels subside from  $w_1$  to  $w_n$ , with unemployment gradually eliminated. ( Here we ignore the impact of this fall in  $w$  on  $D$ , but we take this fully into account in the complete model assembly of Part IX.)

Suppose next that demand shifts upward from  $D_0$ . Physical supply can increase slightly above  $Nh_0^S$ . Then since there is little upward rigidity to wage rates, the market position will move along a nearly vertical curve like  $AKL$ , roughly parallel to the  $S$  curve. Both curve allow for some expansion of hours at high levels of employment.

It appears that  $EAKL$  rather than  $S$  represents labour market positions in the short-run when demand shifts.  $EAKL$  is accordingly like a supply curve, though only applicable to the short -run.

Our labour market analysis has revealed two supply curves, one rather more fundamental and applicable to the long run, the latter based on institutional and historical developments and applicable to the short run. The former is of course the classical labour supply curve, the latter the Keynesian (Cf. [1.6] pp. 10-11, [257; Klein [1.10] p. 74 ). As to which is correct, our theory finds both of them correct, and both necessary to a model which attempts to encompass both short and long run phenomena.

#### 4. An Equation to Explain Short-Run Labour Supply.

To complete our equation model of the labour market we must develop an equation to represent EAKL. The downward viscosity of  $w$  can be represented by relating  $w$  to  $w_{-1}$ . The gradual merging of EAKL into  $S$  in the long run can be treated as a slow process of market adjustment. Here we follow Samuelson [8.14], and Klein [1.11], [9.5] and [9.6].  $w$  will move downward (or upward) under conditions of excess supply (or excess demand). This relates  $w$  to  $(Nh^s - Nh^d)$ . The sharp upturn in EAKL as full employment is approached can be reproduced by adding a pressure effect to  $w$  as unemployment is pressed below a certain critical rate.<sup>1</sup>

Since the wage rate is the principal effect variable in this short-run analysis, the equation will be set up with  $w$  as the variable being explained. But first we must define unemployment.

(1)  $N_u = N_1^s - N_m - N_g - N_i - N_{enp} - N_p^d = N_{lf}^s - N_p^d$ , where  $N_{lf}$  is the effective labour supply available to firms, and  $N_p$  = the number of paid workers or employees in firms. Since our model is in terms of man-hours, it will be preferable for us to work with man-hours of unemployment,

$$(2) Nh_u = (N_1^s - N_m - N_g - N_i - N_{enp}) h^s - N_p^d h^d.$$

The relative or rate of unemployment will be more significant than the absolute level. Our short-run supply or market adjustment equation then

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1. Mr. S.J. May, a former colleague, was the first to use such a term in the labour market adjustment equation. [9.8].



becomes

$$(3) \quad w_{ph} = f \left\{ w_{ph,-1}, \frac{N_{h_u}}{N_{l_f}^s h^s}, \left( k_1 - \frac{N_{h_u}}{N_{l_f}^s h^s} \right) \right\}.$$

$k_1$  is a critical value of the rate of unemployment. It is near full employment and represents the rate at which pressure begins on  $w_{ph}$ . The term involving  $k_1$  is allowed only zero or positive values, being taken as zero when it is negative. This makes  $w_{ph}$  slow moving when the rate of unemployment is above  $k_1$ , but fast moving once this rate is below  $k_1$ .

The unemployment concept used in our analysis requires some brief comment before we conclude. Definitions (1) and (2) above permit unemployment to be negative under a condition of excess demand. Such a condition can occur when the demand curve advances faster than  $w$ . Yet however strong demand seems to be, there is always some excess supply in the labour market. How can this anomaly be explained?

The problem is essentially one of aggregation. The labour market consists of many sub-markets, one for each trade, industry or area, and mobility between these sectors is very limited. There may be excess demand in many of these sectors, but there will almost always be excess supply in some of them. The excess supplies will be observed in labour force surveys, while the excess demands go unrecorded. Because of this the true net position of excess supply and demand is never known, once employment has picked up sufficiently. It can only be inferred by the volume of advertisements for labour, and by movements in wage rates. It could however be estimated with a good econometric model.

##### 5. The Special Case of the Australian Labour Market.

The Australian labour market differs from most others in the western world in that, in addition to being strongly unionized, it is regulated and further protected by the Commonwealth and State Arbitration Courts. One of the functions of these Courts is to set minimum basic wages throughout the major industries plus margins for the various abilities and hardships. The basic wage only was maintained at a fixed real value by quarterly adjustment for changes in the cost of living index, while its real level was occasionally adjusted upward as conditions (unspecified) seemed to warrant. Wage rates could rise to any levels above the minima depending on demand and supply conditions, for no ceilings were set. (Cf. H.P. Brown [1.2]). The quarterly adjustments for cost of living (C Series) changes were suspended by the Commonwealth Arbitration Court in 1953, but some States have since reinstated them.

In view of the differences in the Australian situation, how well is the above theory likely to fit this economy. After an admittedly brief observation of the Australian economy, this writer felt that the type of theory here developed would fit it better than most. This is because the basic motivations seemed to be the same as here described, while the total labour market seemed to be more homogeneous throughout, and to complete its adjustments in shorter time, as a result of the Court system of setting basic wages and margins which became effective right across the country.

PART VTHE FOREIGN TRADE AND FINANCE SECTORChapter 14THE BALANCE OF INTERNATIONAL PAYMENTS AND NATIONAL HOLDINGS OF GOLD  
AND FOREIGN EXCHANGE1. General.

It is perhaps in its international trade activities that the economics of a modern nation comes closest in meaning to the early greek term oikosnomos ( oikos = house ; nomos = management). An individual household produces some of the goods and services it needs - e.g. housekeeping, meals, laundry, repairs, maintenance, gardening - within its own domain. But for the rest of its needs it relies on 'foreign trade'. One or more of its members specialize in the production of particular goods or services for which they have special aptitudes. The products are sold in domestic markets in exchange for money. This money goes into a reserve ( currency, and bank accounts, etc. ) out of which the household spends on its needs. The scale of comfort and material welfare of the household depends very much on the level of its 'exports' ( degree of employment versus unemployment), and its terms of trade ( how the price of one hour of its labour e.g. compares with the prices of the goods it wants to 'import'). The terms of trade in this case depend partly on the level of global productivity of the nation in which the household

finds itself. This productivity is in turn considerably enhanced by the degree of specialization and division of labour, made possible by the 'foreign trade' within the society.

As well as depending on employment and 'terms of trade', the smoothness of household 'imports' and living depends also on the level of its liquid reserves ( $L_h$ ). Without such reserves (stocks of money and liquid securities) any fluctuations in household earnings and any accidental mishaps must be met by corresponding fluctuations in the level of living. A careful household never lets its 'bank account' run too low, except under extreme emergency.

The parallel between the household and the national economy goes a long way. There are few if any self-sufficient countries in the modern world. Almost every country engages in trade with other countries, and the material welfare of many nations depends very much on the level of their trade ( world employment and demand conditions), the terms of their trade ( depending partly on world productivity levels), and the level of their individual international liquid reserves ( their national ' bank accounts' ).

While almost all countries have some reliance on external trade, there are some whose current resources make them either particularly suited to, or only capable of the production of a limited range of goods. Such countries will have a very special reliance on foreign trade for earning their standard of living. Canada and Australia are particularly representative of such countries, and nearly one quarter of the value of goods used domestically in normal times in

these countries is imported, and must be earned ultimately by exports.

Our purpose in the present chapter is to get an overall view of the general flows of goods, services, money and securities into and out of the foreign trade and finance sector of a particular country, A, and of the important financial stocks which are related to this sector.

## 2. The Money of International Transactions.

Money, which is dealt with more fully in Part VII, is essentially a good which will be readily accepted in exchange for all other economic goods. But the internal or domestic money of A is usually not acceptable for purchases from any other country. Consequently some supra-national money is needed to facilitate general international trade. Gold gradually came to fill the need for an ultimate international money, but the awkwardness of gold as a substance to handle caused it to be displaced for day-to-day transaction by the national currency of an important trade centre. In such a centre the internal moneys of all trading nations could be exchanged with each other and with gold, through the medium of the domestic currency of the trade center. Rates of exchange were the prices of other moneys in terms of a domestic money or of gold, and were determined by the usual conditions of market supply and demand.

Because of its pre-eminence in trade from the seventeenth century on, England became the great foreign exchange centre of the world. The pound sterling, on deposit in London, was found to

be a safe and convenient way to hold international money. A nation would tend to sell its exports either for pounds sterling or for money which could be readily exchanged for pounds sterling in London. Likewise, it would purchase its required imports with pounds sterling or would first exchange pounds sterling for the domestic currency of the country from which it planned to buy. During the twentieth century, the U.S. dollar on deposit in New York has also become an important currency for international transactions. The world has gradually become divided into a sterling area and a dollar area. At the present time the international money of world trade and finance would be said to consist of gold, pounds sterling (£S) and U.S. dollars (US \$) . The total holdings of such international money by the government and the residents of any country are defined as its international money IM. That portion held by Government may be defined as  $IM_g$  , while the portion held by individuals, firms and institutions may be defined as  $IM_p$ .

Whenever money holdings in foreign exchange centres are found to be temporarily not needed they are usually converted into liquid short-term earning securities. Since these are so readily turned into money, they combine with IM to give the liquid reserves known as the International Reserves, IR, of the domestic economy.

### 3. The Budget Constraint in International Trade

#### (a) The Balance of Payments.

In the same way that a household and a firm have a

budget constraint which limits their current spending to their net income and their liquid assets, augmented by any further borrowing potential, a whole nation is limited in its international spending by the size of its international reserves and its current income or receipts of international money, plus any further credit potential it may have. Receipts here are interpreted in the broadest sense to include revenue from sales of goods and services, from the sales of securities (capital imports), and from gifts and donations.

A complete picture of a country's international budget constraint is given by the statement of its balance of international payments. An abbreviated summary of such a statement is presented in Table 1 below:

Table 1.

SUMMARY BALANCE OF PAYMENTS STATEMENT.

(For time period  $t$ , and in domestic money of Country A).

Accounts	Flows		Net Flow or Balance	
	Credit +	Debit -	Credit +	Debit -
<u>Current Account</u>				
Trade: Merchandise	$F_{2gm}$	$F_{1gm}$	$F(2-1)gm$	
Services	$F_{2sm}$	$F_{1sm}$	$F(2-1)sm$	
Total	$F_{2gsm}$	$F_{1gsm}$	$F(2-1)gsm$	
Gold	$F_{2go}$	$F_{1go}$	$F(2-1)go$	
Unilateral	$F_{2u}$	$F_{1u}$	$F_u$	
Total Current Account			$F_c$	
<u>Short-term Capital Account</u>				
	$\Delta S_{esdi} + \Delta M_{di}$	$\Delta S_{esid}$	$\Delta S_{es(di-id)} + \Delta M_{di} = F_{ks}$	
<u>Long-term Capital Account</u>				
Portfolio Securities	$\Delta S_{eldi}$	$\Delta S_{elid}$	$\Delta S_{el(di-id)}$	

Direct Investment	$\Delta K_{di}$	$\Delta K_{id}$	$\Delta K_{(di-id)}$
Direct borrowing and Lending	$\Delta B_{di}$	$\Delta B_{id}$	$\Delta B_{(di-id)}$
Total Long-Term Capital (net)			$F_{kl}$
<u>International Money Account</u>			<u>Credit - Debit +</u>
Balance of International Payments			<u><math>e_1 (\Delta IM) = BP</math></u>

Note:  $e_1 (\Delta IM) \text{ (Dr.)} = F_c \text{ (Cr.)} + F_{ks} \text{ (Cr.)} + F_{kl} \text{ (Cr.)}$ .  $e_1$  is the rate of exchange factor or vector which converts international moneys into domestic money.

This statement shows the multiplicity of inflows and outflows of IM through the foreign trade and finance sector of the economy. The net result during a time period is seen to be a change in the level of IM from the end of the previous time period to the end of the current time period.  $\Delta IM$  is seen to be equal to the "balance of payments" (BP) and it is clear that IM itself is the ultimate short-run constraint on a nation's foreign spending.

(b) Fixed versus Variable Exchange Rates.

While the level of its international reserves is a major short-run constraint on A's foreign spending, there is sometimes a further constraint in the form of the exchange rate  $e_1$ . If it is left free to vary according to the supply of IM offered to A and the demand for IM by A, it can by itself afford the necessary encouragement or restraint for A's purchases and sales of IM to bring Table 1 into balance, with no change in IM.



In gold standard days exchange rates were held nearly constant in terms of gold, and the full impact of pressure on the value of A's money was taken up by a loss of IM, mainly gold. This tended to produce deflation and unemployment in A, through its effect on A's internal money supply (Part VII). The advantage of the gold standard was that with fixed exchange rates importers and exporters knew exactly how much international money would cost them or yield them in future transactions. This took some of the risk and uncertainty out of foreign trade. But this advantage was gained at the harsh cost of internal instability.

When the gold standard was abandoned in the 1920's and 1930's, the international economy entered a phase in which it had both variable exchange rates and internal instability, the worst of both worlds. In the post World War II world, a valiant attempt is being made by the International Monetary Fund to bring back fixed exchange rates as a means of eliminating uncertainty and thereby expanding international trade. At the same time the IMF does what it can to maintain internal stability as well. Sometimes however it seems impossible to have the best of both worlds, as was the case in Canada in 1950 when speculators assumed that the Canadian dollar at its fixed exchange rate was undervalued. They began to pour short-term capital into Canada, thereby creating disastrous inflationary pressures. To forestall this situation, Canada forthwith set her exchange rate free to fluctuate under the market influences of supply and demand, and has kept it free ever since.

Canada, accordingly, represents the most general case of the international budgetary constraint. It is constrained in its international spending both by the level of its international reserves ( $IR = IM + S_{esid}$ ), of the Canadian dollar with international money. Australia on the other hand maintains a constant exchange rate  $e_1$  and hence its total restraint and adjustment must be achieved by variation in its IR.

The reserves of course must be sufficient to handle current transaction needs in its international trade and finance plus sufficient precautionary holdings to be able to maintain a country's vital imports in the event of emergency situations. Beyond these needs, a country's holdings of IR will depend upon its affluence, and the yield that can be made in foreign securities and real investments or direct investments, in short on its liquidity preference for foreign exchange.

#### 4. Domestic Versus Foreign Ownership - A's International Balance Sheet.

This financial statement will show the stock values related to foreign finance, the assets and liabilities of A on international account. Its purpose is to show the composition of A's international position, and whether A is on balance a creditor or a debtor nation.

#### Table 2.

#### E. International Balance Sheet of A (As at date $T_1$ )

Assets (Dr.) +Liabilities (Cr.) -1. Liquid or Short-Term Position $M_{go}$   
 $(M_{id})_{el}$  $M_{di}$  $S_{esid}$  $S_{esdi}$ 

IR

IR<sub>di</sub>IR - IR<sub>di</sub> = IR<sub>n</sub> (net reserves)Net Short-term Position2. Long-Term PositionPortfolio $S_{elid}$  $S_{eldi}$  $S_{el(id-di)}$ Direct Borrowing and Lending $B_{id}$  $B_{di}$  $B(id-di)$ Direct Investment $K_{id}$  $K_{di}$  $K(id-di)$  $(S_{el} + B + K)_{id-di} = U_{lid}$ Net Long-Term Position3. International Surplus or Deficit of A with RW

Surplus

 $(IR - IR_{di}) + (S_{el} + B + K)_{id-di} = U_{id}$ 

For a 'new' country in process of economic development we expect  $U_{id}$  to be a large credit (-), and A is then a debtor nation. But as A matures and becomes more fully developed we would expect  $U_{id}$

to swing to a debit balance, and A to become a creditor nation.

5. International Sector Flow and Stock Identities for Economic Model.

(a) Balance of Payments Identity.

$$(1) F_{2gm} + F_{2sm} + F_{2go} + F_{2u} - F_{1gm} - F_{1sm} - F_{1u} + \Delta M_{di} + \Delta S_{esdi} - \\ \Delta S_{esid} + \Delta S_{eldi} - \Delta S_{elid} + \Delta B_{di} - \Delta B_{id} + \Delta K_{di} - \Delta K_{id} = \\ (\Delta IM) e_1$$

(b) International Reserves Identities.

$$(2) \Delta IR = e_1 \Delta IM + \Delta S_{esid}; \Delta IR_n = \Delta IR - \Delta M_{di} - \Delta S_{esdi} \cdot$$

(Note :  $e_1 \Delta IM$  is current value of 'physical' change.)

(c) International Balance Sheet and Surplus or Deficit Identities.

$$(3) M_{go} + (M_{id} - M_{di}) + (S_{esid} - S_{esdi}) + (S_{elid} - S_{eldi}) + (B_{id} - \\ B_{di}) + (K_{id} - K_{di}) = U_{id}$$

The above three equations summarize all of the important flows and stocks involved in the international trade and finance of A. They become an important part of our macroeconomic model. Many of the variables in these equations must be treated as though exogenous, but  $F_{1g}$ ,  $F_{1sg}$ ,  $F_{2g}$ ,  $F_{2sg}$ ,  $\Delta S_{eldi}$ ,  $\Delta K_{di}$ , and  $e_1$  will be treated as endogenous. The behaviour equations for these variables are developed in the next chapter.

Chapter 15.BEHAVIOUR EQUATIONS IN INTERNATIONAL TRADE AND FINANCE.1. Causal Forces Behind Trade.

From the national point of view a country A exports because it needs to import. It has need to import because there are certain goods which it wants but which it either cannot produce at all, or which it can only produce at excessive costs. On the other hand there are usually at least some goods which it can produce at a cost lower than most other countries. Through exchanging any surplus it can develop of its low cost goods, with the low cost surplus goods of some other country B, both countries make enormous gains. Through trade otherwise high cost scarce goods become low cost and abundant. From the point of view of the individual firm of course the relevant force is simply that it can sell a larger output at a better price, or can buy vitally needed materials at a lower price, through canvassing the opportunities in foreign markets. In fact through such price and profitability guides to the actions of individual firms, trade can flourish between nations where one nation has no absolute but only some relative production advantages. A may be able to produce all goods cheaper than B. But both A and B will have certain relative or comparative advantages, and if these are great enough and in different lines A's firms may find it more profitable to produce only the goods in which it has relative advantage, leaving the production of B's relative advantage goods to it.

Why are there differential costs of production for the same goods but in different countries in the world? The answer is largely to be found in the vast differences in resource endowments between countries, at any given time. Resources or factors of production consist of such things as: land and its configurations and arable possibilities, water and waterways, rainfall and climate, minerals; population with its skills, knowledge, health, strength and attitudes; capital equipment and technology. The resource pattern falls naturally into three vectors - land or natural environment (NE), labour (Nh) and capital goods (K). Each product to be produced requires a certain 'recipe' or combination of elements from these vectors (factors of production). There will be some possibilities of substitution within the recipe, thereby converting the recipe to a production function. Let a particular product to be studied be x. Then the production function for x in country A is

(1)  $q_{xA} = f_{xA} (N_{h_x}, m_x, K_x)$ , where q is quantity produced and m represents the raw materials used in producing x. (For the rest of the argument we can drop the subscript x).

It is now easy to show formally what the average cost of production of x will be. To begin with it will depend partly on the rates of remuneration paid to the factors of production. Let  $w_h$  be the average wage rate per hour,  $P_m$  the price of raw materials and r the marginal productivity or 'rent' of fixed capital (normal rate of profit), all with respect to the production of x. Let p be the price charged for x, and  $\Pi$  the total profit earned on x.

Then average cost AC of producing  $x$  in A is given by

$$(2) \quad AC_A = \frac{(N_h \cdot w_h + m \cdot P_m + K \cdot r)_A}{f_A(N_h, m, K)}, \text{ and market price by}$$

$$(3) \quad P_A = \frac{(\pi + N_h \cdot w_h + m \cdot P_m + K \cdot r)_A}{f_A(N_h, m, K)} \cdot$$

Clearly average cost in A depends on the level of factor costs and the level of total productivity ( the production function). Price in A depends on the same elements as average costs plus in addition whatever profit is earned in A on the production of  $x$ . This profit is here counted as surplus earning over costs which include a normal rate of profit. It may arise from monopolistic scarcity due to natural causes or to the contrivances of man; or it may represent the temporary profits of innovation and superior efficiency.

A necessary though not a sufficient condition for trade to flow is that  $p_A < p_B$ . Using (3) as a guide let us now consider how different countries may temporarily vary in resources, productivity, factor costs, and so on, thereby creating or destroying this necessary condition for trade.

(a) Relative Factor Endowments. Few countries are evenly balanced in their resource endowments. One country may have an abundance of labour but a relative scarcity of natural resources and capital ( e.g. India). Another country may have considerable natural resources, but a relative scarcity of labour and of capital (e.g. Australia , Canada). A third country may be well endowed with all three types of resource, but still be left with some scarcities. For example the United States does have a slight relative scarcity of labour, and an absolute scarcity

of some raw materials.

The production function for  $x$  in  $A$  will have an area where the factors are in an optimum proportionality for productivity. If factors occur in  $A$  in roughly this same proportionality, then this high productivity area can be reached without bidding up some factor prices excessively.  $p_A$  should then be relatively low. Thus India could not at present hope to produce cheap surpluses of goods which require a large relative use of land and capital, but could produce such surpluses of goods requiring a high labour content. Canada and Australia could hope to produce cheap surpluses of goods requiring large relative use of natural resources. The U S and the UK on the other hand can export surpluses of manufactured goods requiring extensive use of capital equipment and skilled labour. Each of these country groupings then has a comparative advantage over the other for producing goods whose relative factor requirements parallel the current relative resource endowments of the particular country.

But this is a static picture at an instant of time, or at most a short-run view. It must be supplemented by further considerations, to enable a dynamic or a longer-run theory of trade to be developed.<sup>1</sup> Thus the current factor endowment inherited from the immediate past is by no means immutable. It can be drastically altered by the building or the importation of capital equipment, and by the importation of raw materials from other countries ( Cf. an important

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1. I am indebted to Dr. Wm. C. Hood for drawing these further aspects of the problem to my attention.



article by Romney Robinson [5.14] on this point). It can also be considerably altered by the importation of new training and skills, or even attitudes. Such changes amount to economic development and will tend to be innovational, at least to the developing country, changing and lifting its production functions.

Thus while today's comparative advantages and current factor endowments may help to explain today's pattern of trade, they may not explain tomorrow's. For economic development may have changed them by then. And international trade itself will be likely to have encouraged this change in itself, through enabling the export and import of fixed and raw material capital. It follows that our short-run behaviour equations will require dynamic and long-term evolutionary trends if they are to serve for longer-run analysis. We subsume such changes in the functional forms and parameters.

(b) Relative Factor Prices. In each country relative factor prices  $w_h$ ,  $p_m$  and  $r$  (approximating marginal productivities), will tend to be closely through inversely related to the relative factor endowment. The scarce factor will be relatively high priced (labour in Australia, Canada, the United States, capital and land in India); the abundant factor will be relatively cheap (labour in India, land in Canada and Australia, capital in the United States and the United Kingdom). This price pattern reinforces the comparative advantage already provided on productivity grounds for the current specialization of the production of certain goods in certain countries. For the specialization on goods whose relative resource requirements parallel the relative

resource endowment will not only promote low cost on grounds of productivity; it will also promote low cost on grounds of factor prices and factor costs (Formulae (2) and (3) above.)

(c) Absolute Factor Prices. If A is a rich country with high levels of real income and wealth, while B is a poor country, the absolute level of all factor prices in A will be high. Then despite any comparative advantage A may have in terms of current factor endowment and relative factor prices, if x is produced in both countries,  $p_A$  may still be greater than  $p_B$ . But if B became sufficiently wealthier in the future, then A's comparative advantage might become decisive and trade in x could begin. Thus income and wealth differentials throughout the world mitigate against the world taking full advantage of each country's comparative advantages. And high factor costs, despite high productivity, can price one country out of another's markets.

A second situation which can make A's factor prices too high to permit trade with B is inflation. If A's prices and factor costs are inflated from monetary or other causes A can price itself out of B markets for x, and despite its comparative advantage in production, trade may not flow.

Finally if x is produced under monopolistic conditions in A, the profit component of  $p_A$  may price A out of B markets despite comparative advantage.

(d) Rates of Exchange on A and B Moneys. At some rates of exchange on A money B can take advantage of A's comparative advantage in x, despite A's richness or inflation or monopolistic production. But this might hinge on A wanting to buy sufficient from B (or RW) and

on flexible exchange rates. If A (or B) holds its money at an artificially high (or low) rate of exchange by exchange control, trade in x can be inhibited.

(e) Transportation Costs: Freight and Insurance. A's comparative advantage over B in the production of x can be wiped out by excessive distance or by excessive freight and insurance rates.

(f) Customs Duties and Taxes.  $p_{AB}$  includes the cost of x in A plus all the costs of getting x from A into B. For various reasons the people of B or vested interests in B may want to restrict the entry of x. A common way of doing this is to charge customs duties and other import taxes to the importer in B, thereby raising  $p_{AB}$ .

## 2. Gains from World Trade.

A matrix of elements like  $(p_B - p_{AB})$ , where only positive elements are considered, could be defined as the force of trade. Under free world trade, and with free markets in foreign exchange, production will be located by the "force of trade" on the basis of current comparative advantage, countered only by transportation cost. Resulting world specialization and division of labour paralleling comparative advantages will result in minimum world costs and maximum world real income. Large scale production would be encouraged in specialized areas, with the further advantage of increasing returns and decreasing costs. Aggregate world productivity (truly 'global' productivity) should become a maximum.

In this process a certain amount of redistrib-

ution of world income would take place. Under a system where no world trade was allowed the scarce factors in each country would receive excessively high relative rates of remuneration. But the abundant factors would receive very low rates of pay and would often be unemployed. However under a system of free world trade the "force of trade" would increase the demands on the abundant factors of all countries and decrease the demands on the scarce factors. This would tend to equalize the rewards to different factors within the same country, and the rewards to the same factors in different countries. In fact under a system of free world trade the factor prices of different countries would differ only on the basis of disparate national productivities. A country with higher national productivity could pay its factors more than other countries, and still compete in world trade.

The further dynamic gains from world trade in fixed and raw material capital, as innovation and economic development travel from one country to another, may even outweigh the more traditional gains from world trade [ 5.14 ].

### 3. Resistances to World Trade.

In view of the great world benefits to be gained from completely free world trade, one can at first sight only marvel at all of the restraints on trade that have plagued the world -- the tariffs, physical import quotas, foreign exchange controls and so on. Some of the more obvious reasons for these will be suggested.

To the extent that people and nations pursue purely economic self interest, the scarce factors in any country will

resist any foreign trade which will reduce the strength of the demand facing them -- which will in fact reduce their relative scarcity. They can do this by restraining imports - especially imports having a high content of their particular factor. But these are the very imports which the natural force of trade would encourage. The abundant factor on the other hand will usually oppose any such restraints on trade, for our theory has shown that imports will not be competitive with it or reduce the demand on it, while exports will considerably increase the demand facing it. Other forces which restrain imports in favour of self-sufficiency are the fear of war, the fear of unemployment, and the desire for a more diversified economy.

#### 4. Implications of Trade for Full Employment of Resources.

In our aggregative model of the economy exports will be a force expanding overall employment, while imports will exert a negative influence on employment. This might suggest that as long as exports and imports are roughly equal their influence on employment cancels out; that only the balance of trade  $F_{(2-1)gs}$  influences employment, while the level of trade ( say  $F_{2gs}$  ) does not matter. But our theory of world trade suggests that the level of trade is of the utmost importance. In the first place it is likely that for many countries their abundant factors can only be fully employed when their export trade is high. At the same time their import trade may be equally vital to them in the matter of full employment. For there will usually be certain industrial materials as well as capital equipment which

are necessary ( high complementarity with domestic factors ) to the functioning and development of their productive systems. Yet they may be either unable to produce these goods domestically at all or be able to produce them only at excessive costs. Clearly then if their imports were cut off they could suffer from unemployment of their complementary domestic factors just as severely as when a fall in export trade produces unemployment in their abundant export factors.

#### 5. Special Influences on the Demand for Imports.

Imports combine with domestic production or value added in the production of the final goods which reach domestic markets. Thus,

(4)  $GDS + \Delta H = GDP + F_1$  . They may be finished goods ready for final use, or materials for further processing, and hence fall into the same classes as the goods passing through and out of the domestic productive system -- industrial materials, producers durable capital, finished consumer goods, services. Our theory has already shown the importance of relative price levels in inducing imports. It has also shown the complementary nature of many imports to domestic factors, which suggests an important income effect ( level of domestic activity ) in import demand. With these preliminaries, the dynamic theories of the household and firm should help us to fill in the remainder of the demand equation. There are however certain special global aspects to import demand which need to be added, and which are now considered before assembling the equations.

(a) Components of Price Level of Imports Laid Down Within Domestic Economy.

Let  $(p_{1gi})$  represent the price vector of goods produced in RW and of interest to A as imports. Convert  $(p_{1gi})$  to A money by multiplying by an exchange vector  $e_2$  which converts each price from RW to international money, and then by an exchange conversion factor  $e_1$  which converts each price from international money to A money. Then an index number of vector  $(e_1 e_{2i} p_{1gii}) = P_{1g}$  is now an appropriate deflator of  $F_{1gm}$ , the money value of A's imports f.o.b. Thus  $F_{1gm} = F_{1g} \cdot P_{1g}$ . But  $P_{1g}$  does not completely represent the price level of foreign goods confronting the domestic importer, for it does not include the costs of transportation and of importation - insurance, freight, customs duties and sales, excise, and other import taxes. The importer must still pay these costs before the imports begin to compete with domestic goods. To arrive at the price level confronting the importer let the proportions of the f.o.b. cost of imports that are added by insurance and freight be  $\overline{if}$ , and by customs duties be  $d$ . Let the proportion of the duty paid value of imports added by sales, excise and all other import taxes be  $s$ . Then the price level which finally confronts the importer and against which domestic prices must compete is

$$(5) \quad P_1 = P_{1g} (1 + \overline{if} + d + s + ds) / (1 + \overline{if} + d + s + ds) =$$

$f(e_1, e_{2i}, p_{1gii}, \overline{if}, d, s)$ . Note that  $d$  and  $s$  are under the direct control of the domestic economy, while  $e_1$  may or may not be controlled.  $e_{2i}$  and  $p_{1gii}$  are usually beyond domestic control.

(b) International Reserves.

In the preceding chapter it was argued that the level of IR exerts an influence on a country's purchases from abroad that is parallel to the influence of the liquidity variables  $L_h$  and  $L_f$  on the purchases of households and firms. The variable IR is accordingly added to the import demand equation. This variable however plays another strategic role in the demand equation, for it, along with the exchange rate variables  $e_1$  and  $e_2$ , provides the restraining mechanism which prevents a country from buying more than it sells in the long run, or spending more than its receipts in the intermediate run.

With variable exchange rates influencing  $P_1$ , IR can be held fairly constant and act in a passive role. The main restraint on imports is effected by  $P_1$  (mainly  $e_1$  and  $e_2$ ). But with fixed exchange rates IR must become the major restraining variable. When imports are exceeding exports and causing IR to run down, some critical value in IR must eventually be reached. When this critical value is reached the government will decide to ration the remaining IR on a basis of the priority of national needs. Some combination of foreign exchange and physical import controls will be set into operation. (Gold standard rules calling for deflation and unemployment could be used as the mechanism of restraint. The variables  $P$ , GDP, GI and C would then serve as the ultimate instrumental variables. But no modern government is likely to use this approach to the problem.)

Once controls are installed to preserve IR the causal influence of IR on imports increases. It follows that IR exerts a non-



linear influence on  $F_1^d$ . For values larger than the critical value the influence ( $\partial F_1^d / \partial IR$ ) would be only moderate. But for values at and less than the critical value the influence will be very great. This nonlinear influence can be brought into the demand equation in the following way. To begin with it is probable that the relative size of IR, in comparison to the variable  $F_1$  which it cushions as an inventory, is more important than its absolute value. Thus if  $\bar{F}_1$  represents the average of the last three periods of normal imports, without any import restrictions or depression, then  $IR / \bar{F}_1$  could be used as the causal variable in the equation. A critical value of IR is reached when this ratio reaches some critically low level  $c$ . The variable  $IR / \bar{F}_1 = IR_1$  is used in the equation whenever  $IR / \bar{F}_1 > c$ . But when  $IR / \bar{F}_1 \leq c$ , the variable  $IR_1$  is treated as nonexistent (zero) and a new variable which has previously been nonexistent (zero) is brought into play. This new variable is  $(c - IR / \bar{F}_1) = IR_2$ . The form of the function  $F_1^d$  must then be such that  $\left| \partial F_1^d / \partial IR_2 \right| > \partial F_1^d / \partial IR_1$ .

(c) Lagged Influences .

Because of the longer time interval between the placing of an order and the receipt of the goods when the goods come from a foreign as compared to a domestic source, it is likely that lagged values of causal variables will be important in foreign trade equations. In Australia where a long sea voyage is required for most imports this delivery lag is likely to be quite large<sup>1</sup>. Let the average time between order and delivery be  $\theta$ . Then imports during  $t$

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1. Professor Gerald Firth and Dr. Alfred Hagger of the University of Tasmania have done research on this aspect of Australian trade.

$F_{1gt}$  are the resultant of business men's forecasts for  $t$  made at  $t - \theta$ . These forecasts would be partly influenced by the outflow of exports  $F_{2t-\theta}$ . It follows that  $F_{1t}$  and  $F_{2t}$  can be considerably out of step when  $\theta$  is large, with rather violent swings in IR if  $e_1$  is fixed. This is the situation found in Australia.

(d) Excess Domestic Demand.

When the domestic economy is under excess demand pressure, and production ceilings have been reached in many lines, this will tend to accelerate the demand for imports. Whether the demand will in fact accelerate will depend on the level of IR, and on the exchange rate  $e_1$ . Normally these two variables will soon restrain the import expansion, unless there has been a corresponding excess of earnings from exports. But if there is at the same time an unusual capital inflow the accelerated demand will not be choked off by IR and  $e_1$ . Such a situation did occur in Canada in 1955 and 1956, when a phenomenal acceleration of imports was accompanied by an actual increase in IR and an increase in the exchange value of the Canadian dollar (decrease in  $e_1$ ).

6. Equations of Import Demand.

(a) Raw Materials or Industrial Materials and Fuels.

$$(6) \quad F_{1rm}^d = f_1 \left\{ \left( \frac{P'_{1rm}}{P_{rm}} \right)_{-\theta}, \left( \frac{P'_{1rm}}{P_{wK}} \right)_{-\theta}, \vec{GDF}_{-\theta}, \vec{P}_{-\theta}, \left( \frac{NL_f}{P_{wMK}} \right)_{-\theta}, W_{f,-\theta}, u_{fg,-\theta}, (GDF^d - GDF^s)_{-\theta}, IR_1, IR_2 \right\}, \text{ where } P'_{1rm} \text{ is the industrial materials component of } P_1.$$

(b) Producers' Durable Capital Goods.

$$(7) F_{1GI}^d = f_2 \left\{ \left( \frac{P'_{1GI}}{P_{GI}} \right)^{-e-1}, GI^d, (GDF^d - GDF^s)^{-e-1}, IR_1, IR_2 \right\}.$$

(c) Consumer Goods.

$$(8) F_{1C}^d = f_3 \left\{ (P'_{1C} / P_C)^{-e}, C_{-e}, u_{fg,-e}, IR_1, IR_2 \right\}.$$

(d) Services Related to Goods Imports.

$$(9) F_{1sg}^d = f_4 \left\{ \frac{P_{1sg}}{P_{sg}}, F_{1g} \right\}.$$

$$(10) F_{1g} = F_{1rm} + F_{1GI} + F_{1C}$$

(e) Definition of  $F_1$  and  $F'_1$ .

$$(11) F_1 = F_{1g} + F_{1sg} + F_{1sv}; F'_1 = F_{1g} + F_{1s}$$

(12)  $F_{1s} = F_{1sg} + F_{1sv} + \Pi_{di}$ .  $F_{1sv}$  represents tourist travel of domestic residents abroad = exogenous.  $\Pi_{di}$  = interest and dividends paid abroad plus retained earnings on foreign direct investment.

7. Equations of Import Supply.

For most practical purposes, for all but the largest trading nations, the purchases and sales of a single nation do not affect price levels in the world economy. Hence in the neighbourhood of current trading positions supplies can be taken as infinitely elastic and represented by ,

$$(13) P_{1rmi}, P_{1GIi}, P_{1Ci}, P_{1sgi} : \text{exogenous}$$

8. Demand for Exports.

The forces behind exports are the same as those

behind imports, with the roles of A as importer and RW as source reversed. The current factor supply situation for Australia and Canada predisposes them to the export of predominantly raw materials and hence only one export equation is necessary here. It might however be wise in practice to separate this into farm and non-farm components. We let RW2 represent A's most important customers in RW.

$$(14) \quad F_{2g}^d = f_5 \left\{ \left( \frac{P_2}{P_{RW2}} \right)^{-\theta} \right\}, \quad \begin{matrix} \rightarrow \\ \text{GDF} \end{matrix} \begin{matrix} \rightarrow \\ \text{RW2,} \end{matrix} -\theta, \quad \begin{matrix} \rightarrow \\ P \end{matrix} \begin{matrix} \rightarrow \\ \text{RW2,} \end{matrix} -\theta, \quad \begin{matrix} \rightarrow \\ W \end{matrix} \begin{matrix} \rightarrow \\ \text{RW2,} \end{matrix} -\theta, \quad \begin{matrix} \rightarrow \\ u \end{matrix} \begin{matrix} \rightarrow \\ \text{rmRW2,} \end{matrix} -\theta,$$

$\left. \begin{matrix} IR_{1RW2} \\ IR_{2RW2} \end{matrix} \right\}$ , where  $P_2$  is the total cost of A goods laid down in RW2, analagous to  $P_1$ ; and  $P_{RW2}$  is an index of domestic price levels in RW2.

$$(15) \quad F_{2sg}^d = f_6 \left\{ \frac{P_{2sg}}{(P_{sgRW2})^{e_1} E_2} \right\}, \quad F_{2g}$$

$$(16) \quad F_{2s} = F_{2sg} + F_{2sv} + \Pi_{sd}$$

$$(17) \quad F_2 = F_{2g} + F_{2sg} + F_{2sv} \cdot$$

### 9. Supply of Exports.

While the export pattern is based on A's current resource pattern and world needs, and while A must export in order to be able to pay for vital imports, at the margin export supply can be varied to meet price and profitability changes. In short some export resources must be bid for by RW2 against domestic demands. Supply of exports is accordingly a function of the export price level relative to the global domestic price level. We approximate this situation as follows.

$$(18) \quad F_{2g}^S = a \text{ GDF}^S$$

$$(19) \quad a = b \frac{P_{2g}}{P'} \cdot \left( = b \left( \frac{P_{2g}}{P'} \right) \right)^{-1} \text{ for short time periods } .$$

$$(20) \quad F_{2g}^S = b \frac{P_{2g}}{P'} \cdot GDF^S, \text{ where } P' = P_{GDF} .$$

Shifts in this supply schedule arising from technological and cost developments would be reflected by changes in  $b$ .

#### 10. Long -Term Capital Movements.

If A's real capital is highly productive at the margin, and A's money system is not inflated,  $i_{lf}$  will be high. This would be likely if A is rich in natural resources and labour skills, but short in real savings and capital, and if A is fully employed. Mature economies like the UK and the U S on the other hand are likely to have low  $i_{lf}$ , because they are relatively abundantly supplied with capital. If  $i_{lf}$  in A is far enough above  $i_{lfRW}$  to cover the risks and uncertainties of distance, the dangers of business failure, depression, adverse exchange movements, political disturbances and so on, then long-term capital is likely to flow from RW into A. This flow takes two main forms. Where the provider of capital wishes to be merely a lender, he will purchase a portfolio selection of long-term securities of A companies. But where he wishes to exercise control as well as lend capital he will make "direct investment" by such means as setting up a branch plant, or a wholly owned subsidiary, or by purchasing a controlling proportion of the shares of an existing company ( Cf [ 5.7 ] p. 57 ).

Portfolio Investment.

$$(21) \Delta S_{eldi} = f_7 \left\{ (i_{lf} - i_{lfRW}), E_s \right\}$$

Direct Investment.

$$(22) \Delta K_{di} = f_8 \left\{ (r' - r'_{RW}), E_k \right\}$$

11. The Rate of Exchange.

The net result of all of A's international money transactions are movements in either or both of the reserves IR and the rate of exchange  $e_1$ . The credit elements of Table 1 or the plus elements of equation (1) of the preceding chapter represent a supply of IR. Each of these elements involves the price  $e_1$  of IR. In aggregate we have from the supply side of the balance of payments,

(23)  $IM^S = f_9 (e_1)$ . Similarly the debit elements of Table 1 or the minus elements of (1) of Chapter 14 provide the demand side of A's balance of payments.

(24)  $IM^d = f_{10} (e_1)$ .  $e_1$  is determined by the interaction of these two equations.

The day to day fluctuations in  $e_1$  may be smoothed by government action (e.g. The Exchange Fund Account in Canada) in which part of A's IR inventory are put into the market as supply when demand is heavy, while more IR is purchased to build up the inventory when demand is light.

PART VIGOVERNMENT AND ITS RELATION TO THE ECONOMIC SYSTEMChapter 16.BRIEF SURVEY OF GOVERNMENT FUNCTION AND FINANCE1. General

It is through the institution of government that a multitude of people living in the same area or region and with common background and purposes unite themselves into a society. Having performed this coalescence, government then acts as an overall co-ordinator and manager of the society. In performing these latter functions the government is to the social organism what the ego is to the individual. But in so acting the government of modern western societies tends only to perform collective duties which the individual, as an individual, cannot do for himself. It still leaves individuals to manage their own private affairs and to do all things for themselves as individuals or business firms which do not in any way require collective action.

Government usually consists of three levels - municipal, provincial or state and central or federal. Here we shall aggregate these three areas into one, except when the central government must be dealt with separately.

2. The Agenda of Government

What functions in the social and economic life of

a nation tend to be taken over by government? The criterion in western democracies for any specific function is whether it can be efficiently performed by individual citizens and firms acting individually and producing under competitive conditions, or acting as private groups performing social services with charitable donations. If the good or service cannot be provided to the satisfaction of the society by such private action, then it becomes a candidate for government agenda. As examples, the baking of bread tends to fall naturally into the sphere of private and competitive production, while the provision and management of the streets and water supply of a city fall into the area of government production. Similarly co-ordination, management, laws promoting justice, the administration of the laws, diplomacy, defence and money supply fall into the public sphere. The development, control and conservation of natural resources likewise falls naturally into the collective area.

Where, by its nature, production of a purely economic good must be monopolistic, since only one product is desired, society tends to prefer the monopoly of a democratic government which can be removed by political procedures, to a privately operated monopoly. But in such cases it is not inevitable that government will produce the product. Sometimes it will be content to merely supervise private operations in the public interest, as is sometimes the case in telephone service and electricity production. This method of control still leaves some profit incentive toward efficiency. Under other conditions where there is room for two firms in the production of the



good ( for example - airlines, railways ) the government may achieve its control by operating one of the firms itself.

There may also be goods which could conceivably be provided by the private sector but where the price would keep all of the lower income groups from buying. When the social feeling is that all members should have an equal opportunity to purchase such goods public production may be resorted to. This happens in the case of education, and is beginning to appear in the fields of medical and hospital service.

In connection with the co-ordination and management of the total society modern nations now would expect their governments to keep their economic systems functioning smoothly, free of periodic depressions and unemployment, free of inflation or the debasement of their money supply, and internationally solvent. This relatively recent addition to government agenda is part of the "Keynsian Revolution", and is of course the field toward which the present study is addressed.

### 3. Theory of Social Choice on Content and Level of Products and Services in Government Agenda.

Each decision of whether or not to bring a service into government agenda must be based on some estimate of the benefit to the whole society and of the social cost to be incurred. The social gains and costs include economic, physiological, psychic and cultural values. They may be consciously and carefully worked out, or only 'felt'

on the basis of at least partially unconscious mental processes. Consider for example the elimination of smoke from an urban area, or the provision of a deep water wharf at a certain port. In appraising whether to go ahead with these projects the society must work out the stream of all future benefits as well as present benefits that can be expected from the project and some of these must be brought back to the present through the use of an appropriate rate of interest. Similarly both present and future costs of the operation must be adjusted to the present. If the benefit side appears to well outweigh the cost side then presumably society will decide to undertake the project. In this way the content of government agenda is gradually built up. Next we may consider how the relative and absolute quantities of the activities in the agenda are arrived at. In very general and rough terms and with no pretense at theoretical exactitude it can be said that social welfare will be maximized when each line in the agenda is extended to the point where a falling marginal social benefit is brought into equality with a rising marginal social cost. Marginal social cost can be partly considered as opportunity cost in terms of the alternative goods which individuals might buy from the private sector. At the equilibrium the social benefit from the last unit of social cost is equalized for all lines, and with marginal opportunity cost. There is then no gain in shifting resources within the government agenda, and there is at the same time no advantage in shifting any further resources from the private to the public sector.

When this equilibrium has been reached the further expansion of government production ceases. In countries like Canada and Australia the expansion has continued until general government absorbs from one-sixth to one-fifth of GNP, or up to 20 per cent of the goods available to the domestic economy.

#### 4. Separation of Total Government Agenda into Government Business and Government Non-Business.

As suggested above government sometimes finds it expedient to engage in productive operations which involve the preparation of goods for sale in commercial markets, as well as in its more basic functions of management, coordination and diplomacy for the society as a whole. Because of this dichotomy in government operations it is not advisable to include all of them in the purely government sector of our economic model. This is because those parts of government agenda which involve production for the market have behaviour patterns which are more like those of the firm than they are of the purely administrative side of government. We shall accordingly refer to the market or market-like operations as government business, and to the remainder of the agenda as government non-business. Government business will be included with private sector firms in our model. Rough criteria for the separation are as follows. If the government organization provides a good which is sold on general markets at a market price, such that the public is at liberty to buy or not to buy at this price, and such that the price usually covers at least the full costs of

operation, the organization can be classed as government business. The aggregate profits ( net of losses) of all government businesses are available to the government and the net portion distributed becomes a component of general government revenue.

##### 5. The Valuation of Government Services.

Economic valuation is determined by market price. Market price must cover at least the cost of all inputs in order for the production of a particular good to continue. In fact if market price were only equal to the basic costs of raw materials, labour and the depreciation on capital goods, then production at the given level is not worth while. There must be some gain in value above these costs, if any value added or true production is to occur. This gain over cost is of the essence of productivity and progress. In western democracies it accrues as profit to the owners of capital ( including personal training and skills) and the providers of enterprise, and is an incentive to both saving and initiative. But such profit must exist in any society whatever its economic institutions, if it is to develop and progress at all.

In the case of government non-business (which we shall henceforth refer to simply as government or general government) the product is not sold in a commercial market and hence the productivity or profit in it cannot be measured. It can only be assumed that the society will not expand government until the subjective or 'felt'

profit from these services is eliminated ( see Sec. 3 above). Profit could be imputed using rates of earnings on capital from the private sector. This would assume equilibrium in the social allocation of resources between the public and private sectors. For the present we will not make this assumption, and will instead value government output by the cost of government employed labour and capital only. This means that the government sector can show no productivity improvement in our model, and must accordingly be excluded from GDP in any global productivity estimates.

#### 6. Government Accounts and Identities in the Economic Model.

Government affects the economic system through its taxing, spending and production ( non-business), (fiscal activities), and through its operation of the money system. In this part we shall deal only with fiscal stocks and flows, leaving the money system to Part VII. In its fiscal behaviour government draws money out of the production streams by taxations, and out of the savings pool by borrowing. It pours money back into these through its purchases of goods and services, transfer payments and subsidies, repayment of public debt, and making of loans. It also holds stocks of money, securities, inventories and capital goods.

(a) Government Spending. Government spending separates into current ( short use) and capital ( long use) newly produced goods,  $G_c$ ,  $G_k$ ; plus the net purchase of existing or second hand capital assets from the private sector,  $\Delta K_{PCMLpg} = \Delta K_{pg}$ .

Total government spending is then given by

(1)  $G' = G_c + G_k + \Delta K_{pg}$ . Total spending on new goods and services is

(2)  $G^* = G_c + G_k$ . The stock of government or public capital is

(3)  $K_g = K_{PCMLg} = K_{g,-1} + G_k + \Delta K_{pg} - D_{PCMg}$ . ( $D_{PCMg} = D_g$ ).

Gross cost to the economy of government (non-business) services is

(4)  $G = G_c + G_k + D_g$ , while the net cost of government services is

(5)  $G_n = G_c + D_g$ .

Government purchases of current goods can be separated into materials and services purchased from firms,  $G_{ms}$ , and government wage bills  $w_g N_g$ , and  $w_m N_m$ .

(6)  $G_c = G_{ms} + w_g N_g / P + w_m N_m / P$ .

Government interest payments,  $G'_1$ , present a special problem. Interest in the private sector represents a share of the profit or productivity of capital which is paid to the lender of a corresponding share of the capital. If  $G'_1$  is used as a partial estimate of this surplus, then GNP is no longer invariant with respect to the method of financing operations.  $GNP_f$  is invariant in this respect, and it is desired to that GNP be so likewise. In addition to this difficulty,  $G'_1$  often includes debt incurred as a result of wars and depressions of long ago, which no longer seem relevant to current output. One way of solving this problem is to treat  $G'_1 - D_g = G_1$  as a transfer payment, and this is the course followed here. We also define  $\Pi_{go} = D_g$ .

(b) Government Output, Production or Value Added. This is represented by the income of factors of production employed in government. Since profit and productivity on government capital and enterprise is not known, only the basic costs of these factors can be used for valuation.

$$(7) \quad G_1 = (w_g N_g + w_m N_m) / P + D_g .$$

(c) Government Revenue consists of direct taxes, indirect taxes and miscellaneous or other revenue. Direct taxes are taken out of the income stream or wealth stocks and are paid directly to government by the income earner, or wealth owner or inheritor, on the basis of quantity of income or wealth involved. Income and inheritance taxes are usually progressive, with increasing average and marginal tax rates. Indirect taxes are taken out of the production and import stream and are paid by firms or persons on some basis of volume of production and imports. They are charged on a late stage of production, usually the last before retail selling. These hidden taxes are treated by firms as business expenses, thereby causing the supply function to shift backward ( Part III). Hence in most cases price of a taxed product will be increased, output will be reduced, and profit will be reduced. Where the tax can be completely shifted forward however, ( inelastic demand), output and profit will not be directly influenced. Within the indirect tax pattern are various negative taxes or subsidies through which the government reduces costs, thereby increasing supplies and reducing prices on particular goods.

#### Direct Taxes.

$T_w$  = direct taxes on wage-salary incomes, consisting mainly of

income tax and employer and employee contributions to social insurance and government pension funds.

$T_{\pi p}$  = direct taxes on nonwage income of persons (interest, rent, dividends and profits), plus succession duties on wealth.

$T_{\pi np}$  = direct taxes on corporation profits ( $T_{\pi c}$ ); withholding tax ( $T_{di}$ ); portion of profits of government business (corporations, boards and commissions) transferred to consolidated revenue of government ( $T_{\pi gb}$ ).

#### Indirect Taxes less Subsidies

$T_{i-s}$  = total indirect taxes less subsidies, including sales taxes, excise taxes, excise duties, customs duties, other import taxes, real estate taxes, business taxes, royalties, fees, licenses.

#### Other Revenue

$T_o$  = all other government revenue, including fines, profit on coinage, profits of central bank, sale of miscellaneous goods, gifts from other governments or world organizations, government interest revenue  $G_{ig} \cdot T_o$  = portion of  $T_o$  representing real current production by domestic factors.

#### Total Government Revenue

$$(8) \quad T = T_w + T_{\pi p} + T_{\pi np} + T_{i-s} + T_o .$$

(d) Government Transfer Payments. Most societies give some help to those members who fall into need. In modern countries a growing economic aid is given to wide groups through government transfer payments (negative, direct taxes). These transfer payments, when combined with progressive income taxes, help to produce a massive





income redistribution.

$T_{rw}$  = transfer payments to wage-salary incomes, including unemployment insurance, workmen's compensation, family allowances, mothers' allowances, old age pensions, veterans' allowances. (Complete list with national accounts).

$T_r \Pi_p$  = transfer payments to nonwage personal incomes, including war veterans' rehabilitation into business, assistance to farmers and other noncorporate business, portion of interest on public debt ( $G_{ip}$ ) paid to personal sector.

$T_r \Pi_{np}$  = government aid to nonpersonal, including corporate, part of private sector, plus portion of  $G_i$  paid to nonpersonal sector ( $G_{inp}$ ).

Total Government Transfer Payments

$$(9) T_r = T_{rw} + T_r \Pi_p + T_r \Pi_{np} + G_{ig} \cdot$$

(e) Government Surplus or Deficit.

Government Disposable Income

$$(10) T_d = T - T_r$$

Current Account Surplus

$$(11) T_d - G_n = S_{g1}$$

Cash Surplus

$$(12) T_d - G_c - G_k - \Delta K_{pg} = S_{g2}$$

National Income Surplus, with respect to current new production.

$$(13) T_d' - G_c - G_k = S_{g3} \cdot (T', T_d' = T - T_o + T_o', T_d - T_o + T_o' \cdot)$$

(f) Government Borrowing from Private and International Sectors.

Current government finance is achieved partly from borrowing.

Short-term borrowing ( $\Delta S_{esgp} + \Delta S_{esgi}$ ) is used when cash shortages arise from the difference in timing of revenue and expenditure. The securities sold for such purposes include treasury bills and notes, and deposit certificates. Long-term borrowing is entered into to finance capital projects of long life, or to finance emergencies like wars and depressions. The securities sold are usually bonds and debentures, with fixed interest and usually fixed principal. The net flow during a time period of long-term loans from the savings pool and the foreign sector equals  $\Delta S_{elgp} + \Delta S_{elgi}$ .

(g) Government Borrowing from Commercial or Trading Banks. In the financial aspects of our model we shall consider the commercial or money creating banks (MCB) as a separate sector, distinct from the private sector of households, firms and institutions. Borrowing from the private sector merely transfers existing money to the government, which may activate this money. But borrowing from the MCB involves the creation of new money. We represent this by  $\Delta S_{esgb}$  and  $\Delta S_{elgb}$ . Such borrowing need not be direct. The result is the same if the original security sale is to someone in the private sector, who subsequently sells to an MCB.

(h) Government Borrowing from the Central Bank (CB). The central or federal government (CG) can, if the situation warrants, borrow short or long from the CB. Such transactions (net) are represented by  $\Delta S_{esgc}$  and  $\Delta S_{elgc}$ . Such borrowing may be multiplied by the cash reserve multiplier, (Part VII) which may be of the order of 10 or even 20, and added to the money supply. Hence this type of borrowing

can be very inflationary.

(i) Government Cash Position. Government holdings of money depend upon its cash surplus from budgetary revenues and expenditures, and its net borrowing or repayment with respect to the private economy, RW, the MCB, and the CB.

(13)  $\Delta M_g = T_{dm} - G'_m + \Delta S_{esgp} + \Delta S_{elgp} + \Delta S_{esgi} + \Delta S_{elgi} + \Delta S_{esgb} + \Delta S_{elgb} + \Delta S_{esgc} + \Delta S_{elgc} - \Delta S_{epg}$  (net borrowing and lending abbreviates to  $\Delta S_{egp} + \Delta S_{egi} + \Delta S_{egb} + \Delta S_{egc} - \Delta S_{epg}$ , where  $S_{epg}$  represents loans by government to the private sector).

$$(13a) M_g = M_{g,-1} + \Delta M_g.$$

(j) Impact of Government Fiscal Activities on Private Sector Money Holdings. Government taxing, spending, borrowing and lending changes the money holdings and hence part of the purchasing power of the private economy. The net primary injection or withdrawal of money in the private sector is the obverse of the change in government money holdings, if we exclude from these the amounts financed by the MCB, the CB, and RW.

$$(15) \Delta M_p^g = -\Delta M_g + \Delta S_{egb} + \Delta S_{egc} + \Delta S_{egi}.$$

(k) Redistribution of Earned Incomes to Disposable Incomes by Tax-Transfer Structure.

Disposable Wage-Salary Income

$$(16) (w_{ph}N_{ph} + w_iN_i + w_gN_g + w_mN_m) / P = W_a$$

(16a)  $Y_w = W_a - T_w + T_{rw} + T_{rf1} + T_{rf2}$ .  $T_{rf1}$  = charitable contributions of firms;  $T_{rf2}$  = bad debt losses of firms to wage earners.

Disposable Property-Enterprise or Nonwage Income

(17)  $\Pi = \Pi_f + \Pi_{io} + \Pi_{ri} + \Pi_{go}(=D_g) + \Pi_{id} - \Pi_{di} =$  total nonwage income of residents. ( $T_{rfl}$  added back, and included in  $\Pi$ .)

(18)  $\Pi_{np} = \Pi_c - \Pi_{dfo} + \Pi_{ub} - \Pi_{dub} + \Pi_{gb} =$  nonwage nonpersonal income.  $\Pi_c =$  corporation profits, excluding any transfer payment interest  $G_{ic}$ .

(19)  $\Pi_p = \Pi - \Pi_{np} =$  nonwage income to personal or household sector.

(20)  $Y_{\Pi p} = \Pi_p + T_r \Pi_p - T \Pi_p =$  disposable personal nonwage income.

(21)  $Y_{\Pi np} = \Pi_{np} + T_r \Pi_{np} - T \Pi_c - T_{di} - T \Pi_{gb} - T_{rfl}.$

Government Interest Payments.

(22)  $G_i = D_g + G_{ip} \text{ ( in } T_r \Pi_p \text{ )} + G_{inp} \text{ ( in } T_r \Pi_{np} \text{ )} + G_{ig} \text{ ( in } T_o \text{ and } T_r \text{ )}.$

Income Shares After Redistribution.

(23)  $Y_w + Y_{\Pi p} + Y_{\Pi np} + T_d' - J + D_{f1} + D_{f2} - T_{rf2} = \text{GNP.}$

(24)  $Y_{\Pi} = Y_{\Pi p} + Y_{\Pi np} = Y_{\Pi a} + Y_{\Pi na}.$

7. The Impact of Government on the Economy.

There are perhaps three major areas in which modern governments make profound changes in the nature and operation of the economic system. The first of these is their impact on overall demand. To some extent government/<sup>demand</sup> is a redirection of demand that might have occurred in the private sector, but for taxes. This however is only partially true, and it can be shown that in many circumstances even a balanced budget increases aggregate demand by the total amount of government spending ( c.f. Haavelmo [11.7] ). But modern government demand goes much further than this would suggest,

in that it provides entirely new services which the private sector could not be expected to supply on its own. Many of these services require extensive and large scale capital equipment such as highways, bridges, schools, buildings, laboratories, etc., and spending on these heavy items creates a very large impact on the overall demand and hence on the income flows and employment of the private economy.

The second major area of government impact lies in its massive redistribution of income. The income distribution in a completely 'free' economy would tend to become very unequal due to the uneven distribution of abilities and wealth. The inequality in the wealth distribution generates even greater inequality as wealth grows with compound interest. But democracy and social stability are not likely to survive without a large middle class of property owners (cf. Aristotle [11.1]). Since the Great Depression and World War II western societies have seemed to sense a truth in this theory and have engaged in redistribution and welfare to an extent which marks a quiet revolution in human affairs. This change has been achieved mainly through the progressive income tax and inheritance taxes, the wide system of transfer payments, the provision of many educational and welfare services by government without direct charge, and the encouragement of trade unions (e.g. Wagner Act - Roosevelt).

The third major impact of government occurs with respect to the price system and the allocation of resources. Indirect taxes and subsidies usually change the supplies and hence the market prices and levels of output of the taxed goods. This system can be, and, is used to encourage the output of some goods and to discourage the output

of others, as well as to provide revenue. For example, we might find a subsidy on milk production and excise taxes on the sale of liquor and tobacco. This is a kind of 'direct' impact of government on prices and allocation, coming from the supply side. But the indirect government impact on these from the demand side is probably of much greater dimensions. There is on one hand its direct demand represented by  $G_c + G_k$ , and on the other the changes in demand arising from progressive taxation and transfer payments. The net result of it all is to shift demand away from luxury goods and toward basic and culture needs, welfare services and social capital.

Chapter 17.TAX AND TRANSFER EQUATIONS1. General.

Many of the stocks and flows outlined for the government sector in the preceding chapter are largely exogenously determined by political and social forces. However, in the case of the tax and transfer payments variables, government can only set rates or functions exogenously. The actual tax or transfer flows then become endogenous variables which are functions of the appropriate tax base variables. Our purpose in this chapter is to set out the major tax and transfer functions which are applicable to our economic model. These functions will differ in one major respect from most of the others in the model in that they are likely to change, sometimes much -- sometimes little, from one government budget to the next. Hence they have less stability in the model than is assumed for most of the other equations.

2. Particular Tax and Transfer Equations.

To express these equations, it is merely necessary to state the aggregate tax and transfer variables as functions of the aggregative economic variables which serve as tax bases for them.

$$(1) \quad PT_w = f_1(PW_a)$$

$$(2) \quad PT_{rw} = f_2(N, N_u)$$

$$(3) \quad PT_{\pi p} = f_3 \left\{ P (\pi_p + G_{ip}) \right\}$$

(4)  $\overline{\Pi}_r \overline{\Pi}_p =$  exogenous

$$(5) \quad \overline{\Pi}_{np} = t_c(\overline{\Pi}_c + G_{ic}) + t_{di}r_{di}(K_{di} + S_{edir}) + t_{gb}k_2 \overline{\Pi}_c.$$

$t_c$  = average rate of corporation profits tax.

$t_{di}$  = average rate of withholding tax.

$$(1-t_{di})r_{di}(K_{di} + S_{edir}) = \overline{\Pi}_{di}.$$

$t_{gb}$  = average rate of transfer of government business profits to consolidated revenue.

$k_2$  = ratio of net worth of government business to net worth of other corporations in the private sector. (The same average rate of profit is assumed in each).

$$(5a) \quad \overline{\Pi}_c + \overline{\Pi}_{ub} + \overline{\Pi}_{gb} = \overline{\Pi} - (\overline{\Pi}_p - \overline{\Pi}_{dfo} - \overline{\Pi}_{dub}) = \overline{\Pi}_{fn} = \text{net profit of all firms.}$$

$$(5b) \quad \overline{\Pi}_p - \overline{\Pi}_{dfo} - \overline{\Pi}_{dub} = \overline{\Pi}_{ip} + \overline{\Pi}_{rp}.$$

$\overline{\Pi}_{ip}$  = interest receipts of persons, excluding  $G_{ip}$ .

$\overline{\Pi}_{rp}$  = net rental income of persons or households.

$k_3$  = ratio of net worth of unincorporated business to net worth of non-government corporations. Assuming the same average rate of profit in all three sectors,

$$(5c) \quad \overline{\Pi}_c = \frac{\overline{\Pi} - (\overline{\Pi}_{pi} + \overline{\Pi}_{pr})}{(1 + k_2 + k_3)}.$$

Since  $\overline{\Pi}$  is the most variable component of GNP, absorbing much more of the flows and ebbs of demand than  $W_a$ , and since  $\overline{\Pi}_{pi}$  and  $\overline{\Pi}_{pr}$  are contractual, it follows that the percentage movements of  $\overline{\Pi}_c$  and  $\overline{\Pi}_{fn}$  are likely to be extremely volatile.



(6)  $\Pi_{rnp} = \text{exogenous (often containing only } G_{inp})$ .

The indirect taxes (less subsidies) are charged on a wide range of consumer goods, producers' materials and investment goods. Exports and government purchases are usually exempt, as are also most basic foods, many farm materials, and sometimes machinery and apparatus to be used directly in producing goods. These taxes are usually charged at a late stage in the production process as the goods are finished and just ready to move into retail channels or to final users. They are charged directly on imports as customs duties and import taxes. It follows that the aggregate tax bases of indirect taxes will be imports of merchandise, consumer spending, investment spending, and the general level of domestic activity.

(7)  $T_{i-s} = f_4 \left\{ \frac{P_{lg} F_{lg}}{P}, \frac{P_G C}{P}, \frac{P_{GI} GI}{P}, (GDP - G^* - F_2 - w_i N_i - \Pi_{io} - \Pi_{ri} - W_a^a - \Pi^a) \right\}$  where  $W_a^a + \Pi^a$  is the value added or the production of the agricultural (including pastoral) industry.

### 3. Progressive or Regressive Nature of the Total Tax Structure.

A tax can be considered to be progressive if the proportion that it bears to personal income increases as we move up the income scale. A slightly different and perhaps more useful way of making this definition is to say that a tax is progressive, proportional or regressive depending on whether the per cent increase in tax collected for a one per cent increase in income (the elasticity of the tax) is greater than, equal to or less than one. The progressivity

(elasticity) of the tax may be computed with respect to its own base, with respect to personal income, or with respect to any other relevant variable.

It is of interest to question which taxes are progressive, and which regressive, and also what is the nature of the total tax structure in this respect. The personal income tax is definitely progressive and usually is increasingly progressive ( increasing elasticity) as we move up the income scale. Many business taxes and licenses would tend to be regressive with respect to business income, and with respect to personal income, except perhaps for firms producing luxury goods. Sales and excise taxes are regressive when charged on basic and culture needs, but progressive when charged on luxuries or culture wants. The corporation profits tax presents an especially interesting case. With respect to its own base it is slightly progressive at a low profit level, but becomes proportional once this level is passed. But what is its progressivity with respect to personal income? As long as money is easy and business is expanding this tax will be shifted, to enable firms to obtain a desired yield on their net worth. It then becomes de facto a form of indirect tax added on to the price of goods generally. Even when business is less buoyant a portion of this tax is probably passed on in prices, and the conclusion still holds. Since the savings ratio probably increases with increasing income this tax will probably be regressive with respect to personal income, but progressive with respect to net national income at factor cost ( $W_a + \Pi = \text{NNIFC}$ ). This is because the tax is

related to  $\Pi$  (5c) and because  $\Pi$  increases faster than  $W_a$  during business expansion, but falls faster than  $W_a$  during depression.

What is the nature of the global tax-transfer structure of a modern economy? To answer this question we might consider GNP ( or NNIFC) as the aggregative tax 'base', and the disposable income of Government  $T_d$  as the aggregative tax. Then, if we knew the nature of the function

(8)  $T_d = f_5$  (GNP, or NNIFC), we would be able to describe the global tax structure as progressive, proportional or regressive, at the relevant levels of the base. In view of the income redistribution effects of modern tax-transfer structures, and the progressive nature of the personal income tax, many indirect taxes, and the corporation profits tax ( re NNIFC), one would expect this relationship to be progressive. The buoyancy of government disposable income in recent years in periods of rapid expansion in economic activity seems to confirm this expectation.

PART VIITHE MONEY AND FINANCE SYSTEMChapter 18.A BRIEF DESCRIPTION OF THE MONEY AND FINANCE SYSTEM1. The Meaning and Importance of Money.

Money is one of the great discoveries of man in his progress towards civilization. Its importance lies in the productivity which it makes possible. Without money, economic exchange must be carried on by barter which must usually balance bilaterally. This method of exchange puts a very severe limit on the possibilities of specialization and division of labour both within the domestic economy and internationally, thereby hindering enormously the progress and development of the economy. Money, by contrast, makes possible and encourages an almost infinite multilateralism among the workers and firms - not only of a single country - but over the whole world. What qualities of money enable it to do this?

In order for money to achieve this result, it must possess three main qualities. One is that it must become "generalized purchasing power", so that any economic good produced in the economy will be readily exchanged for it. The second is that it must retain its purchasing power over time, so that economic units are not afraid to hold it into the future. The third is that it must be capable of serving as a unit of value, so that all other goods can be

1. Thus at any instant of time any one unit (pound or dollar) is equal to every other unit.

measured for exchange value in terms of it.

No money in history has ever quite achieved all three of these qualities perfectly, with the second being the most difficult to attain. For long periods the attempt was made to achieve stable value by making money "full-bodied", using for example cattle, gold dust, beaver pelts, or striking coins of metal with their metallic value (gold, silver, bronze) equal to their face value. But changes in the supplies of these metals relative to demands, and changes in their relative values bringing Gresham's law into operation, rendered the results of these attempts far from perfect.

At other times in history, especially in primitive societies, mere token money in the form of shells or stones or any other convenient object has been used. In early Canada playing cards marked by the governor were made money by fiat during shortage of currency from the mother country. It was during that most interesting evolution of some goldsmiths into bankers ( Lombard Street, )<sup>that</sup> notes, which were mere promises to pay gold or silver and written on paper, began to be circulated in commerce in place of metallic money and bullion. Convenience, safety and the trust that these notes could be exchanged at any time into full bodied money, encouraged this development.

In the economic systems of the modern world, coins and paper money are still used for convenience in making the multitude of smaller transactions but no attempt is made to have these forms of money either full-bodied or exchangeable for full-bodied money. These metallic and paper tokens are made money by "fiat" of the central

government (CG) which declares them legal tender for settlement of all obligations - public and private. Such money is "fiduciary" depending ultimately on public trust and confidence in its value. It may prove easier to stabilize the value of such money, properly managed by the CG through its central bank (CB), than to stabilize the value of either full-bodied money or money backed by precious metals. In fact such stabilization is the second most important of the economic problems to which the present study is addressed.

Full bodied money has almost passed into history, but there is one important residual example of its use. This is the use of gold as a component of international money.

The evolution of modern money continued when the early goldsmiths and their clients discovered that it was inconvenient, costly and dangerous not only to transport gold ( or silver) about the country in settlement of payments, but also to carry large sums of paper notes payable ( in gold or silver) to bearer. They eventually lit upon the idea of transferring this value merely by changing the credits in the accounts of the various depositors of gold, debiting one depositor and crediting another , on order from the debtor. Presently they speeded up even this process by inventing the cheque, a written order from a depositor to a goldsmith ordering him to transfer gold credit to the payee of the cheque. With these developments precious metals moved very little, and mainly only to settle accounts between goldsmiths.

Some further developments bring us to modern money.

Since most of their gold and silver lay idle the goldsmiths began to lend it charging a rent ( interest) for its use. But they tended to lend only a deposit credit for the gold, since the borrowers found this more convenient than the gold itself. The deposit credit became the real money of use, and since the gold reserve behind it was rarely demanded the deposits could exceed it by a considerable margin without danger. The goldsmiths found they could lend deposits, thereby creating money by a "stroke of the pen", until total deposits climbed to the order of ten times their metallic reserves. The only danger was that a crisis and lack of confidence would bring all depositors clamoring for gold at once, thereby ruining the goldsmith and many of his clients.

Modern money is almost the same as goldsmith money, differing only in that token coins and paper money, made legal tender by government fiat, replace the use of precious metals as part of bankers reserves. The balance of their reserves are claims to such legal tender from the CB. The domestic money of a modern economy consists of public holdings of such legal tender, used for most small and retail transactions; plus deposits with the commercial, trading or money creating banks (MCB), used for almost all large and business transactions. Deposit money forms the bulk of the effective money supply. It is not legal tender, and its popularity rests on its convenience, safety, and proof of payment when transferred by cheque.

We may symbolize the stocks and indicate the method of issue of the various forms of money, as follows.

$M_m$  = coin or metallic money manufactured by national "Mint", and sold

at a profit to CB. The Mint is usually a part of the Treasury or Department of Finance of the CG. CB sells  $M_m$  to MCB who in turn sell it to general public.

$M_n$  = paper note money, manufactured for the CB, and treated as a liability of the CB when issued; issued to MCB and to CG in exchange for  $M_m$ , or securities of the CG, or CB deposit liabilities.

$M_b$  = bank deposit money consisting only of credit entries in the ledgers of the MCB; transferable by cheque; written up in exchange for  $M_n$ ,  $M_m$ , or CG securities, or as proceeds of bank loan in exchange for borrower's promissory note or other security.

Coin and note money which 'runs' from hand to hand is defined as currency,  $M_c = M_m + M_n$ . Total money  $M = M_c + M_b$  is sometimes referred to as cash. The cash reserves of the MCB however are defined to include their till holdings of  $M_c$  plus deposits to their credit with the CB, which can be converted to  $M_c$  on demand.

Deposits in true savings banks are not money within our definitions. They are essentially short-term securities sold by the savings banks in exchange for 'true' money. The savings banks, like mortgage and loan companies and insurance companies, then use this money to purchase long-term securities with a higher yield than the interest they pay on savings accounts. True savings deposits often are not transferable by cheque and can only be withdrawn after notice. In Canada the savings departments of the commercial or chartered banks are not true savings banks. The deposits in them are essentially money created by the MCB, often by granting personal loans, and can be



transferred by cheque or withdrawn ( in practice) without notice.

The holders of domestic money fall into four main sectors - households, firms, government and international - which we symbolize by the subscripts h, f, g, i. For this part of our study institutions are included with firms.

## 2. The Functions of Money.

Three main functions of money can be observed.

The first is of course its use to facilitate exchange between producers who have specialized, each in one particular line of activity. Money thus integrates all individual producers into a composite aggregate productive system providing fully multilateral exchange and enhanced productivity.

Coupled with this function of medium of exchange money also serves as a measuring rod of economic value. Each product in the economy is valued in terms of money units. Each money unit in turn represents generalized purchasing power, the real value of a composite 'unit' of all goods at a particular instant of time. As a measure of value money facilitates economic calculations, providing a further aid to exchange and productivity.

The third major function of money is its ability to store value. As was observed in our studies of the household and the firm, no economic unit wants to spend all of its available resources immediately. All units want to keep a cushion or inventory of value for future uses and needs. An easy way to store this future purchasing

power is to hold money.

The third function is a dynamic one, and cannot be adequately or equitably performed if money loses its value through inflation of prices. Nor can money perform its second function for inter-temporal or dynamic comparisons if the price level rises or falls significantly; for now the unit of measure, represented by the real composite unit of goods purchasable by a pound or a dollar, shrinks or expands.

### 3. Description of Money Flows and Stocks in the Economic System.

Money tends to be continuously shifting from one holding sector to another and these shifts in holdings are referred to as flows. A unit inter-sector flow is a transfer of one pound or dollar from one holding sector to another in one time unit. The net effect of the flows of money between the different sectors is of course changes in their money holdings. Our previous theory suggests that these holdings affect what the sectors do in the subsequent time period.

It is helpful in seeing the whole money system to distinguish certain broad and basic flows. The first one is the flow of money from the purchasers of final goods to the producing firms. Here we have the spending of households and government, the investment spending of firms,<sup>and</sup> the export spending of foreigners. There is next a secondary money flow down through the productive system from final goods stages of production through the intermediate stages and

ultimately reaching primary extractions from the natural environment, and imports. Here the flow is from industry to industry, and filaments out through the whole system including the energy and linking industry systems. The money flow is in the opposite direction to the corresponding flow of goods and services.

While this downward flow is taking place there is a simultaneous horizontal flow out of the productive system and into households, as industry purchases labour and the use of property and enterprise. At the same time there are flows out of households and firms to Government in payment of taxes, while opposite flows from government to households and firms occur as transfer payments.

All of the net or disposable money flows into households, firms and government do not flow back to firms as payment for final goods. Instead some of this money is saved for future uses as household savings, undistributed profits and depreciation reserves of firms, and surpluses of government. Let us construe all such savings as money flowing into a savings pool, SP.

If all of the money which flowed into the SP were left there for any appreciable length of time, the modern economic system would tend to grind to a subsistence level of output with zero savings. It usually happens however that firms require more money for investment purposes than their accumulated savings would permit, and that government sometimes wants to spend or invest money in excess of its cash and current income. Through the mediation of the various firms in the finance industry - banks, insurance companies, trust companies,

mortgage and loan companies, stock and bond brokers and jobbers, and so on - firms and governments arranged to borrow the money savings of the other sectors and use them for their investment needs. Thus there is established a simultaneous outflow of money from the SP at the same time as money flows in. Some of this outflow goes back to individual households who arrange consumer credit with the financial institutions. But on balance the aggregate household sector provides an inflow to the SP rather than an outflow.

The remaining important money flows of the system have to do with the flows of domestic money into and out of what we may call the foreign exchange pool (FEP). The FEP is operated by the commercial or trading banks, foreign exchange brokers and jobbers, and the CG usually with the aid of the CB. Each time a firm makes payment for an import of goods it must surrender domestic money within the FEP in exchange for international money. It usually obtains the IM from the MCB. They provide it from their transactions holdings, but may need to replenish these by purchases from the CG, CB or from broker or jobber. Such purchases involve direct loss of cash reserves, or at least a weakening of the cash reserve ratio ( Chap. 19). Where the IM is obtained from the official reserves ( CG or CB), the domestic money supply is decreased in the first instance by the amount of the import, but this is followed by a secondary contraction in the total money supply equal to the amount of the import multiplied by the cash reserve multiplier of the banks ( Chap. 19). Thus the FEP acts to some extent like a money vacuum with regard to imports. But the opposite occurs

in the case of exports. In this case the foreign buyer surrenders IM to the banks in exchange for domestic money, and the cash reserves of the banks become increased as they sell IM to brokers, the CG or the CB. Again there is the money multiplier effect, so that exports of goods or securities turn the FEP into a pressure pump adding to the domestic money supply.

#### 4. The Securities Markets and the Savings Pool

It is through the device of securities that it is possible for as much money to flow out of the savings pool and back into the productive system, as flows in from savings. This helps to keep the economic system running evenly and at full employment. Thus a household A may be saving for something it wants ten years hence. Its failure to spend its full income now will contribute to unemployment. But firm B wants a new plant now and does not have sufficient savings to buy it. B can obtain the use of A's savings by giving him in exchange a security which promises to repay the principle ( as cash or equity) plus a current rent ( interest or dividend) for the use of the money until repayment. This interest or dividend pays A for loss of liquidity, risk of capital loss, and the trouble of the transaction. It is paid out of the real productivity of the new plant.

Securities consist essentially of two main types. In the first type, the fixed interest security  $S_{ei}$ , the issuer of the security promises to pay fixed amounts of principal at specified fut-

ure times and, at the same time, a fixed rate of interest on the unpaid principal in the interim. The second main type of security makes the purchaser a part owner of the corporation issuing the security, with his ownership proportionate to the number of security shares he has purchased. These are equity securities,  $S_{ee}$ . They entitle the owner to a share in the profits in the form of dividends, to a vote in the affairs of the corporation, and to a proportionate share of the net worth of the corporation should it be liquidated or wound up.

Almost every institution in the SP is engaged in one way or another in the exchange of securities for money or of money for securities. In this respect the whole SP is a vast securities market. However, there have developed various specialized markets for certain types of securities. The best known of these is the share stock market where the shares of various corporations are bought and sold. In company with this there is usually a bond and debenture market. In large financial centres like London and New York there are also markets for short-term securities such as bills of exchange and treasury bills.

Through differential security prices the securities markets tend to ration the limited money in the SP in what it considers to be the most fruitful investment directions. They make the cost of money cheap to favoured firms or industries, and dear to those considered to be less productive. This is a most important function with respect to the productivity and the economic development of a nation. If it is not well done resources will be misdirected,

value is likely to be destroyed rather than created, and economic development will be impaired. ( Cf. Keynes re " gambling casino" [1.6] p. 159).

##### 5. Securities as a Store of Value.

Since securities represent promises to pay money in the future they, like money, serve as a store of value. Assuming that money is not losing its value through inflation it will be preferred to a security of equal market value, because of its ready liquidity. Hence it requires something extra, a yield, to induce the individual to exchange money for securities as a store of value. Through this yield the provider of money savings gains the advantage of having both a store of value, and a share in the productivity of real capital.

##### 6. Summary of Money and Security Holdings in the Economy

To gain a complete view of all of the money and security holdings of the economy, we divide the economy up into the basic issuing and holding sectors. Let these be households ( including all personal consumer units), firms, government, money creating banks, central bank, and foreign countries or RW; and let them be symbolized by the subscripts : h, f, g, b, c and i. Let the first subscript represent the issuer of the money or security, and the second subscript the holder. To the issuer the money or security is a liability, while to the holder it is an asset. Thus  $M_{bh}$  represents bank deposit

money held by households,  $S_{esif}$  represents short-term foreign securities held by firms, and so on. In our model it is necessary to consider four main types of financial assets - money  $M$ , short-term fixed interest securities  $S_{es}$  (including accounts receivable and payable), long-term fixed interest securities  $S_{eil}$ , and equity securities  $S_{ee}$ . We can represent these completely either by setting up four different finance matrices  $M_{jk}$ ,  $S_{esjk}$ ,  $S_{eiljk}$ ,  $S_{eejk}$ , ( $i, k = h, f, g, b, c, i$ ), or we can use only one matrix where each element is divided into four compartments. Let us assume that the latter has been done. Then the columns and column sums of this four-dimensional matrix will represent the holdings of each sector of all money and security assets, while the rows and row sums will then show the money and security issues of each sector, and hence the financial liabilities of the sector. There will be many zeros in this matrix since, for example, only the CB, CG and the MCB issue  $M$ .

The total assets in this system can be represented by column sums  $V_h$ , ---,  $V_i$ , while total liabilities are the row sums  $D_{eh}$ , ---,  $D_{ei}$ . In the complete matrix  $\sum V = \sum D_e$ . If however, we delete the column on international holdings of assets (here we take  $M_{ii}$ , etc. to be zero) and the row of A holdings of RW liabilities, the sum of the remaining columns gives total domestic financial assets  $V_d$ , while the sum of the remaining rows gives total domestic liabilities  $D_{ed}$ . It follows that

(1)  $V_d - D_{ed} = U_{id} - M_{go}$ . ( See Chap. 14). In another form this basic identity is



$$(2) \quad V_h - D_{eh} + V_f - D_{ef} + V_g - D_{eg} + V_b - D_{eb} + V_c - D_{ec} + V_{ei} - D_{ei} \\ = 0. \quad \text{It is within this identity and the matrix}$$

$M_{jk}$ , ---,  $S_{eejk}$  that the liquidity variables  $L_h$ ,  $NL_h$ ,  $L_f$ ,  $NL_f$  are formed. These are defined as

$$(3) \quad L_h = M_h + S_{egh}; \quad NL_h = L_h + S_{eshh} - D_{esh} \cdot$$

$$(4) \quad L_f = M_f + S_{ef} - S_{emf}; \quad NL_f = L_f - D_{esf} \cdot$$

An interesting question is how the volume of securities relates to other magnitudes in the economy. Since each security must usually be serviced by real capital, it follows that total securities issued by firms plus  $K_{ub}$  should be very roughly equal to total producers' real capital. In the household sector securities will usually be issued against real capital, but in total will be well below total real capital. In government, because of wars and depressions securities issued will often be greater than real capital. We cannot tie total securities in with any more precise relationship to real capital than these, because in a completely aggregated balance sheet of the domestic economy they and money cancel out. We are left with the identity

$$(5) \quad K_h + K_{d+1} + H + K_f + K_g + M_m + U_{id} = NW_h + NW_g$$

$NW_g$  will usually be negative because of past wars and depressions.

## 7. The Level of Money in the Savings Pool and the Business Cycle.

On balance household money savings provide a positive flow of money into the SP, while firms produce a net outflow by for investment/using their own funds and by obtaining household funds

through the sale of securities. Government may run a surplus (inflow to SP) at one time, and a deficit (outflow from SP) at another. The banking system injects and withdraws money with respect to the total system, but this will usually affect the level of money in the productive system in the first instance. Because of its costly rent it is usually put directly to work.

Since the savers and investors are usually completely different sets of people acting under different motivations, there is no reason why the inflows and outflows, with respect to the SP, should be exactly equal. If they were exactly equal, the level of money in the SP inventory would never change. However, we know from practical experience that there are times when money is "easy", when there is an abundance of money in the SP and available for investment purposes. There are other times when money is "tight", when there is very little money left in the SP available for investment.

The inflows and outflows are not completely unrelated of course. A fast inflow and a high level of money in the SP increases its availability and reduces its interest cost to firms wanting loans. Conversely the high cost of money when the SP is nearly dry tends to encourage a greater saving inflow. But this price and inventory "feed-back" or control mechanism is not strong enough to keep the level of money in the SP constant, as witnessed by cycles of filling and emptying. It is however assisted in the longer run by income changes, which do bring longer run savings and investment

into equality.

The short-run behaviour of the system can be explained by a simple dynamic model which incorporates the short time lags between spending and income formation, and between income receipt and spending. We ignore the first of these lags in the interest of brevity. Let  $Y_t$  represent disposable money income of all economic units during the time period  $t$ , while  $C_t$  is the spending of all units on current goods and materials, and  $I_t$  is the spending on all durable capital goods. The spending flows create new income after a short time lag, here omitted.

(6)  $Y_t = C_t + I_t$ . In analysing the dynamic nature of these flows, we know that spending  $C$  and  $I$  always create income. But this income need not create subsequent spending. The act of spending is discretionary, partly based on causal relations and partly dependent on exogenous and random factors. Usually spending occurs out of income (production) after it is received (produced) and there is hence a necessary time lag between income (production) and spending (CF. D. H. Robertson [7.18], [7.19]; Franco Modigliani [7.15]). This conclusion is only significantly true when we are working with short time periods - day, week, month. In this case  $C_{t+1}$  and  $I_{t+1}$  are spent out of the income (production) flow  $Y_t$ . Hence the saving flow into the SP in  $t + 1$  is

(7)  $S_{t+1} = Y_t - C_{t+1} = \Delta_1 M_s$ , where  $M_s$  is the level of money in the SP. Investment in  $t+1$  is

(8)  $I_{t+1} = \Delta_2 M_s$

The difference between saving and investment is

$$(9) \quad S_{t+1} - I_{t+1} = Y_t - C_{t+1} - I_{t+1} = Y_t - Y_{t+1} = \Delta M_{st+1} \cdot$$

In this model changes in  $Y$  occur only as a result of difference between  $S$  and  $I$  in the same (short) time period. These differences arise out of fluctuations and cycles in both  $C$  and  $I$ , but our theory and experience lead us to expect wider variations in  $I$ , and hence greater causation from the capital goods side. Movements once begun influence their continuance or become autocorrelated because they generate expectations about future profits, thereby influencing  $I$ . But if the quantity of money in the total system is held constant, except for growth, the rises and falls of  $M_s$  in the SP tends to turn movements into cycles. The upswing is eventually partly checked by lack of availability of  $M_s$  and a high charge for its use, and conversely.

#### 8. Active and Inactive Money.

The money which is circulating through the productive system and aiding production is active, and we call it  $M_a$ . Conversely money lying idle in the SP is inactive and we call it  $M_{s2}$ . There is a third portion of total money which is actively at work serving to exchange the various forms of wealth holding in the SP - mainly securities and real estate. The money engaged in this active swirl in the SP we shall call  $M_{s1}$ .  $M_s = M_{s1} + M_{s2}$ .  $M_a$  and  $M_{s1}$  represent money held for transactions needs, while  $M_{s2}$  is money held for the precautionary and speculative motives ( Cf. Keynes [1.6] chap. 15), but note

that our  $M_{s2}$  is essentially money held inactive for lengthy periods while Keynes'  $M_2$  is money held for the speculative motive only). The precautionary motive involves accumulation for further needs, and the provision of a cushion against unforeseen contingencies. Speculative holdings on the other hand imply that the current yield on securities or real estate or business operations is not as great to the holder as that from holding money itself. This is especially the case if business is slack and prices are falling. For now the real yield on  $M_{s2}$  may be quite high, while that on securities or business operations may be less. Thus  $M_{s2}$  is "liquidity preference" over other methods of storing value. The argument here is that all  $M_{s2}$  is held for the purpose of storing value for future use, and that the quantity of it is chosen in competition with securities and real estate depending on relative real yields. There is however a minimum residual of  $M_{s2}$  which must be kept as cash, except under extreme inflation, and this must be the portion which Keynes allocated as "precautionary" to his  $M_1$ . But this portion is still inactive for long periods, though related to P.GDP. ( See Chap. 20).

When longer time periods are used, it seems preferable to analyse the level of  $M_s$  in terms of stocks of  $M_a$  and  $M_s$  rather than in terms of income flows. In a long time period  $C_t + I_t = Y_t$  and  $C_t + S_t = Y_t$ , so that  $S_t = I_t$ .  $Y_{t+1}$  may differ from  $Y_t$  because of different values of  $C$  and  $I$ , with no close dynamic sequence. But  $S_{t+1} = I_{t+1}$ . However with constant  $M = M_a + M_s$ , apart from  $M_{at}$  needed, if  $Y_{t+1} > Y_t$  then  $M_{at+1} > M_{at}^1$ . The extra  $M_a$  must be drawn

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1. To the extent that  $M_a$  is not given greater velocity, or money work per money unit.

from the SP ( see (9) above) and hence  $M_{st+1} < M_{st}$  . This will usually only occur if  $r$  in the productive system increases its margin over yields from money and securities in the savings pool.

Chapter 19.THE SUPPLY OF MONEY1. Introductory.

The money supply mechanics of the modern economy has developed partly out of the money and banking discoveries of the goldsmiths, mentioned briefly above, and partly from the institutional evolution which has followed. The money systems of the nineteenth century were based on national gold and foreign exchange (IM) holdings, with the prices of currencies fixed in terms of gold, and with domestic money freely exchangeable into gold. With fixed exchange rates an excess domestic demand for IM (balance of payments disequilibrium) caused a running down of IM and IR, and hence a reduction in the domestic money base. Ensuing deflation and unemployment presently corrected the excess demand, possibly converting it to an excess supply of IM, a disequilibrium in the opposite direction. Unofficial central banks used "bank rate" as a stabilizer, but the ship still continued to roll with the ebbs and flows of world trade.

In the twentieth century gold standards were abandoned one by one, usually with little choice, as World War I and its aftermath drained gold from most countries to the highly productive and undamaged US. These countries were then forced to use a managed fiat and fiduciary money supply. Then as the Great Depression left its scars most countries became determined to manage their money supplies in accordance with internal full employment and development needs

and independently of foreign trade and finance. They set up central banks to do this. Modern money systems are still significantly affected by balance of payments developments, but there is much that central banks can now do to offset these when they are adverse.

## 2. The Central Bank.

The CB is a modern institution which evolved out of 'bankers' banks "like " The Old Lady of Threadneedle Street". It is publicly owned, a semi-autonomous body, coming sometimes under the Minister of Finance or the Treasury, sometimes under the Cabinet, or the Executive or Privy Council. It is enjoined to regulate the money supply of the nation in such a way as to protect the external value of the monetary unit, to maintain full employment, and to promote the general financial and economic welfare in as much as this can be achieved with monetary action.

The CB provides the money supply base in the modern economy partly through its powers as the sole issuer of currency. It has paper note money  $M_n$  made to its order, and it buys coin money  $M_m$  from the national Mint at face value. It is further empowered to grant deposits to the CG and to the MCB in exchange for currency, securities of the CG, and IM ( including gold, of course). Thus we see that the money base is emitted from the CB through the lending-spending activities of the CG, and through foreign trade and finance.

In Canada at the present time the CB buys and sells some IM on its own account, and in addition operates an Exchange Fund



Account (EFA) for the Minister of Finance. Since the domestic money of the EFA is on deposit with the CB, for our analysis it is as though the EFA were part of the CB. Both CB and EFA are empowered to buy and sell short-term securities in foreign exchange centers.

The CB is also empowered to lend deposits to the MCB at Bank Rate  $i_c$ .

### 3. The Creation of Money and its Issue to the Public.

There are three basic ways in which the money supply  $M = M_m + M_n + M_b$  of a modern economy becomes issued. The first is the borrowing of money by the CG from the CB in exchange for CG securities. As the CG spends this money it flows into the private sector. The second is where an exporter having obtained IM sells it to the MCB in exchange for  $M_b$ . If the MCB can count the IM as cash reserves (they can't in Canada now) they may be content to hold it, and perhaps invest it in short-term securities in a foreign exchange center. If the IM cannot be so treated, and if it is above transactions needs the MCB will sell it to the CB or the EFA, receiving deposits with the CB in exchange. MCB cash reserves (CR) are thereby increased by exports and decreased by imports. Should however a CG fund operating with  $M_b$  domestic money buy the IM, the net effect is a transfer of  $M_b$  from the government to the exporter, with no change in CR.

The third way in which new money is created and issued is the granting of deposits by the MCB to individuals, firms or government in exchange for promissory notes, bills of exchange,

short or long term securities, and in some cases mortgages.

#### 4. Limitations on the Issue and Supply of Money.

In the previous century the volume of IR provided a base which set limits to the supply of money, for gold and other IR were the cash reserves of the banking system. To some extent the IR still provide a rather general limitation of the money supply. For if  $M$  is increased too much imports and capital outflow may deplete IR or increase  $e_1$  beyond bearable levels, forcing either reduction of  $M$  or some form of CG control of imports and capital exports. But the proximate limitation on the money supply in gold standard days and today is the CR of the MCB. These consist of  $M_{mb} + M_{nb} + M_{dcb}$ , where  $M_{dcb}$  are deposits of MCB with CB. These cash reserves are created by the CG and CB, and are on the surface not connected with IR. But as we have seen, they are both directly and indirectly related to IR. The difference with modern money is that it is not rigidly related to IR, as it was in gold standard days.

Since the bulk of business is transacted with  $M_b$ , the MCB find that only relatively small  $CR = M_{cb}$  (currency) +  $M_{dcb}$  are necessary for safety. A cash reserve ratio  $cr = CR / M_b$  of 5 per cent is adequate in normal times. However banks have tended to use  $cr = 10$  per cent or thereabouts, to be on the safer side. The legal minimum ratio in Canada is now 8 per cent. There is no legal ratio in Australia and  $cr$  may be changed from time to time by CB action calling part of  $M_{dcb}$  into impounded Special Accounts ( $M_{dcbs}$ ).

These are not available as CR and hence the operating cr is reduced. The Australian MCB used to operate on a cr of about 10 per cent. With the introduction of the Special Accounts their cr has become about 45 per cent, while their operating cr,  $cr_1 = \frac{CR - M_{dcbs}}{M_b}$ , has become about 5 per cent.

Given a temporarily fixed cr (or  $cr_1$ ), an important money supply relation follows:

(1)  $M_b \leq CR / cr$ , so that  $1/cr$  becomes a "cash reserve multiplier" in the money supply. In Canada this used to be 10, is now near 12.5, while in Australia it is around 20 in the short run, in between CB changes in the Special Accounts, but only about 2.2, in the longer run.

A further limitation to the money supply is the proportion of its money that the public decides to hold as currency.

Let  $M_{cp}$  be public holdings of currency. Then

(2)  $M = M_{cp} + M_b$ ;  $M_{cp} = m_1 M$ , with  $m_1$  a constant for given habits and institutions.

(3)  $M_b \leq \frac{1}{cr} (M_{cb} + M_{dcb})$ ;  $M_{cb} = m_2 M_{cp}$ , with  $m_2$  a constant discovered by banking practice.

(4)  $M \leq \frac{M_{dcb}}{cr - m_1(cr + m_2)}$  Thus the money supply multiplier on changes

in MCB deposits with CB is less than the CR multiplier, because of the public's adjustment of its currency to the new money supply. The multiplier for initial changes in total CR, taking account of subsequent public currency adjustments is

$$(4a) \quad dM / dCR = \frac{1}{cr} \left[ \frac{1}{1 - m_1 \left(1 - \frac{1}{cr}\right)} \right] < \frac{1}{cr} .$$

Thus the influence of CR on total M is diminished by increases in  $m_1$ , the public's propensity to use currency. This is because increases in  $M_{cp}$  must be drawn from CR.<sup>1.</sup>

##### 5. Factors Which Cause CR to Change.

If we assume that  $cr$ ,  $m_1$ , and  $m_2$  are given constants in the short-run, then M changes only as a result of changes in CR. What are the influences which change CR, or  $M_{dcb}$ ?

(a) Balance of International Payments Surplus  $\Delta$  IM creates

$$\Delta_1 M_{dcb} = b_1 \Delta IM = \Delta IR_g .$$

In Australia  $b_1 = 1$  for all  $\Delta$  IM can be sold to the CB for  $\Delta M_{dcb}$ . In Canada this is also substantially the case, since the Exchange Fund Account keeps its account with the CB. Included in  $\Delta$  IM is an increase in the domestic mining of gold, which is then sold to the CB or the EFA.

(b) CG Borrows from CB and Spends Part of Proceeds in Private Sector.  $\Delta_2 M_{dcb} = \Delta S_{esgc} + \Delta S_{elgc} - \Delta M_{dgc}$

(c) CB Buys Securities of CG in the Open Market.

$$\Delta_3 M_{dcb} = \Delta S_{egc} . \text{ (Open Market Operations)}$$

(d) MCB Borrow from CB.

$$\Delta_4 M_{dcb} = \Delta S_{esbc} .$$

(e) Public Changes its Propensity to Use Currency.

1. Thus cheque charges and other costs charged on the use of  $M_b$  tend to increase the public's use of currency, thereby increasing the costs of providing currency, decreasing the money supply, and decreasing the productivity of money in facilitating transactions.

$$\Delta_5 M_{dcb} = - (1 + m_2) \Delta M_{cp}.$$

$$\Delta_5 CR = - \Delta M_{cp}.$$

(f) Special Accounts of MCB Increased (Australia Only).

$$\Delta_6 M_{dcb} = - \Delta M_{dcbs}.$$

## 6. The Supply of Money.

We are now ready to develop a supply function for money. We do this by first building in the variables that cause CR to change.

$$(5) \quad CR = CR_{-1} - \Delta M_{cp} + \Delta IR_g + \Delta S_{egc} - \Delta M_{dcb} + \Delta S_{esbc} - \Delta M_{dcbs}$$

(In case the MCB are allowed to treat their IR holdings as cash reserves,  $IR_g$  in this formula becomes simply IR.)

Next we consider the MCB responses in the money market to changes in the demands to rent the  $M_b$  which they create. The banks make most of their profit from these loans. A study of their balance sheets shows that they have a variety of ways of holding earning assets in return for the use of their money. Their most common borrower is usually the business firm to whom they lend money in return for short-term securities  $S_{esfb}$  - promissory notes, bills of exchange, overdraft agreements. They also make some loans to households on securities  $S_{eshb}$ . These loans produce the highest interest yield to the banks. But they do not put all of their eggs in this basket, and in addition they exchange their  $M_b$  for long term securities of firms, short and long term securities of government, and for mortgages.

When times are good the MCB tend to shift out of the lower yield - higher safety securities ( in effect long-term loans to government and firms) to the higher yield short-term loans which are their traditional field of lending. Note that  $i_b$  must be greater than the yield  $i_l$  on these, or individuals and firms could profit by making bank loans to finance their holdings of longs. Should the demand for bank short loans fall, thereby reducing  $i_b$  below  $i_l$ , the banks would reduce their supply of short loans by buying more longs of government and firms themselves. Bank shorter loans are primarily used for financing inventories and the current operations of business. As business expands the demand for these shorter loans increases,  $i_b$  can move higher until it reaches its legal maximum, and the MCB gradually reduce their holdings of longs in favour of short loans. Where there may have been excess reserves in the MCB in poorer times, now they will expand  $M_b$  until  $cr$  reaches its legal minimum. At this point they may even find it worth while to borrow additional CR from the CB at Bank Rate  $i_c$ .

This we see that the banks' supply of their product is responsive to changes in its price or rent. A deterring influence is the volume of business failures, which we represent by  $BF$ , the value of defaulted liabilities in these. Our supply equation becomes

$$(6) M^s = f ( CR / cr, m_1, i_b, i'_b, i_c, BF ).$$

$i'_b$  in this equation is the average yield on all bank loans ( securities held). As the banks shift out of lower yield securities into  $i_b$  loans,  $i'_b$  is of course increased. Since the market is not one of

pure competition ( see next chapter and Part VIII) this equation can be taken as representing middle and long-run supply.

### 7. The Short Money Market and Bank Rate.

In working with low cr the MCB must keep some of their non-CR assets in fairly liquid form. They achieve this by buying a certain proportion of very short-term securities - call money loans, Treasury Bills and discounted bills of exchange. These can be sold for cash almost immediately with little or no capital loss, since their due date is usually within three months. Yet they bring a positive yield, as well as providing a secondary cash reserve ( SCR) for the MCB. The yield on these short securities  $i_s$  is usually lowest in the pattern of interest rates, for liquidity is given up for the shortest time, and the risk of capital loss is also least. But  $i_s$  moves up and down as the total pattern of interest rates moves under money market conditions. Bank Rate  $i_c$  must be kept above  $i_c$  because otherwise the banks could make profit by borrowing  $M_{dcb}$  and investing  $(1/cr) \Delta M_{dcb}$  in the very shorts. This could disrupt the whole system of interest rates, the banks traditional function, and the money supply. Note that when  $i_b$  is high and there is heavy demand pressure for  $M_b$  the banks can increase  $i_b$  by reducing SCR to a bare minimum.

### 8. Mechanics of Monetary Policy.

The above analysis reveals the methods available to CG and CB for controlling the money supply  $M$  and  $M_p$  in the interests

of economic health. These may be listed as:

- (a) CG increases (retires) debt to CB, injecting (withdrawing) money with respect to private sector ( money multiplier approx.  $1/cr$ );
- (b) CB open market operations ( multiplier approx.  $1/cr$ );
- (c) CB raises or lowers Bank Rate;
- (d) CB varies the legal minimum CR ratio  $cr$ ;
- (e) CG and CB free exchange rate  $e_1$  to keep  $IR_g$  from fluctuating;
- (f) CB varies Special Accounts of MCB ( Australia);
- (g) CB induces MCB to maintain a minimum secondary cash reserve ratio  $scr = (CR + SCR) / M_b$ .



## Chapter 20.

THE DEMAND FOR MONEY, AND THE MONEY AND SECURITIES MARKETS.1. A Preliminary View.

In our previous study of households and firms we saw them deciding on the separation of income and wealth into saving and wealth. In the first instance saving tended to take the form of an increase in money,  $M_s$ . Having made this first decision now a second major choice must be made. Saving is designed to increase wealth, which will in turn serve as a store of future purchasing power. But money is only one means of storing wealth. The other important means are to hold securities, real estate and unincorporated business. Economic units must choose among these forms of wealth on the basis of how well they think each will perform the storage function.

In making this choice of the pattern of holding wealth some money will be chosen, especially if prices are falling thereby giving money a real yield. This will be the bulk of  $M_{s2}$ , and provides part of the total demand for money. The small remainder of  $M_{s2}$  (inactive money) is held to provide ready liquidity in case of emergency, or in preparation for a bulk purchase.

The remainder of the demand for money arises from its usefulness in active exchange.  $M_{s1}$  is held in the SP for making the many reallocations of wealth which economic units decide upon. This is the second main component of the overall demand for money. Finally there is of course the active money,  $M_a$ , which is held by economic

units for purchasing the final goods of the productive system, and for performing all of the intermediate exchanges within this real system. This is the third part of the total demand for money ( Note that our money categories are not quite the same as the Keynesian  $M_1$  and  $M_2$  ( [1.6] Chap. 15).  $M_2$  of Keynes ( speculative-motive) includes only that part of  $M_{s2}$  which would be exchanged for other wealth forms if the real yields of these increased sufficiently.)

These demands for money interact with its aggregate supply in what we may designate as the primary money market of the economy, producing the ex post money supply  $M$  and the interest rate  $i_b$ . But since the demand for money is in part a choice between money and other forms of holding wealth, the markets for securities and real estate become rather intimately related to the money market. In fact to a large extent these markets can be thought of as existing or "second-hand" money markets. The MCB are not directly involved, and in each of them a supply of securities or other wealth is a demand for money, while a demand for securities is a supply of largely 'second-hand' money.

The wealth which is to be rearranged after initial savings has taken place is summed up in the following identities

$$(1) \quad N W_{hm} = M_{ah} + M_{s1h} + M_{s2h} + S_{egh} + S_{ei(fi)h} + S_{ee(fi)h} + S_{emhh} \\ + K_{sdhdam} + K_{d+1,hm} + K_{ubm} - S_{esho} - S_{emho}$$

$$(2) \quad N W_{fm} = M_{af} + M_{s1f} + M_{s2f} + S_{esff} + S_{eshf} + S_{egf} + S_{eil(fi)f} \\ + S_{ee(fi)f} + S_{emhf} + H_{fm} + K_{fm} + K_{d+1,fm} - S_{esfo} - S_{eilfo} = S_{eefo} \\ + K_{ubm} \cdot$$

The demand for money can only be fully achieved by analysing the demands for each of the elements within the wealth complex, most of which is represented in (1) and (2). The problem is somewhat similar to the theoretical consumer demand analysis of Chap. 7. But here  $NW$  replaces income and yields replace prices. (Cf. savings analysis Chap. 8). Also here we have the peculiarity of negative elements, which become supplies rather than demands. We deduce independent  $NW$  effects, plus  $NW$  and substitution effects arising from yield changes. Presumably the demands and supplies of each economic unit will be such as to attempt to maximize the aggregate yield of its wealth.

## 2. Demands for $M_a$ .

Money is held out of income for planned current spending in the productive system. The quantity held out is sufficient to last until the next income date. Hence on an average day in the economy  $M_a$  will be roughly one half of the planned spending between income dates. We anticipate some substitution toward economising on  $M_a$  should its cost  $i_b$  or its opportunity cost  $i_h$  increase. Consequently

$$(3) \quad M_{ah}^d = f_1(C_m^d, i_h)$$

(4)  $M_{af}^d = f_2(W_{am}, \Delta H_m, GIPCM_m, P'.GDF, i_f)$ , where  $i_f$  is a weighted average of  $i_s, i_l, i_b, i_m, i_e$ , appropriate to the firm.  $i_e$  is the yield on equity securities.

$$(5) \quad M_{bg}^d = \text{exog.} ; M_{bi}^d = \text{exog.}$$

### 3. The Securities Markets.

Here we shall consider a representative securities market, and the general results obtained will then be used for the individual markets.

#### (a) Supply of Security.

The current daily supply on the market comes from two sources. The first is from firms or individuals who want more money to spend in the productive system, and who issue new securities and put them up for sale in the market. The 'interest cost' for servicing this security must be less than the marginal efficiency or yield of active money and capital in their businesses, or less than the time preference for goods in the case of households. The second supply source are those units who are rearranging their wealth holdings, and who believe that the prospective yield of this type of security is low relative to yields from alternative forms of storing wealth. These are the "bears" of the market.

The yield on a security is a gain to the purchaser and a cost to the supplier. If its present selling price is  $S_{e0}$ , its expected future selling price or final payment in period  $n$  is  $S_{en}$ , and the stream of interim service payments is  $d_1, d_2, \dots, d_n$ , yield "y" is given by the formula

$$(6) \quad S_{e0} = \frac{d_1}{1+y} + \frac{d_2}{(1+y)^2} + \dots + \frac{d_n}{(1+y)^n} + \frac{S_{en}}{(1+y)^n} \quad \cdot \text{Solving for } y$$

gives

$$(7) \quad y = f_3(S_{e0}, S_{en}, d_1, \dots, d_n). \text{ Note that } y \text{ takes its value}$$

partly from capital gain, and partly from the stream  $d$ . Corresponding to money yield  $y$ , is a real yield  $y_r$ , found by converting all values in (6) and (7) to real purchasing power.

$$(8) \quad y_r = f_3 \left( S_{eo}/P_o, S_{en}/P_n, d_1/P_1, \dots, d_n/P_n \right) .$$

Where there is complete absence of money illusion it will be real yields that will be compared in choosing among wealth forms.

In comparing the yields on securities with the yields on money we must consider how the yields on money can be appraised. Presumably  $M_{ah}$  has yield  $i_h$  at the margin in equilibrium, while  $M_{af}$  has yield  $r$ . Similar conclusions can be drawn for  $M_{s1}$ . In the case of  $M_{s2}$  however real yields become of more relevance. Apart from other values or utility from holding  $M_{s2}$ , it has a real yield arising from price changes only ( see (8) ). This is positive when prices are falling and may be greater than the real yields on other forms of wealth. But this real yield for money is negative when prices are rising, while that for other wealth forms is positive.

In summary the very short-run supply of a security depends on spending plans in the real system ( net new issues ) and on the number of 'bear' holders, who believe its yield to have dropped below that of other wealth alternatives. Let us now consider the longer run supply. For this we can ignore the day-to-day shifts of ownership, and simply treat the total issued amount plus net new issues as the supply. Net new issues of course still depend on spending plans, and servicing cost, while bearishness will depend on relative prices and yields.

$$(9) \Delta S_e^s = f_4( X, P, P_{se}, P_{so}, y_{se}, y_{so}, r )$$

(10)  $S_e^s = S_{e,-1} + \Delta S_e^s$ . Here X is the economic activity to which the proceeds of  $\Delta S_e$  will be applied,  $P_{se}$  is the current price level of  $S_e$ ,  $P_o$  is the corresponding price levels of other forms of storing value,  $y_{se}$  is the expected yield on  $S_e$ , and  $y_o$  is the corresponding yields on competitive wealth forms.

(b) Demand for Security.

As the multitude of economic units consider rearranging their wealth holdings to maximize their aggregate yields, some of them will have favourable opinions about the prospective yield of the security in question. They become the "bulls" in the market for this security. Those bulls who already hold some of the security will continue to hold, and some of these will demand more of it. Other bulls, not as yet holders will exert demand and become holders.

It is in fact the dispersion of opinion between bulls and bears that gives the market some stability. Should all marketers become bulls or bears at once, price would move violently up and down.

A further important influence on both demand and supply is the recent price movements of the market. These set up price and hence yield expectations and cause price autocorrelation in these markets.

The suggested demand equation is

$$(11) S_e^d = f_5 ( W, NW, P_{se}, P_o, y_{ser}, y_{or}, \vec{P}_{se}, \vec{P}_o, \vec{P}, E ).$$

We now consider specific securities markets.

(c) Market for Short-Term Securities.

- (12)  $S_{esgo}^s = \text{exog.}$
- (13)  $S_{esfo}^s = f_6 \left\{ (\Delta H_m), P, GDF, \vec{P}, i_s, r \right\}$
- (14)  $S_{esho}^s = f_7 \left\{ C_{hdm}, C_{am}, \vec{P}, i_{sh}, B_s(i, d, t) \right\} - (\text{consumer credit})$
- (15)  $S_{esdi}^d = f_8 \left\{ (i_s - i_{SRW}), E_s \right\}$
- (16)  $S_{esof}^d = f_9 (L_f, NW_f, \vec{P}, i_s, i)$

(d) Market for Long-Term Government Securities.

- (17)  $S_{elgo}^s = \text{exog.}$
- (18)  $S_{elgc}^d = \text{exog.}$
- (19)  $S_{elgh}^d = f_{10}(i_{lg}, i_{lf}, i_m, i_e, rd, r, P_{slg}, P_{slf}, P_e, P_{d+1}, \vec{P}, \vec{P}, W_h, NW_h, L_h) = f_{10}(V_h).$
- (20)  $S_{elgf}^d = f_{11}(V_f)$
- (21)  $S_{elgb}^d = f_{12} \left\{ i_{lg}, i_{lf}, i_m, i_e, i_b, P_{slg}, P_{slf}, P_e, (M^s - M^d)_{-1} \right\} = f_{12}(V_{1b})$
- (22)  $S_{elgi}^d = f_{13}(i_{lg}, i_{lf}, i_e; i_{lgrw}, i_{lfrw}, i_{erw}; E_i) = f_{13}(V_{2i})$

(e) Market for Long-Term Fixed Interest Securities of Firms.

- (23)  $\Delta S_{eilfo}^s = f_{14}(G_{PCMLm}^d, P_{slf}, P_e, i_{lf}, i_e)$
- (23a)  $S_{eil(fi)-1}^s = f_{14a}(i_{lg}, i_{lf}, i_e, i_m, rd, r, P_{slg}, P_{slf}, P_e, P_{d+1}, \vec{P}, \vec{P})$  (holders and bears).
- (23b)  $S_{eil(fi)}^s = S_{eil(fi)-1}^s + \Delta S_{eilfo}^s + \Delta S_{eilid}^s.$
- (24)  $S_{eilfih}^d = f_{15}(V_h)$
- (25)  $S_{eilfif}^d = f_{16}(V_f)$
- (26)  $S_{eilfib}^d = f_{17}(V_{1b})$
- (27)  $S_{eilfi}^d = f_{18}(V_{2i})$

(f) Market for Equity Share Stocks.

(28) --- (32) in  $S_{ee}^s$ ,  $S_{eeh}^d$ ,  $S_{eef}^d$ ,  $S_{eeb}^d$ ,  $S_{eei}^d$ , and functions  $f_{19}$ , ---,  $f_{23}$ , in pattern identical to (e).

(g) Market for Residential Mortgages.

(33)  $\Delta S_{emho}^s = f_{24}(C_{d+1}^d, i_m, i_h, B_m(i,d,t))$ .

(34)  $S_{emho}^s = S_{emho,-1}^s + \Delta S_{emho}^s$ .

(35)  $S_{emhh}^d = f_{25}(V_h)$

(36)  $S_{emhf}^d = f_{26}(V_f)$

(37)  $S_{emhb} = f_{27}(V_{1b})$

(h) Market for Residential Real Estate as Store of Value Investment.

(38)  $K_{d+1}^s = K_{d+1,-1}^s + C_{d+1}^d - D_d$ .

(39)  $K_{(d+1)hm}^d = f_{28}(V_h)$

(40)  $K_{(d+1)fm}^d = f_{29}(V_f)$

(i) Unincorporated Business as Store of Values Investment.

(41)  $K_{ubm}^s = K_{ubm}^d = f_{30}(V_h)$ .

4. Demand for  $M_{sl}$ .

Having covered the main markets in which value stores can be reallocated to maximize aggregate yield, we can now consider the impact these markets have on the demand for transactions money in the SP.  $M_{sl}^d$  will depend on the volume of transactions in the various markets of the SP. This volume will tend to increase as the differences between price levels and yields in  $V, V_1$  and  $V_2$  become greater, and conversely. It may also increase with increasing wealth.



Let  $V^*$  be an index of the relevant differences in the elements of  $V$ .

$$(42) \quad M_{slh}^d = f_{28}(V_h^*, i_h, W_h, NW_h, E_h) .$$

$$(43) \quad M_{slf}^d = f_{29}(V_f^*, i_f, W_f, NW_f, E_f) .$$

As for  $M_a^d$ , there will be some substitution to economise  $M_{s1}$  as  $i$  increases.

#### 5. Demand for Inactive Money $M_{s2}$ .

Inactive money is now the only remaining element of the net wealth identities (1) and (2) which has not as yet been analysed. Since it is part of an identity in which  $NW$  has been determined by saving, and all other elements have been determined in their respective markets, independent demand equations cannot be set up for  $M_{s2}$ . Should such independent equations be set up the system would be overdetermined.

However there is a sense in which  $M_{s2}^d$  is as independent as any of the other elements of (1) and (2). All store of value choices have been choices among all of the various elements, including  $M_{s2}$ . Part of the value of holding  $M_{s2}$  is established by  $P$  movements in the demand equations. The remainder finds its way into the parameters of (12) --- (43). Thus the residual element, in this case  $M_{s2}$ , is in effect already chosen by time it is reached. But any element in (1), (2) could be chosen as the residual. In the present model  $M_{s2}$  is chosen for this role because it is more like a residual, in that much original saving takes place in this form, and hence it is probably the easiest wealth item to shift into.

It is possible to express demand equations for  $M_{s2}$  by combining the identities (1) and (2) with the demand and supply equations of all other elements in (1) and (2). Thus

$$(44) \quad M_{s2h}^d = NW_{hm}^d - M_{ah}^d - M_{slh}^d - S_{elgh}^d - S_{eiloh}^d - S_{eeh}^d + S_{emhh}^d - K_{sdhdam}^d \\ - K_{d+1,hm}^d - K_{ubm}^d + S_{esho}^s + S_{emho}^s = f_{31}(C_m^d, C_{d+1,m}^d, \vec{P}, P, \\ i_h, B_s(i,d,t), B_m(i,d,t), V_h, V_h^*)$$

$$(45) \quad M_{s2f}^d = NW_{fm}^d - M_{af}^d + S_{eilfo}^s \\ = f_{32}(P, GDF, \Delta H_m, GI_{PCMLm}^d, \vec{P}_{GI}, P, P, i_f, V_f, V_f^*)$$

## 6. Completion of the Analysis of the Money Market.

In Chapter 19 we developed the supply side of the money production market. In this chapter we have followed the demands for this money in active production of goods, in active use in various SP exchange markets between money and other value storage forms, and finally in its own capacity as an inactive store of value. The aggregate of these several demands gives us the demand side of the money production market.

$$(46) \quad M^d = M_{ah}^d + M_{af}^d + M_{slh}^d + M_{slf}^d + M_{s2h}^d + M_{s2f}^d + M_{bg}^d + M_{bi}^d$$

This demand equation combines with (9) of Chapter 19 to determine the ex post equilibrium quantity of money and interest rate  $i_b$ . It is not likely however that this equilibrium market position will be often reached. This market is not usually an atomistic one of pure competition (See Part VIII). Hence market price  $i_b$  tends to change only slowly, in orderly fashion, but toward the equilibrium position. The short-run market path will then follow a market adjustment and short-run supply

relation like

$$(47) \quad i_b = f_{33} \left\{ i_{b,-1}, (M^d - M^s) \right\}.$$

### 7. The Pattern of Yields in Wealth Stores.

There seem to be two rather basic yields in the system, to which the others adjust in a logical pattern. The first of these is the marginal efficiency of real capital and enterprise,  $r$ . This must be the highest yield in the system, and out of it the other yields are paid to the extent that they <sup>represent</sup> contributed property in the form of purchasing power. The second basic yield is  $i_b$ , the rent on newly created money. It must be below  $r$ , for it would be uneconomic for firms to rent  $M_b$  at a higher cost than it could yield to them. Some consumers might be willing to drive  $i_b$  above  $r$ , for their time preference rates may at times be greater than  $r$ . But this is prevented by the legal maximum usually set for  $i_b$ .

The other yields in the system tend to adjust to levels related to these two by the degrees of loss of liquidity and risk involved. Loss of liquidity depends on time to repayment and marketability. Risk is related to the danger of capital loss should liquidity be required when market values are low, and to the danger of default and bankruptcy (BF). In each instance the buyer must appraise the probability distributions and the expected value, and allow a safety margin based on dispersions. The following normal pattern of yields is suggested, but with the caution that distortions can arise where excessive illiquidity pushes a yield above its position in a

tight money situation ( e.g.  $i_m > i_b$ ) or where the market has not had time to correct an abnormal pattern ( e.g.  $i_s > i_l$ ).

The following yield pattern is suggested as roughly normal, in ascending order :

(48)  $i_s, i_c, i_{lg}, i_{lf}, i_m, i_e, i_b, r$  .

## 7. Interactions Between the Money and the Real Systems.

The money and real systems are both major areas of the complete economic organism. Hence what is happening in one has important effects on the other. For example if the real system becomes more active  $M_a$  will increase, and then  $M_s$  must decrease. But now  $M^d$  will probably advance and thereby increase both  $M$  and  $i_b$  unless the monetary authorities decide against this. Initially security prices will fall and yields rise, until the money supply increases and slows this trend, perhaps even reversing it. Equities should begin to increase in price as  $r$  and dividends rise, with a relative decline in fixed interest security prices. As money becomes tight this decline will become absolute.  $M_{s1}$  may increase, but  $M_{s2}$  is likely to become quite low.

At the same time developments in the money system will have important reactions on the real system. As  $M$  expands, and as more securities are issued and demanded,  $L_h$  and  $L_f$  will increase, and  $NL_h$  and  $NL_f$  will probably expand as well. The money system has now fully determined these variables ( cf. their definitions (3) and (4) of Chap. 18). But they in turn influence demands for real goods, and

hence the levels of real output and prices. As the boom proceeds  $i_p$ , and the complex of interest rates which are related to it by the SP financial markets, will increase and perhaps narrow the range  $i-r$ , eventually inhibiting investment. When  $i_p$  reaches its maximum there may be considerable excess demand in the money market, so that bank lending must be rationed. Now security prices in the 'second-hand' money markets may fall very low as the demand for second-hand or existing money grows, and yields on all securities rise. The actual shortage of  $M$ , combined with high  $i$  and eventually reducing  $r$  will in time cause some firms to reduce plans, and a downturn may begin.

It is then the variables  $i$ ,  $L$  and  $NL$  which gear the money system back into the real system, and through which the money sector reacts on real activity of the total organism. Monetary policy can influence these variables through its ability to alter components of the money supply.

Part VIIITHE PRICE SYSTEM AND THE PRICE LEVEL.Chapter 21TYPES OF MARKET BEHAVIOUR1. Purpose.

In this section we attempt to get a working picture of the function of prices in the total economic organism, and of how the level of these prices is formed. Up until now the price level variables  $P$  and  $P'$  have appeared in many equations as causal variables. Now we must treat one of these (they are interrelated) as an effect variable, and see what the proximate forces are that determine it. Since all of the important final goods demands, and final goods supply have already been explained it might appear that  $P'$  is already determined by the interaction of these. This would be true if we were dealing with long time periods, or if all prices were flexible. Otherwise, as we have already been warned by the labour and money markets, a short-run market analysis may be necessary.

2. The Nature and Function of the Price and Profit System.

Prices are the rates of exchange between money (generalized purchasing power) and individual economic goods. An increase in an individual price influences supplying units (cells) to increase supply, and using or demanding units to diminish purchases.

Supplying units find themselves with greater profits which they attempt to increase still further by drawing more resources into their line of production. The converse tends to be true for a falling price. Thus the price pattern in an economy acts as a system of 'slow down' and 'speed up' indicators, which act as 'feed-back' mechanisms to bring resource use and final output into agreement with social preferences as constrained by costs.

Such agreement is only transitory, for the social organism is constantly in a state of change with respect to its resource base, techniques of production, pattern of wants and preferences, and distributions of income and wealth. Any change in these calls for a different evaluation and use of resources, and by and large the new adjustment is achieved through the price - profit regulator mechanism.

Although this system has great social value, it also has some imperfections. For example: it may redirect resources too slowly during national emergencies such as war; it tends to give priority to economic values, which are often in conflict with broader social values; it tends to generate a distribution of income and wealth which becomes increasingly uneven as wealth accumulates at compound interest, and which eventually becomes socially unacceptable; the control mechanisms tend to generate oscillatory or cyclical movements as they bring about adjustments; where production has become monopolistic, the price system may not induce the socially desired shift in resources and output, and cost of production may remain well above the minimum average cost to which an ideal price system and pure competition would

tend.

These imperfections make the complete "laissez-faire" operation of the system unsatisfactory. Conscious control and an appropriate social policy is needed to steer the system toward social goals. Each of the above imperfections becomes a component of government agenda, in the department of economic policy. Given good control and regulation, the system still retains its own automatic, decentralized, and democratic regulatory mechanisms which must contribute a great deal to the attitudes and the productivity of the total economy. It can still reward efficiency and innovation and penalize inefficiency and rigidity.

One would expect government intervention in the system to alter the price pattern at any time considerably. In Part VI it was suggested how the tax and transfer system and the pattern of government demand itself can give greater equality to the 'votes' which each individual preference system can cast, in determining prices and aggregate resource uses in the economy.

### 3. The Role of Inventory Stocks in the System.

As will be discussed later many prices in the system do not move as rapidly as shifts in demand and supply conditions occur. Where there is price sluggishness, price cannot provide quick signals to decrease purchases and increase production, or conversely. In such cases a supporting control mechanism comes into play. This is the system of inventory 'fat' in the system. If demand falls off in a



particular line, and price is slow to fall, producers' inventories will pile up above desired levels. This will rapidly create storage availability and cost problems, which will force production to be reduced, even though prices have not yet fallen. Conversely when an increase in demand occurs in a line where price is sluggish, purchases will not be at all restrained by price increase. Consequently inventories will rapidly run down. They will move well below the level of desired inventories  $H^d$ , eventually reaching a minimum level  $H_{\min}$  below which production schedules of both supplying firms and their customers would be jeopardized. Once all inventory slack has been used up purchases become restrained to the level of output. At the same time the inventory shortage will induce suppliers to increase output, if they have any excess capacity. If they have no excess capacity the market will be in a state of excess demand with unfilled order backlogs. This condition will persist until gradual price increases, coupled with enlargement of capacity, eventually wipe out the excess demand.

Thus in an economic system where some markets are not perfectly competitive, it is a combination of the price and inventory systems which provide the short-run feed-back mechanisms to adjust resource uses to changing preference systems and changing technology. Where price is slow moving inventories provide the early signals and adjustments, while price will take care of the final or longer-run adaptation.

#### 4. Markets and the Time Factor in Market Adjustment.

A market may be defined as the physical and instit-

utional arrangements whereby buyers and sellers come together, bargain over prices, quality and quantity, and consummate sales contracts. It may consist of a specific area and buildings, or it may consist only of the offices of salesmen and purchasing agents in firms. For our present purposes however the aspect of interest in any market is its tendency to be in equilibrium or disequilibrium, or its speed of adjustment to changes in demand and supply conditions. In the case of predominant equilibrium  $q^s$  and  $q^d$  equality is achieved mainly by price movements. In the disequilibrium case inventories take the initial shock of changes, and  $q^s$  and  $q^d$  may rarely reach equality. This is because the time intervals between  $q^s$  and  $q^d$  shifts may be less than the time required for a complete adjustment to equilibrium.

If however time is aggregated into long time periods, longer than the time required for equilibrium adjustments, basically disequilibrium markets may appear as equilibrium markets in longer-run analysis.

Various types of markets will now be considered from the point of view of the time and the nature of the adjustment to changing demand and supply conditions.

(a) A Market of Pure Competition. In markets of this type the numbers of buying and selling units must be large, thereby rendering the market "atomistic" and impersonal. The product sold must be nearly homogeneous, so that one producer is as good as another. Competitive advertising and predatory competition will be non-existent. Examples of such markets are those for some farm products; primary

raw materials like rubber, wool, cotton; and the large securities markets of financial centers. Such markets move to their equilibrium points with great speed, following any shift in the  $q^s$  or  $q^d$  functions. Inventories need not be reduced to  $H_{\min}$ , and excess demand or supply on the market is wiped out by price movements in a matter of hours or days. Prices are accordingly inclined to be volatile.

(b) Monopsonistic Market - Few Buyers, Large Number of Sellers.

An example of such a market would be a large number of crude oil well operators selling their output to only a few large refineries. The few large buyers will likely form some kind of association, and will have considerable power over the small sellers. They will be able to set prices, and will tend to set them low, but at the minimum necessary to draw forth sufficient supply. Prices will tend to be steady, only changing after long intervals, as the buyers find it convenient or necessary.

Although they possess considerable power, the refiners cannot completely disregard the feelings of the well operators or of the general public. Hostile suppliers could prove costly, and an aroused public opinion could demand government control or nationalization.

(c) Monopolistic Market - Few Sellers, Large Number of Buyers.

This is the situation in heavy, large-scale industries like newsprint, steel, chemicals, automobiles. Here the sellers have some power over the buyers. They will usually group into some form of association to advance their interests both generally, and with respect to market

power. Prices will be set by some form of price leadership, and will usually be steady for long intervals. The sellers need not worry much about offending individual buyers, since customer power is usually diffuse. But they must be wary of offending the buyers as a group, or of offending public opinion at large. Price increases must not be too frequent or too large, and will usually be <sup>prepared for by</sup> suitable "explanations" in terms of costs (mainly prime costs, of labour and materials). Thus, for fear of spoiling customer and public goodwill, the sellers do not take immediate advantage of every demand increase. Their policy is rather to slowly 'catch up' in the longer run. A further reason for not charging all that the traffic will bear in the short run is the fear of attracting new capacity into the field and thereby "spoiling the market".

In this type of market price may be even more sluggish in its downward than in its upward movements, especially if demand is rather inelastic. This is because average costs contain a high proportion of fixed overheads, which become an ever higher proportion as volume falls.

Personal relationships in this class of market are weak (impersonal) as between sellers and buyers, and among buyers, but strong (personal) as between sellers. Hence each seller must consider carefully the effects of his price-quantity behaviour on the other sellers, should he attempt to act independently. This tends to preserve the group solidarity and the monopolistic character of the

supply side of the market.

A representative market of this type is now described graphically, taking into account its price rigidity, the possibility of using inventories for supply in the short-run, and the influence of short-run boundary values on capacity and output. Let  $S_0$  in Fig. 1 represent middle or long-run supply, allowing for capacity changes.

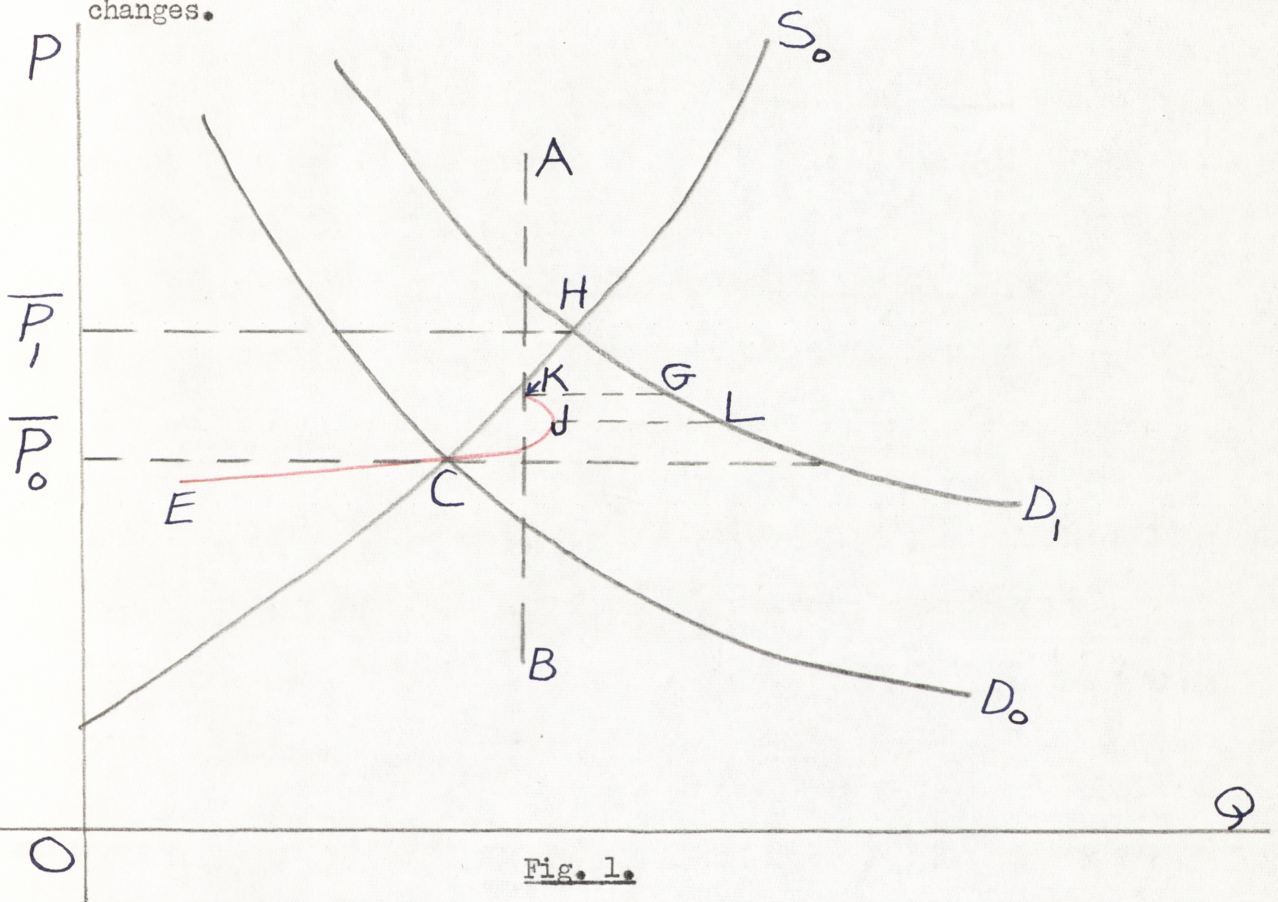


Fig. 1.

$\bar{P}_0$  and  $\bar{P}_1$  are equilibrium values of price for demand positions  $D_0$  and  $D_1$ .  $AB$  represents a short-run boundary value of supply, assuming that  $C$  is a normal full employment (capacity) position, but that with overtime work and excess use of capacity, output can be pushed beyond  $C$ . We now assume that demand fluctuates upward and downward from  $D_0$ , and



consider the likely short-run market path or possibilities. Because of price viscosity the path both left and right of C will be nearly infinitely elastic. As demand increases beyond  $D_0$  inventories can be run down to  $H_{\min}$ , at which time the point J is reached. These inventories can only be used up once in the short run, and hence the market position must rapidly curve up and back to reach AB. Price will increase as overtime pay and other costs rise under over-capacity operation. Since EC JK represents the price-quantity positions of the market for a demand curve shifting from C, it is the short-run market supply relation.

Assuming that demand shifted rapidly from  $D_0$  to  $D_1$ , excess demand when the market reaches K will be KG. This would show up mainly as unfilled orders in the suppliers' books. Should KG persist, suppliers will be induced to increase capacity, so that in the middle and longer run the market position will slowly trace out a path to H, with new equilibrium price  $\bar{P}_1$ .

For short-run market analysis we must add the short-run supply relation ECJK to D and S. In expressing this relationship we are enormously indebted to Professor P. A. Samuelson [8.14], [2.27], and to Dr. Lawrence R. Klein [1.11, pp. 50-57]. Following their lead, a short-run equation which expresses price viscosity, the influence of excess demand or supply, and the boundary conditions on supply, is

$$(1) \quad P_t = f \left\{ P_{t-1}, (D - S - H + H_{\min})_t \right\}; S_t \leq S_t \max .$$

The complete market situation is now described by three equations, the other two being the demand function

(2)  $q_t^d = D_t(P_t)$ , and the middle and long-run supply function.

(3)  $q_t^s = S_t(P_t)$ . With these three equations we can solve for  $P_t$ ,  $q_t^d$ , and  $q_t^s$ . We no longer expect  $q_t^d$  and  $q_t^s$  to be equal, except at equilibrium points like C and H.

(d) Small Market with Few Sellers and Few Buyers. This is the situation where a relatively small number of firms supply materials, parts or tools for another relatively small group of customers. There may be a formal or informal association of the sellers or buyers, but by and large, relations tend to be on a firm to firm 'personal' basis. Each firm must be wary of its relations with all the others in the market. Thus while sellers develop some power over buyers in periods of excess demand, and buyers develop power over sellers during excess supply, they will all be chary of using this power. In the long run good relationships are more valuable to them than any short-run gains.

This market situation implies slow moving prices in the short-run. No selling firm will want to step out of line with the other sellers for fear of poor relations and retaliation. With regard to its customers it will also be cautious in increasing prices, even though demand has increased creating an equilibrium price well above the current price. Sellers will prefer to accumulate order backlogs, and only raise prices gradually as their current prime costs increase, thereby providing them with satisfactory 'explanations' of price increase for their customers. ( This in no way implies any

constant markup. See Part III).

In the case of declining demand the combination of fixed past overhead costs with reduced output, plus the group pressure from the other sellers, will keep prices from falling much in the short-run.

Because of the small number of buyers and sellers in this kind of market, the very personal relationships, and hence the great importance of good relations and goodwill, there will be the same price viscosity, and use of inventories as buffers, as in the previous market. The analysis depicted in Fig. 1 will serve here as well.

(e) Retail Markets - Especially for Durable Goods. The market in this case consists of all of the retail stores within an area that can be conveniently reached by cars, public transport and mail order. For any particular line there will usually be few sellers and many buyers. But the buyers now represent public opinion, and hence the sellers must be more wary than those of (c) above with respect to the reactions of customers. In this respect the situation is a bit more like (d). For public goodwill is all important to the sellers.

There are usually enough sellers that no individual one has much power over the market. But sellers may combine formally or informally and agree on prices and market services. Prices, especially for large items, will move only slowly, and then usually with the accompaniment of suitable explanations.

In the case of consumer durables the manufacturing



firms in combination with the retail outlets must be thought of as the sellers. In this field model changes present a useful device for softening the impact of price increases. These and increased prime costs provide the 'explanation' of price increases. But the connection between increased unit cost and price increase need not be too precise, and firms can move closer to middle and long-run equilibrium positions at model change times, with a minimum of disturbance to public relations.

Thus the almost infinitely elastic supply situation in the short-run, as depicted in Fig. 1, seems to fit many retail markets as well.

(f) Markets for Trade and Professional Services. Under pure and atomistic competition the demand facing any producer is infinitely elastic at an impersonally set market price. Output tends to be at or above minimum average cost output (socially optimum output). In most trade and professional service markets however the sellers, because of common training, interests and problems, tend to form associations. Here are at work similar sociological laws to those that are behind labour unions. Once such an association is formed and agreement is reached on market practices and price, the association is in effect a monopoly. The trade or profession is then confronted by a downward sloping demand curve. Its price will usually be above the pure competition price, and its output will be below the pure competition output, at a market position which will tend to maximize profit for the existing association. The more inelastic the demand the greater the divergence from the pure

competition situation, and the more society loses absolutely, but the association gains relatively.

In order for the association to keep sufficient control over the market it must be able to regulate or fix either supply or price. To make things more certain associations usually work at both of these. They restrict supply by imposing stringent and lengthy training and apprenticeship, costly initiation, and restraint on the geographical movement of trained personnel. They also fix the price, so that should supply have been misjudged there will be underemployment, or overemployment. The latter can be slowly cured by further price increases. Individual members can increase their incomes by restricting still further the size of the association (membership) whose profits are to be maximized. The only limitation on such monopolistic practice is public opinion and social resentment.

Because prices are fixed in this market for rather long periods it will have infinitely elastic supply in the short-run, up to some maximum or boundary value.

##### 5. The Global Market for Final Goods.

When we put all of the markets for final goods (including inventory materials) together, what would we expect to be the average characteristics of this aggregate? Since only a few markets can be classed as purely competitive, we can conclude that the situation depicted in Fig. 1 will give a picture of the average nature of this global market in the economy.

Chapter 22.THE AGGREGATE MARKET FOR ALL FINAL GOODS1. Global Demand in Domestic Markets .

The various demands for final goods in the economy have been analysed in the earlier parts of this study, and it is merely necessary to assemble these now into a composite aggregate demand. To shorten the theory for this chapter we shall aggregate consumer minor groups into a major group of final demand. But in large scale practical work this final aggregation need not be used, and the pattern here developed will be equally applicable to the more detailed layout.

$$(1) P_p C_p + P_s C_s + P_{sd} C_{sd} + P_{hd} C_{hd} + P_a C_a = P_C C.$$

$$(2) C_p + C_s + C_{sd} + C_{hd} + C_a = C .$$

$$(3) P_d C_d = P_d G_d .$$

$$(4) H_{rm} + H_{gp} + H_{fg} = H .$$

$$(5) C^d + G_d^d + G^d + G_l^d + H^d - H_{-1} + F_2^d = GDF^d .$$

GDF is a function of the large number of variables which influence C,  $G_d$  ----  $F_2$  , but for the present we shall show only the price variables, grouping all of the other variables into the vector  $X_1$ .

$$(6) GDF^d = f_1 ( P_C , \text{---}, P_{2gs}, P', X_1 ). \text{ By definition}$$

$$(7) P' = \frac{P_C C + \text{---} + P_{2gs} F_2}{C + \text{---} + F_2} , \text{ a weighted arithmetic mean; or}$$

$$(7a) P' = \frac{C_m + \text{---} + F_{2m}}{C_m/P_C + \text{---} + F_{2m}/P_{2gs}} , \text{ a weighted harmonic mean, of the major group price levels.}$$

## 2. Middle and Long-Run Global Supply to Domestic Markets.

The ex post flow consists of domestic production plus imports. The supply function is

(8)  $GDF^S = GDP^S + F_1^d = f_2 (P', X_2)$ , where  $X_2$  is a vector containing mainly factors of production and costs. Because of boundary limitations on factors of production and the average price viscidly deduced for the aggregate economy in Chap. 21, solution of (6) and (8) will only explain the price-quantity situation of the global market in the middle and long run, and when the time aggregation is to long time periods, such as a year and more. (8) is accordingly to be interpreted as a middle and long-run supply function.

## 3. The Global Market in the Short Run.

The short-run market analysis differs from the long run, mainly in the matter of supply. In particular three special attributes must be built into a short period supply relation: viscid prices, boundary maximum of factors and output, the possibility of running inventory stocks down to a minimum, of augmenting, supply by temporarily living off 'fat'.

$$(9) \quad GDF_1^S = GDP^S + H - H_{\min}; \quad GDF_{1t}^S \leq GDF_{1t\max}.$$

$$(10) \quad P' = f_3 \left\{ P'_{-1}, (GDF^d - GDF_1^S)_{-1} \right\}$$

Equations (6), (8), (9), and (10) provide us with a dynamic short-run market path for the global economy similar to that depicted in Fig. 1 of Chap. 21. Supply will be very elastic over the range E C J, until H reaches  $H_{\min}$ , and will then double back to K at

at overtime, over capacity output, with excess demand KG.

The three equation system (6), (9) and (10) will provide us with successive time period solutions of  $P'$ ,  $\dot{GDF}^d$  and  $\dot{GDF}^s$ . Only when a stationary equilibrium like C and H is reached will  $P'_t = P'_{t+1} = P'_{t+2} = \dots$ , and  $\dot{GDF}_t^d = \dot{GDF}_t^s = \dots$ . Solution values which are not at stationary equilibria, are still a form of equilibrium, based on the simultaneous interaction of many variables and functions. We may refer to these as short-run, dynamic or moving equilibria.

#### 4. Further Analysis of Excess Demand and Supply.

Using the symbolism that GDF represents an ex post observed value of the flow of supply during a time period, we have  $GDF = \dot{GDF}^s$  when there is excess demand, and  $GDF = \dot{GDF}^d + u_h$  when there is excess supply. ( $u_h = H \dot{H}^d =$  undesired inventories.)

Excess demand analyses into two parts, one of which can be observed as unfilled orders,  $Z_1$ , while the remainder  $Z_2$  is latent, based on opinions that it is not worth while to order while there is a supply shortage. Thus

(11)  $GNF^d - GNF_1^s = Z_1 + Z_2$ . In Fig. 1 of Chap. 21 JL and K G represent  $Z_1 + Z_2$  at different times.

How does the concept of undesired inventories relate to that of excess demand? We shall now attempt to bring these two rather elusive concepts into a more complete picture of excess demand and supply. We begin by considering the stocks and flows which prod-

uce H.

$$(12) \quad H = H_{-1} + GDF - (C + GI_d + G + GI + F_2) = H_{-1} + GDF - GDS.$$

$$(13) \quad (u_h) = H - H^d = H_{-1} + GDF - GDS - H^d$$

$$(14) \quad H^d - H_{-1} = (\Delta H)^d ;$$

$$(15) \quad GDS^d + (\Delta H)^d = GDF^d .$$

$$(16) \quad u_h = \cdot GDF^s - \cdot GDF^d .$$

Thus positive undesired inventories (at end of any time period equals the excess supply flow during the period. When there is excess supply, the demand for inventory investment  $(\Delta H)^d$  will usually be negative.

While (16) gives a satisfactory relation between undesired inventories and excess supply, the same relation will not transpose completely into the excess demand case. This is because while inventories can accumulate indefinitely, they cannot be run down indefinitely without bringing production to a halt. As long as  $H > H_{min}$  however there is complete symmetry between the excess demand and supply cases.

(16a)  $-u_h = \cdot GDF^d - \cdot GDF^s$  ( $H > H_{min}$ ).  $u_h$  is now negative and  $(\Delta H)^d$  is positive. But once H has been run down to  $H_{min}$ , unfilled, unsatisfied demand begins to exist.

$$(17) \quad -u_{hmax} = \cdot H^d - H_{min} .$$

$$(18) \quad -u_{hmax} + Z_1 + Z_2 = \cdot H^d - H + \cdot GDS^d - \cdot GDF^s = \cdot GDF^d - \cdot GDF^s .$$

$$(H = H_{min}) .$$

Note that  $Z_1$  is directly observable, while  $Z_2$  like  $u_h$  could only be estimated from an econometric model or possibly by sample survey methods.

### 5. Separation of P' and Global Supply into their Major Components.

Our model has been separated into many demand categories in order to achieve as much homogeneity of behaviour within aggregates as possible. Each of these demand categories has its own price level. If we wish to explain each of these individual price levels, we must somehow separate global supply into corresponding categories. To do this properly would involve separating the economy into all of its industry groups, and studying each industry in the same way that we have studied the total economy. Inter-industry relations would then be determined by the 'foreign trade' of each industry with all others (including RW). Industry exports to the final demand sectors of our economy would provide us with the desired supply relations.

Such a model would contain a very large number of equations and would be very difficult to handle mathematically because of this. However the fact that changes in final demand and in technology do not sweep through such an industrial complex 'instantaneously' when short time periods are being used would simplify short-run solutions, enabling them to move forward in dynamic sequences, and suggests iterative approximations for longer run solutions. With the growing use of electronic computers it may well be possible to handle such large systems in the not too distant future.

A preliminary stage of this type of industrial analysis is of course Professor Leontieff's inter-industry input-output model of production [3.24]. It may be possible to relate

supply to price for each final demand category in the way required for our present model, using such an input-output table.

For the present model however an even simpler approach is suggested as a temporary approximate solution. It is the method used earlier in arriving at the supply of exports. The assumption is made that the proportion of any demand category which flows out of global supply can be increased over its value in the preceding period by increasing its price more than other categories have their prices increased. Thus, from (5) and (7), we have

$$(19) \quad \frac{P_C}{P'} \cdot \frac{C}{GDF} + \frac{P_{2gs}}{P'} \cdot \frac{F_2}{GDF} = 1.$$

Let the proportions of total supply flowing to the different demand categories be

$$(20) \quad \frac{C}{GDF} = r_c ; \quad \frac{F_2}{GDF} = r_f$$

$$(21) \quad r_c + \frac{P_{2gs}}{P'} r_f = 1, \text{ from (5).}$$

$$(22) \quad r_c = q_c \cdot \frac{P_C}{P'} ; \quad r_f = q_f \cdot \frac{P_{2gs}}{P'}$$

$$(23) \quad C^s = q_c \cdot \frac{P_C}{P'} \cdot GDF^s ; \quad F_2^s = q_f \cdot \frac{P_{2gs}}{P'} \cdot GDF^s$$

$$(24) \quad C^s + F_2^s = GDF^s$$

The system (23) of supply relations can be solved in conjunction with the corresponding demand functions to provide ex post values of the major group prices and volumes. There are however two constraints which affect the solution. One is that  $P_{2gs}$  is



exogenous and hence predetermined by world conditions. The second is that the identities (19) and (24) must explain two variables in the system, selected as residuals. Those variables which are most nearly of the nature of residuals are  $\Delta H$  and  $P_{\Delta H}$ , and hence it is suggested that  $\Delta H$  be determined by (24) and  $P_{\Delta H}$  by (19). Then  $(\Delta H)^S$  in (23) can be eliminated.

In a solution of the model we shall have  $C^d = C^S$ ;   
 -----  $F_2^d = F_2^S$ ; but  $(\Delta H)^d \neq (\Delta H)^S$  and  $GDF^d \neq GDF^S$ . It follows that  $GDF^S - GDF^d = (\Delta H)^S - (\Delta H)^d = H - H_{-1} - H^d + H_{-1} = u_h$

The  $q_i$  of (22) and (23) are determined for each time period from the appropriate data of the previous time period, using the formulas (20), (21) and (22). They will accordingly vary through time as conditions of demand and supply change. They should of course be adjusted from their  $t-1$  values for period  $t$  whenever any important technological or factor cost developments are known to be taking place in  $t$  with sufficient speed to cause even  $t-1$  values to be out of date.

Equations (23) and (24) bring to a conclusion the attempt in this study to develop a complete model of how the total economic system works. The model developed is summarized in Part IX, and subsequent parts are devoted briefly to the uses and applications of such a model, mainly in applied and empirical economics. A further chapter in this section however carries on with the price level concept for a brief discussion of inflation and deflation.

Chapter 23.INFLATION AND DEFLATION.1. Definition of Terms Inflation and Deflation.

These terms are commonly used with varying shades of meaning. For example inflation has sometimes been used to mean an expansion of the money supply, while deflation has been used to mean unemployment and reduction of business activity below capacity. Also we sometimes hear such terms as price inflation, cost inflation, demand inflation, and so on. It seems important then that we should define precisely what meanings are to be attached to the terms for this study. The heart of the matter has to do with the real value of money. The real value of money has always been its current purchasing power, and its ability to store such value. Inflation is that which weakens both of these powers, while deflation is the converse. It follows that inflation is the result of an upward movement in the prices of goods on which money is spent, and deflation is the converse. Inflation accordingly has to do with upward movements in  $P'$ ,  $P$ ,  $P_w$ ,  $P_m$ ,  $P_k$ .

Should we then combine all of these price indexes into some very aggregative index of all final and intermediate goods prices? This could be done, with much justification, but here we shall argue differently. Increases in  $P_{wmk}$  shift supply schedules backward and hence are a possible cause of increases in  $P$  and  $P'$ . But they need not do this if productivity improvements succeed in shifting supply

schedules forward by an equal or greater amount. No user of final goods would complain of inflation under such circumstances. Here we shall take the final goods point of view, and consider  $P'$  as the relevant price level.

We accordingly define inflation for this study as either equivalent to or the resultant of an increase in  $P'$ , while deflation is either the same as or the resultant of a downward movement in  $P'$ . When the rates of change of  $P'$  are small, medium and large we shall apply the qualifying adjectives mild, strong, and severe, extreme ( or hyper-).

## 2. Causes of Inflation ( Deflation).

Movements in  $P'$  which we have here defined as inflation or deflation arise as a result of shifts in both global demand and the middle and long run supply function. If the net effects of these movements are such that they leave global excess demand, inflation will follow, and if they result in global excess supply, deflation will result.<sup>1</sup> These results can also be produced by movements to excess demand in some sectors of final demand where prices are flexible, with compensating shifts to excess supply in other sectors where prices are rigid. ( Our model would not handle these latter situations particularly well.) The causes of the inflation or deflation are the causes which produced the shifts in global demand or supply. These can be traced through the model which we have developed.

1. According to our definitions of course inflation (usually mild) can occur without excess demand, and mild deflation could occur without excess supply.

It may be useful however to list some of the major causes likely to be found, singly or in groups, in any particular inflation or deflation. Inflationary causes only will be listed, with the understanding that usually the opposite causes will contribute to deflation.

(a) Demand Changes:

(1) Increased exogenous demands from government or from foreign trade.

(2) Desire of households to expand levels of living or to increase stocks at a faster rate than consumer goods are being produced or imported.

(3) Desire of firms to expand fixed investment and inventories at a faster rate than society is saving, or at a faster rate than such investment goods are being produced or imported.

(4) Money supply increasing faster than growth needs warrant, and causing demands to expand because of increased liquidity and reduced interest rates. The money expansion might arise from government deficits, especially when financed by MCB or CB credit, or from a balance of payments surplus.

(5) Expansion of private sector disposable incomes by government fiscal policy.

(6) Tendency for expectations that a price trend will continue to develop and strengthen the trend.

(b) Supply Changes:

(1) If average hourly earnings increase faster than real output per man-hour, supply will be reduced. Unit labour costs increase and the profit share of output is reduced, until prices are increased.

(2) Declining productivity, such as might follow from failure to maintain capital equipment, or from a deterioration of human health, skills or attitudes which are important to productivity.

(3) Increasing costs of non-labour factors of production, including increased profit margins, causing supply to shift backward.

(4) Increase in indirect taxes less subsidies.

(5) Increase in prices of imported goods.

(6) Increase in rate of exchange  $e_1$ .

(7) In some cases, an increase in income tax on profits.

The ex post relationship of these various supply influences on the price level is revealed by the identity (somewhat abbreviated here)

$$(1) \quad P' = \frac{N_h \cdot w_h + (P_{HH} + P_{KK})r + T_{i-s,m} + D_m + e_1 P_{lgsi} F_l}{f(GDP_f, N_h, H, K) + F_l + G_l + I + \prod_{ri}}$$

This relationship is derived from various equations in Parts III and VIII. Note that the denominator in (1) is equal to

$$(2) \quad C + GI_d + G + GI + \Delta H + F_2, \text{ and that}$$

$$(3) \quad (P_{HH} + P_{KK})r = \prod_m + \prod_{dim} - \prod_{idm}.$$

Actually in the complete model in general equilibrium (simultaneous solution) a change in any variable or function will exert some influence on  $P'$ . Here we have attempted to list only the major causes of inflation or deflation, which we now summarize under the headings;

demand (both autonomous, arising from exogenous and disturbance fact-

ors; and induced but with growth or cyclical components); monetary; exchange rate; foreign prices; factor cost; productivity; expectations; fiscal.

### 3. Evils of Inflation and Deflation.

These evils are too well known to require elaboration. During severe inflation money can no longer perform its function as a store of value, and is exchanged, as rapidly as received, for real wealth.  $M_{s2}$  is reduced to zero. Social relations are embittered, especially as between contract income receivers and the entrepreneurial class. Productivity is reduced as attitudes are eroded. Pensioners and those living off past monetary savings find their real level of living falling disastrously. Creditors find their loans repaid at far less real value than that loaned. They are not so willing to make further loans except at very high interest rates. Long range investment is curtailed.

Deflation would not be at all so disastrous as inflation, were it not for its dynamic effects on the level of activity. Its direct effect is to force debtors to repay more real value than they borrowed. Indirectly however it encourages everyone to hold more and more money as a form of wealth, because of the high real yield on money as prices fall. This causes a disastrous fall in current demand, in economic activity and in employment. Unemployment and business failure grow dangerously in this dynamic process, which feeds upon itself as price declines beget expectations of further price declines.

#### 4. Concluding Remarks.

The diseases of inflation and deflation and their accompanying complications are not easy to cure, especially when the cause can be traced to the monopoly power of labour or capital, or to world economic conditions. Further discussion on these points is deferred however to Part XI. In this part we have completed our analysis of the 'anatomy' and 'physiology' of the total economic organism. In subsequent parts we can now deal with prediction, disease, good health, and methods of curing the one and of preserving the other.

Part IXSUMMARY OF THE ECONOMIC MODELChapter 24.FINAL ASSEMBLY OF THE ECONOMIC MODEL1. The Large Model.

In this chapter the various behaviour relations, institutional and legal relations, definitions and accounting identities, for each sector of the economy studied above, are assembled to produce a complete model of the economy. This final assembly has the dual advantage of serving as a ready reference, and of giving a kind of bird's eye view or map of the total economy and all of its inter-relations.

Almost every equation in the model explains one particular variable in terms of the direct or proximate causes which influence it. The only exception to this rule is where a demand and a supply equation produce an equilibrium price as a by-product, as in equations (152). There are no separate equations for  $P_p$ ,  $P_s$ , and so on. Where there is a disequilibrium market situation a separate equation is required for each of the three major variables - demand, supply and price.

Since we have this one to one correspondence between variables being explained and equations, it follows that there are the same number of equations as there are current variables treated as endogenous. All other variables in the model are lagged, exogenous,



or treated as exogenous as an approximation.

Once the equations are all assembled together into an operating and interrelated model something new in the way of explanation has been created. A current endogenous variable is no longer explained just by the direct and proximate influences in its own equation. It is related to every other variable and parameter in the complete model. We may refer to all variables and parameters other than those in its own equation ( or small group of equations within which it is determined as a by-product) as remote influences. As we shall see later the relative magnitudes of different proximate and remote influences on any particular development can be estimated, once the model has been converted into numerical form.

The complete theoretical model which has been gradually developed in the preceding sections now follows.

#### A. THE HOUSEHOLD

##### (a) Demand for Consumer Goods.

$$(1) C_p^d = f_1 \left\{ \frac{Y_{wm}}{P_h}, \frac{Y_{\pi nam}}{P_h}, \frac{Y_{\pi am}}{P_h}, \frac{P_p}{P_h}, C_{p,-1}, \frac{NW_{nm}}{P_h} \right\}$$

$$(2) C_s^d = f_2 \left\{ \frac{Y_{wm}}{P_h}, \frac{Y_{\pi nam}}{P_h}, \frac{Y_{\pi am}}{P_h}, \frac{P_s}{P_h}, C_{s,-1}, \frac{NW_{nm}}{P_h}, E_h \right\}$$

$$(3) C_{sd}^d = f_3 \left\{ \frac{Y_{wm}}{P_h}, \frac{Y_{\pi nam}}{P_h}, \frac{Y_{\pi am}}{P_h}, \frac{P_{sd}}{P_h}, \frac{L_h}{P_h}, \frac{NW_{nm}}{P_h}, (A_{sd} - K_{sd}), E_h \right\}$$

$$(4) C_{hd}^d = f_4 \left\{ \frac{Y_{wm}}{P_h}, \frac{Y_{\pi nam}}{P_h}, \frac{Y_{\pi am}}{P_h}, \frac{P_{hd}}{P_h}, \frac{L_h}{P_h}, \frac{D_{eshf}}{P_h}, \frac{NW_{nm}}{P_h}, (A_{hd} - K_{hd}), \right.$$

$$\left. B_s(i,d,t), K_{hds}, \frac{P_{hds}}{P_{hd}}, C_{d+1}, \Delta N_f, E_h \right\}.$$

$$(5) C_a^d = f_5 \left\{ \frac{Y_{wm}}{P_h}, \frac{Y_{\pi nam}}{P_h}, \frac{Y_{\pi am}}{P_h}, \frac{P_a}{P_h}, \frac{L_h}{P_h}, \frac{D_{eshf}}{P_h}, \frac{NW_{hm}}{P_h}, (A_a - K_a), \right. \\ \left. B_s(i, d, t), K_{as}, \frac{P_{as}}{P_a}, E_h \right\}$$

$$(6) C_{d+1}^d = f_6 \left\{ \frac{Y_{wm}}{P_h}, \frac{Y_{\pi nam}}{P_h}, \frac{Y_{\pi am}}{P_h}, \frac{P_{d+1}}{P_h}, \frac{P_r}{P_{d+1}}, \frac{P_{(d+1)s}}{P_{d+1}}, B_m(i, d, t), \right. \\ \left. B_{ms}(i, d, t), \Delta N_f, \frac{L_h}{P_h}, \frac{NW_{hm}}{P_h}, (A_{d+1} - K_{d+1}), \vec{P}_h, E_h \right\}.$$

$$(7) C_r^d = f_7 \left\{ (Y_w, Y_{\pi na}, Y_{\pi a}, \frac{P_r}{P_h}, NW_h) \right\}.$$

$$(7a) P_r = f_{7a}(P_{r,-1}, E_{dr}).$$

$$(8) C_l = f_8(C_{d+1}, P_l).$$

(b) Household Identities.

$$(9) Y_h = Y_w + Y_{\pi p}.$$

$$(10) - (13) \begin{cases} K_{sd} = K_{sd,-1} + C_{sd} - D_{sd} \text{ (Similarly for hd, a, d+1)} \\ D_{sd} = C_{sd} - \Delta K_{sd}. \quad K_h = K_{sd} + K_{hd} + K_a + K_{d+1}. \end{cases}$$

$$(14) Y_h + NW_{h,-1} = C_{sr} + C_p + C_{sd} + C_{hd} + C_a + C_{d+1} - \Delta K_{sd} - \Delta K_{hd} \\ - \Delta K_a - \Delta K_{d+1} + NW_h. \quad C_{sr} = C_s + C_r.$$

$$(15) S_h = Y_h - C_s - C_p - C_{sd} - C_{hd} - C_a = Y_h - C \text{ (conventional saving).}$$

$$(16) S_h' = Y_h - C_s - C_p - C_{sd} - C_{hd} - C_a - C_{d+1} + \Delta K_{sd} + \Delta K_{hd} + \\ \Delta K_a + \Delta K_{d+1} \text{ (True saving).}$$

$$(17) Y_h = C_s + C_p + C_{sd} + C_{hd} + C_a + C_{d+1} + \Delta M_{hr} + \Delta S_{ehr} + \Delta K_{ub} - \\ \Delta S_{eshfr} - \Delta S_{emhr} \text{ (Total economic activity during time period).}$$

$$(17a) L_h = M_h + S_{egh}.$$

$$(17b) W_h = M_{hr} + S_{ehr} + K_{sd} + K_{hd} + K_a + K_{d+1} \\ + K_{ub}$$

$$(17c) \quad NW_h = W_h - S_{eshf} - S_{emhf} - S_{emhh} \cdot$$

### B THE FIRM

#### (a) Demand for Factors of Production.

$$(18) \quad \frac{N_{ph}^d w_{ph}}{P} = f_{18} \left\{ \overrightarrow{GDP}, GDP, \frac{P_w}{P_{mK}}, \frac{NL_f}{P_{wmK}}, W_f, u_{fg,-1}, E_f \right\}.$$

$$(19) \quad h^d = f_{20} \left\{ h_s, (GDP_f^d - GDP_f^s), \frac{w_{ph}}{P} \right\}$$

$$(20) \quad H_{rm}^d = f_{20} \left\{ \frac{P_{rm}}{P}, \frac{P_{rm}}{P_{wK}}, \overrightarrow{GDP}, GDP, \overrightarrow{P}, \frac{NL_f}{P_{wmK}}, W_f, u_{fg,-1}, E_f \right\}.$$

$$(21) \quad H_{gp}^d = f_{21} \left\{ \frac{P_{gp}}{P}, \frac{P_{gp}}{P_{wK}}, \overrightarrow{GDP}, GDP, \overrightarrow{P}, \frac{NL_f}{P_{wmK}}, W_f, u_{fg,-1}, E_f \right\}$$

$$(22) \quad H_{fg}^d = f_{22} \left\{ \frac{P_{fg}}{P}, \frac{P_{fg}}{P_{wK}}, \overrightarrow{GDP}, GDP, \overrightarrow{P}, \frac{NL_f}{P_{wmK}}, W_f, E_f \right\}.$$

$$(23) \quad u_{fgt} = H_{fgt} - \cdot H_{fgt}^d \quad (\text{Undesired inventories}).$$

$$(24) \quad GI^d = f_{24} \left\{ \frac{P_{GI}}{P}, \overrightarrow{GDP}, GDP, \frac{P_{GI}}{P_{wm}}, \frac{L_f}{P_{wmK}}, \frac{NL_f}{P_{wmK}}, W_f, (r' - i_{1f}), \right. \\ \left. (r' - i_{1f})_{-1}, (GDP^d - GDP^s)_{-1}, P_{e,-1}, (\Delta S_{elfo})_{-1}, \right. \\ \left. (\Delta K_{di})_{-1}, GI_{-1} \right\}.$$

$$(25) \quad K_f^d = K_{f,-1} + GI^d - D_f + \Delta K_{1f}.$$

#### (b) Supply.

$$(26) \quad GDP_f^s = f_{26} (GDP_f^o, (N_p^d + N_{enp}) h^d, H^d, K_f^d).$$

$$(27) \quad GDP^s = GDP_f^s + G_1 + I + \prod_{ri}.$$

#### (c) Identities of Production and Finance.

##### Total Domestic Sales.

$$(28) \quad GDS = C + GI_d + GI + G + F_2.$$

Gross Domestic Supply Flow.

$$(28a) \quad GDF = GDS + \Delta H = GDP + F_1 .$$

Gross Domestic Production or Value Added.

$$(29) \quad GDP = GDS + \Delta H - F_1 .$$

Gross National Product.

$$(30) \quad GNP = GDP + \pi_{id} - \pi_{di} .$$

Total Income Payments to Factors of Production.

$$(31) \quad GDP = ( w_{ph} N_{ph} + w_i N_i + w_g N_g + w_m N_m ) / P + \pi_f - J + \pi_{go} \\ + \pi_{io} + \pi_{ri} + T_{i+s} + D_{f1} + D_{f2} .$$

$$(31a) \quad r' = \pi_f / (H + K_f) \stackrel{=} {=} r$$

Gross and Net Savings of Firms.

$$(32) \quad S_f = \pi_f - J + D_{f1} + D_{f2} - \pi_{ifo} - \pi_{rfo} - \pi_{dfo} - \pi_{dub} .$$

$$(33) \quad NS_f' = S_f - D_f - D_{f2} \quad ( NS_f = S_f - D_{f1} - D_{f2} ) . \text{ True (Conventional)}$$

Net Worth and Reallocation of Wealth.

$$(34) \quad \Delta NW_f = NS_f' + \Delta_{lK_{ub}} + \Delta_{lS_{eeor}} .$$

$$(35) \quad NS_f' = \Delta M_{fr} + \Delta S_{efr} + \Delta H + \Delta K_f + - \Delta S_{eifor} - \Delta_{lK_{ub}} - \\ \Delta_{lS_{eeor}} .$$

$$(36) \quad W_f = M_{fr} + S_{efr} + H + K_f .$$

$$(37) \quad L_f = M_f + S_{ef} - S_{emf} .$$

$$(38) \quad NL_f = L_f - D_{esf} .$$

The Finance of Investment.

$$(39) \quad \Delta H + GI = S_f - \Delta M_{fr} - \Delta S_{efr} + \Delta S_{eifor} + \Delta_{lK_{ub}} + \Delta_{lS_{eeor}} .$$

C. THE LABOUR MARKET.

(a) Middle and Long-Run Supply of Labour.

$$(40) \quad N_1^s = b^s a' ; b^s = f_{40} \left\{ b_{-1}, w_{ph}/P_h, W_h/N, (A_h - K_h) \right\}.$$

$$(41) \quad h^s = f_{41} \left\{ h_{s,-1}, w_{ph}/P_h, W_h/N, (A_h - K_h) \right\}.$$

(b) Unemployment.

$$(42) \quad N_u = N_1^s - N_m - N_g - N_i - N_{enp} - N_p^d = N_{lf}^s - N_p^d.$$

$$(43) \quad N_{h_u} = N_{lfh}^s - N_{ph}^d.$$

(c) Short-Run Supply or Market Adjustment.

$$(44) \quad w_{ph} = f_{44} \left\{ w_{ph,-1}, \frac{N_{h_u}}{N_{lfh}^s}, \left( k_1 - \frac{N_{h_u}}{N_{lfh}^s} \right) \right\}$$

D. THE FOREIGN TRADE AND FINANCE SECTOR(a) Flow and Stock Identities.Balance of Payments.

$$(45) \quad (\Delta IM) e_1 = F_{2gm} + F_{2sm} + F_{2go} + F_{2u} - F_{1gm} - F_{1sm} - F_{1u} + \Delta M_{di} \\ + \Delta S_{esdi} - \Delta S_{esid} + \Delta S_{eldi} - \Delta S_{elid} + \Delta B_{di} - \Delta B_{id} + \Delta K_{di} \\ - \Delta K_{id}.$$

International Reserves.

$$(46) \quad IR = e_1 IM + S_{esid}; IR_n = IR - M_{di} - S_{esdi}.$$

International Balance Sheet.

$$(47) \quad M_{go} + (M_{id} - M_{di}) + (S_{esid} - S_{esdi}) + (S_{eldi} - S_{elid}) + (B_{id} - B_{di}) \\ + (K_{id} - K_{di}) = U_{id}.$$

(b) Price Levels.

$$(48) \quad P_1 = P_{1g} (1 + \bar{i}f + d + s + ds) / (i + \bar{i}f + d + s + ds)_0 = \\ f_{48} (e_1, e_{2i}, P_{1gii}, \bar{i}f, s, d).$$

$$(49) P'_{1rm} = f'_{49}(e_1, \dots, d). \text{ Similarly } P'_{1GI}, P'_{1C}.$$

$$(50) P_2 = \frac{P_{2g}}{E_2} \cdot \frac{1}{e_1/e_{1o}} \cdot \left[ \frac{1 + \overline{if} + d + s + \overline{ds}}{(1 + \overline{if} + d + s + \overline{ds})_o} \right]_{RW2}$$

(c) Import Demand.

$$(51) F'_{1rm} = f_{51} \left\{ \left( \frac{P'_{1rm}}{P_{rm}} \right)_{-e}, \left( \frac{P'_{1rm}}{P_{wk}} \right)_{-e}, \overrightarrow{GDF}_{-e}, \overrightarrow{P}_{-e}, \left( \frac{NL_f}{P_{vmK}} \right)_{-e}, \right. \\ \left. u_{f,-e}, u_{fg,-e}, (GDF^d - GDF^s)_{-e}, IR_1, IR_2, E_F \right\}.$$

$$(52) F'_{1GI} = f_{52} \left\{ \left( \frac{P'_{1GI}}{P_{GI}} \right)_{-e-1}, GI^d, (GDF^d - GDF^s)_{-e-1}, IR_1, IR_2, E_F \right\}.$$

$$(53) F'_{1C} = f_{53} \left\{ \left( \frac{P'_{1C}}{P_C} \right)_{-e}, \overrightarrow{C}_{-e}, u_{fg,-e}, IR_1, IR_2, E_F \right\}.$$

$$(54) F'_{1sg} = f_{54} \left\{ \frac{e_1 P_{1sgi}}{P_{sg}}, F_{1g} \right\}.$$

$$(55) F_{1g} = F'_{1rm} + F'_{1GI} + F'_{1C}.$$

$$(56) F_{1s} = F'_{1sg} + F_{1sv} + \pi_{di}.$$

$$(57) F_1 = F_{1g} + F_{1sg} + F_{1sv}.$$

$$(57a) IR_1 = IR / F_1; IR_2 = (c - IR / F_1)$$

(d) Import Supply.

$$(58) P_{1rmi}, P_{1GIi}, P_{1Ci}, P_{1sgi} : \text{exogenous.}$$

(e) Export Demand.

$$(59) F'_{2g} = f_{59} \left\{ \left( \frac{P_2}{P_{RW2}} \right)_{-e}, \overrightarrow{GDF}_{RW2,-e}, \overrightarrow{P}_{RW2,-e}, W_{RW2,-e}, \right. \\ \left. u_{rmRW2,-e}, IR_{1RW2}, IR_{2RW2}, E_{FRW2} \right\}.$$

$$(60) F'_{2sg} = f_{60} \left\{ \frac{P_{2sg}}{(P_{sgRW})e_1E_2}, F_{2g} \right\}.$$

$$(61) F_{2s} = F'_{2sg} + F_{2sv} + \pi_{id}.$$

$$(62) F_2 = F_{2g} + F_{2sg} + F_{2sv}.$$

(f) Export Supply

$$(63) \quad F_2^S = q_f \cdot \frac{P_{2gs}}{P} \cdot GDF^S \cdot$$

(g) Long-Term Capital InflowPortfolio Investment

$$(64) \quad \Delta S_{eldi} = f_{64} \left\{ (i_{lf} - i_{lfrw}), E_s \right\}$$

Direct Investment

$$(65) \quad \Delta K_{di} = f_{65} \left\{ (r' - r'_{RW}), E_k \right\}$$

(h) The Exchange Rate

$$(66) \quad F_{2g}^d P_{2g} + F_{2s}^d P_{2s} + F_{2go} - \Delta M_{go} + F_{2um} + \Delta M_{di} + \Delta S_{esdi} + \Delta B_{di} \\ + \Delta K_{di} = IM^S = f_{66}(e_1).$$

$$(67) \quad F_{1g}^d P_{1g} + F_{1s}^d P_{1s} + \Delta S_{esid} + \Delta S_{elid} + \Delta B_{id} + \Delta K_{id} = IM^d = f_{67}(e_1).$$

E. THE GOVERNMENT SECTOR(a) IdentitiesSpending

$$(68) \quad G' = G_c + G_k + \Delta K_{pg}; \quad G^* = G_c + G_k; \quad G = G_c + G_k + D_g;$$

$$G_n = G_c + D_g; \quad G_c = G_{ms} + (w_g N_g + w_m N_m) / P.$$

Stock of Capital

$$(69) \quad K_g = K_{PCMLg} = K_{g,-1} + G_k + \Delta K_{pg} - D_{PCMg}; \quad D_{PCMg} = D_g \cdot$$

Output or Value Added

$$(70) \quad G_1 = (w_g N_g + w_m N_m) / P + D_g \cdot$$

Revenue

$$(71) \quad T = T_w + T_{\pi p} + T_{\pi np} + T_{i-s} + T_o \cdot$$



Transfer Payments

$$(72) \quad T_r = T_{rw} + T_r \pi_p + T_r \pi_{np} + G_{ig}$$

Surplus and Deficit Accounts

$$(73) \quad T - T_r = T_d ; \quad \text{Current Account : } T_d - G_n = S_{g1} ;$$

$$\text{Cash: } T_d - G' = S_{g2} ; \quad \text{National Income: } T_d' - G^* = S_{g3} ;$$

$$T_d' = T_d - T_o + T_o' .$$

Cash Position

$$(74) \quad \Delta M_g = T_{dm} - G_m' + \Delta S_{egp} + \Delta S_{egb} + \Delta S_{egc} + \Delta S_{egi} - \Delta S_{epg}$$

$$(74a) \quad \Delta M_p^g = - \Delta M_g + \Delta S_{egb} + \Delta S_{egc} + \Delta S_{egi} .$$

Redistribution of Earned Incomes

$$(75) \quad (w_{ph}N_{ph} + w_iN_i + w_gN_g + w_mN_m) / P = W_a .$$

$$(75a) \quad Y_w = W_a + T_{rw} - T_w + T_{rf1} + T_{rf2}$$

$$(76) \quad \pi = \pi_f + \pi_{io} + \pi_{ri} + \pi_{go} + \pi_{id} - \pi_{di}$$

$$(76a) \quad \pi_{np} = \pi_c - \pi_{dfo} + \pi_{ub} - \pi_{dub} + \pi_{gb}$$

$$(76b) \quad \pi_p = \pi - \pi_{np}$$

$$(77) \quad Y_{\pi p} = \pi_p + T_r \pi_p - T_{\pi p}$$

$$(78) \quad Y_{\pi np} = \pi_{np} + T_r \pi_{np} - T_{\pi c} - T_{di} - T_{\pi gb} - T_{rf1}$$

$$(79) \quad Y_w + Y_{\pi p} + Y_{\pi np} + T_d' - J + D_{f1} + D_{f2} - T_{rf2} = GNP .$$

$$(80) \quad Y_{\pi} = Y_{\pi p} + Y_{\pi np} = Y_{\pi a} + Y_{\pi na}$$

(b) Tax and Transfer Equations

$$(81) \quad PT_w = f_{g1}(PW_a)$$

$$(82) \quad PT_{rw} = f_{g2}(N, N_u)$$



$$(83) \quad PT_{\pi p} = f_{83} \left\{ P (\pi_p + G_{ip}) \right\} .$$

$$(84) \quad PT_r \pi_p = \text{exogenous} .$$

$$(85) \quad T_{\pi np} = t_c (\pi_c + G_{ic}) + t_{di} r_{di} (K_{di} + S_{edir}) + t_{gb} k_2 \pi_c .$$

$$(85a) \quad (1 - t_{di}) r_{di} (K_{di} + S_{edir}) = \pi_{di} .$$

$$(85b) \quad \pi_c + \pi_{ub} + \pi_{gb} = \pi_{fn} = \pi - (\pi_p - \pi_{dfo} - \pi_{dub}) .$$

$$(85c) \quad \pi_p - \pi_{dfo} - \pi_{dub} = \pi_{ip} + \pi_{rp} .$$

$$(85d) \quad \pi_c = \frac{\pi - (\pi_{ip} + \pi_{rp})}{(1 + k_2 + k_3)}$$

$$(86) \quad PT_r \pi_{np} = \text{exogenous (mainly } G_{inp}) .$$

$$(87) \quad T_{i-s} = f_{87} \left\{ \frac{P_{lg} F_{lg}}{P}, \frac{P_C C}{P}, \frac{P_{GI} GI}{P}, \left( GDP - G^* - F_2 - w_1 N_1 - \pi_{io} - \pi_{ri} - W_a^a - \pi^a \right) \right\} .$$

$$(88) \quad T_d = f(\text{GNP, or NNIFC}) \quad (\text{side relation}) .$$

#### F. THE MONEY AND FINANCE SYSTEM

(a) Money Supply (Middle and Long Run)

$$(89) \quad M = M_{cp} + M_b ; M_{cp} = m_1 M .$$

$$(90) \quad CR = M_{cb} + M_{dcb} ; M_{cb} = m_2 M_{cp} .$$

$$(91) \quad CR = CR_{-1} - \Delta M_{cp} + \Delta IR_g + \Delta S_{egc} - \Delta M_{dcg} + \Delta S_{esbc} - \Delta M_{dcb} .$$

$$(92) \quad M^s = f_{92} \left\{ CR / cr, m_1, i_b, i'_b, i_c, BF \right\} .$$

$$(92a) \quad M_p = M_{cp} + S_{egc} - M_{dcg} + S_{egb} - M_{bg} + S_{eab} + IR_c + IR_b + X .$$

(b) Value Storage Reallocations.

$$(93) \quad NW_{hm} = M_{ah} + M_{slh} + M_{s2h} + S_{egh} + S_{ei(fi)h} + S_{ee(fi)h} + S_{emhh} + K_{sdhdam} + K_{d+1,hm} + K_{ubm} - S_{esho} - S_{emho} .$$

$$(94) \quad NW_f = M_{af} + M_{slf} + M_{s2f} + S_{esff} + S_{eshf} + S_{egf} + S_{eil(fi)f} + \\ S_{ee(fi)f} + S_{emhf} + H_{fm} + K_{fm} + K_{d+1, fm} - S_{esfo} - S_{eilfo} = S_{eefo} \\ + K_{ubm} .$$

$$(95) \quad M_{ah}^d = f_{95}(C_m^d, i_h) .$$

$$(96) \quad M_{af}^d = f_{96}(W_{am}, \Delta H_m, GI_{PCMLm}, P'.GDF, i_f) .$$

(3) Securities Markets

$$(97) \quad S_{esgo}^s = \text{exogenous} .$$

$$(98) \quad S_{esfo}^s = f_{98}(\Delta H_m, P'.GDF, \vec{P}, i_s, r) .$$

$$(99) \quad S_{esho}^s = f_{99} \left\{ C_{ndm}, C_{am}, \vec{P}, i_{sh}, B_s(i, d, t) \right\} .$$

$$(100) \quad S_{esdi}^d = f_{100} \left\{ (i_s - i_{sRW}), E_s \right\} .$$

$$(101) \quad S_{esof}^d = f_{101}(L_f, NW_f, \vec{P}, i_s, i) \cdot i = f_{101a}(i_s, i_{lg}, i_{lf}, i_m, i_b) .$$

$$(102) \quad S_{elgo}^s = \text{exog} . \quad (103) \quad S_{elgc}^d = \text{exog} .$$

$$(104) \quad S_{elgh}^d = f_{104}(i_{lg}, i_{lf}, i_m, i_e, r_d, r, P_{slg}, P_{slf}, P_e, \\ P_{d+1}, P, \vec{P}, W_h, NW_h, L_h, E_h) = f_{104}(V_h) .$$

$$(105) \quad S_{elgf}^d = f_{105}(V_f) .$$

$$(106) \quad S_{elgb}^d = f_{106} \left\{ i_{lg}, i_{lf}, i_m, i_e, i_b, P_{slg}, P_{slf}, P_e, \right. \\ \left. (M^s - M^d)_{-1} \right\} = f_{106}(V_{lb}) .$$

$$(107) \quad S_{elgi}^d = f_{107}(i_{lg}, i_{lf}, i_e; i_{lgRW}, i_{lfRW}, i_{eRW}; E_i) \\ = f_{107}(V_{2i}) .$$

$$(108) \quad \Delta S_{eilfo}^s = f_{108}(GI_{PCMLm}^d, P_{slf}, P_e, i_{lf}, i_e) . (\text{Net new issues}) .$$

$$(109) \quad s_{eil(fi)-1}^s = f_{109}(i_{lg}, i_{lf}, i_e, i_m, r_d, r, P_{slg}, P_{slf}, \\ P_e, P_{d+1}, P, \vec{P}) . (\text{Market supply of existing issues}) .$$

$$(110) S_{eil(fi)}^s = S_{eil(fi)}^s + \Delta S_{eilfo}^s \Delta S_{eilid} \cdot$$

$$(111) S_{eilfi}^d = f_{111}(V_h) \cdot$$

$$(112) S_{eilfif}^d = f_{112}(V_f) \cdot$$

$$(113) S_{eilfib}^d = f_{113}(V_{1b}) \cdot$$

$$(114) S_{eilfii}^d = f_{114}(V_{2i}) \cdot$$

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$$(115) - (121) S_{ee}^s, S_{eeh}^d, S_{eef}^d, S_{eeb}^d, S_{eei} \text{ identical in layout to } (108) - (114)$$

$$(122) \Delta_1 S_{emho}^s = f_{122} ( C_{d+1}^d, i_m, i_h, B_m(i,d,t) ) \cdot$$

$$(123) S_{emho}^s = S_{emho,-1} + \Delta_1 S_{emho}^s - \Delta_2 S_{emho}^s \quad (\Delta_2 \text{ refers to repayments of principal}).$$

$$(124) S_{emhh}^d = f_{124}(V_h) \cdot$$

$$(125) S_{emhf} = f_{125}(V_f) \cdot$$

$$(126) S_{emhb} = f_{126}(V_{1b}) \cdot$$

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$$(127) K_{d+1}^s = K_{d+1,-1} + C_{d+1}^d - D_d \cdot$$

$$(128) K_{d+1,hm}^d = f_{128}(V_h) \cdot$$

$$(129) K_{d+1,fm}^d = f_{129}(V_f) \cdot$$

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$$(130) K_{ubm}^s = K_{ubm}^d = f_{130}(V_h) \cdot$$

$$(131) M_{slh}^d = f_{131}( V_h^*, i_h, W_h, NW_h, E_h ) \cdot$$

$$(132) M_{slf}^d = f_{132}( V_f^*, i_f, W_f, NW_f, E_f ) \cdot$$

$$(133) V_h^* = f_{133a}(V_h) ; V_f^* = f_{133b}(V_f) \cdot$$

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$$(134) M_{s2h}^d = NW_{hm}^d - M_{ah}^d - M_{slh}^d - S_{elgh}^d - S_{eiloh}^d - S_{eeh}^d - S_{emhh}^d - K_{sdhdam} - K_{d+1,hm}^d - K_{ubm}^d + S_{esho}^s + S_{emho}^s = f_{134}( C_m^d, C_{d+1,m}^d, \vec{P}, P, i_h, B_s(i,d,t), B_m(i,d,t), V_h, V_h^* ) \cdot$$

$$(135) M_{s2f}^d = NW_{fm} - M_{af}^d - \dots + S_{eilfo}^s = f_{135}(P', GDF, \Delta H_m, \\ GI_{PCMLm}, \vec{P}_{GI}, \vec{P}, P, i_f, V_f, V_f^*) \cdot i_f = f_{135}(i_s, \\ i_{1f}, i_b, i_m, i_e).$$

$$(136) M_{bg}^d, M_{bi}^d = \text{exogenous.}$$

(c) Money Demand.

$$(137) M^d = M_{ah}^d + M_{af}^d + M_{slh}^d + M_{slf}^d + M_{s2h}^d + M_{s2f}^d + M_{bg}^d + M_{bi}^d.$$

(d) Market Adjustment and Short-Run Money Supply.

$$(138) i_b = f_{138} \left\{ i_{b,-1}, (M^d - M^s) \right\}.$$

G. AGGREGATE MARKET FOR FINAL GOODS AND GLOBAL PRICE LEVEL.(a) Definitions

$$(139) P_p C_p + P_s C_s + P_r C_r + P_{sd} C_{sd} + P_{hd} C_{hd} + P_a C_a = P_C C.$$

$$(140) C_p + C_{sr} + C_{sd} + C_{hd} + C_a = C.$$

$$(141) C_d = GI_d; (142) H_{rm} + H_{gp} + H_{fg} = H.$$

$$(142) C^d + GI_d^d + G^d + GI^d + H^d - H_{-1}^d + F_2^d = GDF^d = f_{142}(P_C, \dots, P_{2gs},$$

$$P', X_1)$$

$$(143) P' = \frac{P_C C + P_d GI_d + P_G G + P_{GI} GI + P_{\Delta H} \Delta H + P_{2gs} F_2}{C + GI_d + G + GI + \Delta H + F_2}$$

$$(144) GDF^s = GDP^s + F_1^d = f_{144}(P', X_2).$$

$$(145) GDF_1^s = GDF^s + H - H_{min}; GDF_{1t}^s \leq GDF_{1tmax}.$$

(b) Short-Run Supply and Market Adjustment.

$$(146) P' = f_{146} \left\{ P'_{-1}, (GDF^d - GDF_1^s), \left( \frac{GDF^d - GDF_1^s}{GDF_1^s} \right)_{-1} \right\}.$$

$$(147) GDF^d - GDF^s = -u_{hj}(H > H_{min})$$

$$= -u_h \max + Z_1 + Z_2 (H = H_{min}).$$

$$(148) \quad u_h = H - H^d ; u_{fg} = H_{fg} - H_{fg}^d ; u_{h \max} = H_{\min} \cdot H^d .$$

(c) Separation of the Global Supply and Price Level to Match Individual Demand Categories.

$$(149) \quad GDF^S = C_p^S + C_s^S + C_r^S + C_{sd}^S + C_{hd}^S + C_a^S + GI_d^S + G + (\Delta H_{rm})^S + (\Delta H_{gp})^S + (\Delta H_{fg})^S + GI^S + F_2^S .$$

$$(150) \quad \frac{C_p}{GDF} = r_p ; \frac{G}{GDF} = r_g ; \frac{GI}{GDF} = r_I ; \frac{F_2}{GDF} = r_f .$$

$$(151) \quad r_p = q_p \cdot \frac{P_p}{P'} ; r_s = q_s \cdot \frac{P_s}{P'} ; r_{sd} = q_{sd} \cdot \frac{P_{sd}}{P'} ; r_f = q_f \cdot \frac{P_{2gs}}{P'}$$

$$(152) \quad C_p^S = q_p \cdot \frac{P_p}{P'} \cdot GDF^S ; C_s^S = q_s \cdot \frac{P_s}{P'} \cdot GDF^S ; C_r^S = f_{152}(P_r, P', K_{d+1}) ; C_{sd}^S = q_{sd} \cdot \frac{P_{sd}}{P'} \cdot GDF^S ; C_{hd}^S = q_{hd} \cdot \frac{P_{hd}}{P'} \cdot GDF^S ; C_a^S = q_a \cdot \frac{P_a}{P'} \cdot GDF^S ;$$

$$GI_d^S = q_d \cdot \frac{P_d}{P'} \cdot GDF^S ; G^S = q_g \cdot \frac{P_g}{P'} \cdot GDF^S ; (\Delta H_{rm})^S = q_{rm} \cdot$$

$$\frac{P_{\Delta H_{rm}}}{P'} \cdot GDF^S ; (\Delta H_{gp})^S = q_{gp} \cdot \frac{P_{\Delta H_{gp}}}{P'} \cdot GDF^S ; GI^S = q_I \cdot$$

$$\frac{P_{GI}}{P'} \cdot GDF^S ; F_2^S = q_f \cdot \frac{P_{2gs}}{P'} \cdot GDF^S .$$

$(\Delta H_{fg})^S$  is determined as a residual from (149);  $P_{\Delta H_{fg}}$  is determined from (139), (143) and (149)

$$(153) \quad P_{\Delta H} \Delta H = P_{\Delta H_{rm}} \Delta H_{rm} + P_{\Delta H_{gp}} \Delta H_{gp} + P_{\Delta H_{fg}} \Delta H_{fg} .$$

$$(154) \quad P^* = \frac{P' \cdot GDF - P_{LGS} \cdot F_1}{GDF - F_1} ; P = \frac{P^* \cdot GDF - P_{GI} \cdot D_{f1}}{GDF - D_{f1}} .$$

$$(155) \quad P_h = \frac{P_C \cdot P \cdot Y_h}{P_C \cdot C + P_{ih}(P \cdot Y_h - P_C \cdot C)} ; P_{ih} = \frac{1 + i_{ht}}{1 + i_{ho}} ;$$

$$i_h = f_{155}(i_{lg}, i_{lf}, i_e, i_m, r_d, r) .$$

( P represents the price level of Net Domestic Expenditure NDE. As a first approximation it can be used to deflate all of the income side of GDP or of GNP, except for  $D_{f1m}$ , which is more appropriately deflated by  $P_{GI}$ . A more precise deflation of the income side would be to deflate each component of the income side, in final disposable form, by the price level of the goods on which it is currently spent. Such an income deflation will balance with the expenditure side deflation depicted here.)

(d) Derived Relations for Aiding in the Analysis of the Causes

of Inflation.

$$(156) \quad P' = \frac{(w_{ph}N_{ph} + w_iN_i + w_gN_g + w_mN_m) + (\pi - \pi_{id} + \pi_{di})_m + \frac{D_m + e_1 P_{lgsi} F_1}{+T_{i-s,m}}}{GDP_f + G_1 + I + \pi_{ri} + F_1}$$

$$(157) \quad P^* = \frac{W_{am} + \pi'_m + T_{i-sm} + D_m}{GDP} .$$

$$(158) \quad \frac{P_f \cdot GDP_f}{w_{ph}(N_{penp})} \stackrel{-1}{=} S_L ; \quad \frac{P_f \cdot GDP_f}{\pi'_m} = S_{\pi}^{-1} .$$

$$(159) \quad \frac{GDP_f}{N_{penph}} = S_L^{-1} \times \frac{w_{ph}}{P_f} = P_L ; \quad \frac{GDP_f}{H_f + K_f} = S_{\pi}^{-1} \times r = P_{\pi} = P_K .$$

$$(160) \quad S_L = \frac{w_{phr}}{P_L} ; \quad S_{\pi} = \frac{r}{P_K} .$$

One basic industrial separation should be made in the above model when used for practical work. This is the separation between agricultural ( including pastoral) and non-agricultural production. The nature of the production function is vastly different in these two sectors ( cf. Zimmerman [1.24] ), and in addition their market characteristics vary widely. It follows that the aggregation of these two dissimilar sectors may at times produce a large aggregation

error. The correct theoretical solution to this problem is to separate the economy into two industrial sectors, each similar to the macro model above, with each engaging in 'foreign trade' with the other, and the sectors of final demand including RW. This procedure would however expand our model considerably without requiring the development of any new principles. Because of this and to save space, we shall adopt the temporary expedient and approximation of treating agricultural output, sales and inventories as exogenous.

$$(161) \quad \text{GDP}_f = \text{GDP}_f^{\text{na}} + \text{GDP}_f^{\text{a}} \quad .$$

$$(162) \quad \text{GDP}^{\text{a}} + \text{X}^{\text{a}} = \text{C}^{\text{a}} + \text{F}_2^{\text{a}} + \Delta \text{H}^{\text{a}} \quad .$$

## 2. Further Aggregation.

The large model summarized above was designed for two specific purposes. One was to give a description and outline of all of the main relations and interrelations that comprise a complete economic system. This outline can then be used as an aid in current and historical analyses using literary descriptive methods supplemented by statistical data. Its other specific purpose was to set up a priori hypotheses for econometric testing, followed by the conversion of the model into a system of numerical equations. In this latter form it can provide quantitative as well as qualitative answers to specific questions.

The model outlined above however is too large for practical econometric model construction at the present time. Such obstacles as the lack of availability of data and the problems of

numerical computation intervene. Hence it is necessary to aggregate this model down to a size that is currently workable. Examples of models of the size that can be currently handled are found in the bibliography to this section. Why then has such a large theoretical model been constructed here?

The large model has been constructed as the only means of seeing in sufficient detail how the total economy works. It was believed that only with such a detailed view as a guide could completely satisfactory smaller aggregations be reached.

As economic data and electronic computers become increasingly available, numerical models of the size of our theoretical model can undoubtedly be handled, and toward these will very likely be the trend taken by the economics of the future. It is hoped that the present model will make a modest contribution in this direction, as well as helping currently with general macroeconomic analysis and econometric model construction.



Part XECONOMIC CHANGE AND FORECASTINGChapter 25.ON ECONOMIC FORECASTING1. Preliminary.

If a theory of the behaviour of some particular aspect or sector of the universe truly describes and explains this sector, then it must be able to predict future developments in the sector, subject to certain outside conditions. If it cannot do this then it is faulty theory, lacking in adequate cause-effect relationships. It follows that if the economic macro-theory developed in this study and summarized in the preceding chapter is an adequate theory of how the total economic system works, it should enable us to predict the future course of the economy, again subject to certain exogenous conditions. In this chapter we shall explore this possibility and the problems involved, attempting to provide a workable solution.

In any applied science the purpose is mainly to control the relevant aspects of the universe in the interests of human welfare. This control requires that it is possible to predict from cause to effect, so that if certain causes are manipulated it will be known that certain effects will be achieved. Prediction is a necessary part of control; it requires a knowledge of the cause-effect relationships,

and a knowledge of what will happen to the causes up to the period for which control is desired.

The general outlines of the forecasting problem begin to emerge, but before going further with it a short digression on the meaning of "cause" and "effect" as used in this study is necessary.

## 2. On "Cause" and "Effect".

No attempt will be made here to contribute to the philosophical argument about the ultimate meaning of these terms. Only usable, workable 'definitions' will be attempted. An "empirical regularity" such as "if A then B ; if B then not A" can be treated as a law of nature and can be used to deduce and predict, even though a full explanation of how A produces B cannot as yet be found. Scientists continue to refer to A as the cause of B in many such cases, as for example when a body falls under the influence of 'gravity'. 'Gravity' cannot as yet be 'explained'. Dr. H. A. Simon [10.30] [10.31] finds this to be still a good use of the term "cause", since the relation between A and B is asymmetric. He shows that the above 'causal' relation between A and B cannot be fully described by a functional relation of interdependence  $B = f(A)$ , for such an equation does not specify that B occurs as a result of A, while A does not occur as a result of B.

In our present study we shall attempt to get around this latter objection by stating that each autonomous equation of our model will usually be set up so that the variables on the right

hand side ( RHS) are the proximate causes of change in the variable on the LHS. But the equations in an economic model do not stand alone. They are all interrelated, so that in some other equation B may act as a proximate cause on A. In this sense there is interdependence between A and B. In the complete model acting as a whole all of the endogenous variables are interdependent in this way, while at the same time each is produced by a proximate, causal, asymmetric relation - its own autonomous explanatory equation.

There are however sets of causal variables in a complete model which are only causal, and in no sense interdependent with respect to the rest of the system. Their relationship to the system is asymmetric both in the proximate and in the ultimate sense. They affect the system ( if A then B), but the system does not affect them (if B not necessarily A).

In order to isolate these 'ultimate causes' let us set up a complete economic model comparable to that of the preceding chapter but in linear form.

Let  $Y = (Y_1, \dots, Y_n)$  be the vector of endogenous variables,  $Z_e = (Z_1, \dots, Z_m)$  the vector of exogenous variables, and  $Z_l = (Y_{a,-1}, \dots, Y_{r,-g}, Z_{b,-1}, \dots, Z_{s,-h})$  the vector of lagged variables. Let  $(Z_e Z_l) = Z$  be the vector of  $k$  predetermined variables. A representative autonomous or structural behaviour equation in the complete system might be

$$(1) \quad Y_i = b_{ia}Y_a + b_{id}Y_d + c_{im}Z_m + c_{ip}Z_p + c_{io} + u_{pi}$$

where  $u_{pi}$  is a random disturbance. This disturbance implies that (1) is not an exact relation, since it may not be possible to include in it all variables which exert a direct influence on  $Y_i$ , and since behaviour patterns themselves may vary from time to time. In the proximate sense the variables  $Y_a$ ,  $Y_d$ ,  $Z_m$ ,  $Z_p$ , the parameter vector  $(b_i c_i)$ , and the random disturbance  $u_{pi}$  are the causal influences which produce  $Y_i$ . The complete economy is represented by all of the  $n$  equations like (1), which together produce a complete model. In vector and matrix form, this model is represented by

(2)  $BY' + CZ' = u_p'$ .  $B$  and  $C$  are the matrices of structural parameters, like  $b_{ia}$  and  $c_{im}$ , associated with the endogenous and predetermined variables of the system respectively.  $B$  is of order  $n \times n$ , and  $C$  is of order  $n \times k$ .  $u_p$  is a vector of the partial or proximate disturbances associated with the separate structural equations like (1).

In the system (2) it is easy to see which variables are ultimately interrelated, and which variables are ultimately causal in Simon's sense. Through the system of equations the  $Y$  variables influence each other and are ultimately interdependent. But the variables  $Z$ ,  $u_p$ ,  $B$  and  $C$  all have an asymmetric relation with respect to  $Y$ .

This relation is expressed by solving (2) for  $Y$ ,

$$(3) Y' = -B^{-1}CZ' + B^{-1}u_p'; \quad Y' = -B^{-1}CZ'.$$

(3) is a reduced form of the basic or autonomous structure (2) and can henceforth be referred to as the structural reduced form. The disturbances  $B^{-1}u_p' = u_s'$  are related to the total model rather than to any particular structural equation. They are accordingly defined as the total disturbances of the model (Cf. Brown [10.5] p. 365).

Thus  $u_{si}$  is the total disturbance associated with  $Y_i$  after a solution (3) of the complete model. It is equal to  $Y_i - {}^*Y_i$ , and is the ultimate effect of all of the individual equation disturbances  $u_p$  on  $Y_i$ .

All of the variables on the RHS of (3) affect  $Y$ , but each of them is independent of and is unaffected in return by the values taken by  $Y$ . Because of this asymmetry the variables  $B, C, Z$  and  $u_p$  are defined to be ultimately causal with respect to  $Y$ . Whatever terminology we use, it is clear that if we wish to forecast  $Y$  we must first forecast the matrices and vectors  $B, C, Z$  and  $u_p$ .

### 3. Economic Change.

How does economic change occur? What are its sources?

How can a knowledge of it guide us with the problem of economic prediction? To answer these questions we must make use of the model of the economy in Part IX, and of its schematic representation in (2) or in (2a)  $BY' + C_1Z_1' + C_eZ_e' = u_p'$ , where  $C_1C_e = C$ .

Let us begin by supposing that there are no lagged variables in the model, that the exogenous variables remain constant, and that the disturbances are all zero. Solution using (3) will provide us with a set of values  ${}^*Y = Y$  which, along with the constant  $Z$  values, will satisfy all of the structural equations like (1) simultaneously.

It is of course these ex post solution values  ${}^*Y$  which correspond to the observed values  $Y$  of the real world, for each of these too is simultaneously the same in all of its different roles of proximate cause and ultimate effect. Given our assumptions,  ${}^*Y_t = {}^*Y_{t+1} = \dots = {}^*Y_{t+T}$ ,

and there is no change. What we have is a static solution, as well as a very simple type of stationary equilibrium.

Suppose now that we relax our assumptions about the causal variables in the economic system, one by one. We begin by allowing the random disturbances  $u_p$  to take non-zero values. Each  $u_{pit}$  will now be a random drawing from a universe with a probability distribution. Since  $u_{pit}$  is random each element of its universe is equally likely to be drawn in any instance or time period. The corresponding  $\cdot Y_t$  will now continue at constant levels as before, but the observed  $Y_t = \cdot Y_t + u_{pt}B^{-1}$  will reveal time paths or graphs at constant levels but with small random fluctuations about their horizontal trends.

Next we suppose that the exogenous variables begin to take on the rather large variations which they sometimes do in the real world. These variations will be transmitted first to proximate effect endogenous variables in the structural relations like (1), and will then spread to the other structural equations which contain these first proximate Y's as proximate causal variables, until a final equilibrium is reached, as in (3). Now the time paths of the  $Y_t$  will display rather large often erratic fluctuations, with the small random fluctuations attributable to  $u_p$  superimposed on these.

Next we introduce lagged variables  $Z_1$  into our  $Z$  vector. We know from our previous work that there are many such lagged variables in the real economy. We momentarily assume  $Z_e$  to remain constant and  $u_p$  to be zero. Now we shall no longer get constant values of  $Y_t$  from one time period to the next. The lagged variables will

usually change in successive time periods, and hence the solutions of (3) will change with dynamic time paths. What kinds of time path can we expect?

What we have now is a system of difference equations, and these tend to produce endogenous cycles and trends. The several stock variables in the system are particularly likely to add to the cyclical behaviour of the system. When the trends are asymptotic and the cycles are damped our system may eventually reach a stationary equilibrium where the  $Y$  values begin to repeat themselves from one time period to the next. This would correspond to the "stationary state" of economic theory.

Suppose next that we permit all of the causal variables  $u_p$ ,  $Z_e$ , and  $Z_l$  to change from time period to time period. What kind of time paths will the economic variables  $Y_t$  now follow? We shall observe a complex combination of trends, cycles, and large fluctuations, with small random fluctuations superimposed on the composite of these larger movements. Is this not the way the observed time series of economic life in fact behave?

There is one further source of economic change which needs to be considered. This is the slow evolution which we assume in the economic structure itself, here represented by  $B$  and  $C$ , to which we may add  $D$ , the multivariate probability distribution of the  $u_p$ . These changes occur with evolution in total preference systems, culture patterns, knowledge, technology and resources. It may be these in company with some changes in exogenous factors which produce some

of the very long range trends of economic life.

When all of these basic types of movement - endogenous trend and cycle, exogenous, disturbance and structure - happen to coincide in direction, there will be mutual reinforcement, 'resonance', and amplification. The economy will tend to move violently up or down. At other times basic movements may oppose each other, largely cancelling out, and providing a measure of overall stability. The general situation will be something in between these situations.

#### 4. Economic Forecasting - Principles and Procedures.

There have been many famous failures in economic prediction. Mainly this is because an immensely difficult job has been tackled with tools that were far from adequate. The present study has shown that the total economic system is vast and complex. Hence any attempts to project its movements into the future with only one equation such as an aggregate consumption function, or with a system of indicators which are not tied together in a system of autonomous structural cause-effect relationships, are doomed to failure from the start.

The logical approach to economic forecasting is surely to make full use of all the autonomous cause-effect relations possible for the macroeconomy, and to project these forward from ultimate causes to final effects in the forecast period. The causal variables  $B, C, Z, u_p$  can be called the predictors, the effect variables  $Y$  the predictands ( Hurwicz, [10.21], Chap. VI ). The relation-



ships (1), (2) and (3) are the laws or structure within which the predictors (causes) produce the predictands (effects). Since (3) is the form best suited for forecasting, we can use it to lay out a procedure for economic forecasting.

#### A Suggested Forecast Procedure

(a) Estimate Past Structure of Economy. This involves building a numerical model of the total economy, patterned after Part IX, but more aggregative. This structure may be linear or nonlinear, but must be autonomous or basic (Chap. 6). Structural parameters, here symbolized by B, C and D ( $u_p$ ) are estimated by a simultaneous procedure which takes into account the interrelatedness of all of the equations, using methods and references given in Klein [1.12], Koopmans [10.21] and Hood and Koopmans [10.15]. The present writer is working on simplification of the full maximum likelihood (FML) procedures.

The FML program includes estimates of the variance-covariance (VC) matrix of  $\text{vec}^* [B \ C] = a$ , the structure vector with ones and zeros omitted. We represent this VC matrix by  $S(a)$ . It also produces the full VC matrix  $S_p = S(u_p)$  of the disturbances.

#### (b) Forecast Structure for Forecast Period $t + 1$

Estimates of past structure should not be slavishly applied to the future. Through following closely the very current pulse of the economy the forecast group may detect structural changes applicable to the forecast period. These should be cautiously and carefully applied to B, C, and S(a), producing  $B_{Ft+1}$ ,  $C_{Ft+1}$ ,  $S_{Ft+1}(a)$ .  $B_F$  and  $C_F$  are the expected values, while  $S_F$  represents the dispersions of a multivariate probability distribution.

(c) Forecast of Disturbances  $u_p$ . We recall that the disturbances arise from omitted variables and small, temporary changes in structure. They are random and have expected value zero. Hence they should usually be forecast as zero. But there will be occasions when an omitted variable is changing drastically, or when there is a known temporary change in a structural equation which is applicable to the whole equation rather than to any particular parameter. In such cases the elements of  $u_p$  involved should be forecast at appropriate nonzero values. At the same time any changes that this may make appropriate to  $S_p$  should be cautiously applied. The product of this stage of the forecast is  $u_{pT+1}$  ,  $S_{pT+1}$  •

(d) Forecast of Exogenous Data. It is the exogenous variables of an economy that gear it into the total stream of history and change. The economic system is a part of the total social and physical universe, and the exogenous variables provide the links between the economic subsystem and the total. In the long run perhaps there is little that is exogenous to this greater system other than perhaps new ideas, discoveries and aspirations.

It follows that the forecasting of exogenous data requires a much broader view of the streams of current change than can be obtained from a study of economics alone. The forecast group must keep abreast of current international as well as domestic economic trends, and both domestic and international developments and trends in politics, social structures and conditions, technology and resources, demography and migrations, and finally conditions in the physical en-

vironment affecting production, health, weather, soils, and crops.

Literary, descriptive techniques must be used for most of the analysis here, and prediction is assisted by the projections of trends, many of which are stabilized in the short run by inertia.

Predictions in this field are particularly probabilistic, and the forecaster must estimate the expected values, variances and covariances of the variables he is predicting. 'Keeping score' of previous forecasts and subsequent observations and errors will gradually build up sufficient data and experience to enable the VC estimation. Error covariances can be computed from the formula

(4)  $S(Z_{1F}Z_{2F}) = r_{12} S(Z_{1F})S(Z_{2F})$ , where  $r_{12}$  is the estimated coefficient of correlation between forecast errors of  $Z_{1F}$  and  $Z_{2F}$ ; and  $S(Z_{1F})$  is the standard deviation of errors in forecasting  $Z_1$ .

In this area, as with the disturbances, the unexpected is to be expected, and allowed for within the probability distributions. Even with the best of search for trends unexpected events and unexpected timing as in the case of the Korean War, the Suez Crisis, and so on will continue to occur. Technological and other types of developments are usually slower to produce change, and can be allowed for more easily. The product of this stage of the forecast is the vector and matrix  $Z_{eFt+1}$  and  $S(Z_{eFt+1})$ .

(e) Forecast of Lagged Variables. When the first forecast of period  $t+1$  is being prepared it will usually be around the end of period  $t$ . Hence the elements of  $Y_t$ ,  $Z_{et}$ , and  $u_{sh,-1}$  or  $u_{sfg,-1}$  which are

required for  $Z_{1t+1}$  are not yet completely observed. However, with the help of the econometric model, current economic indicators, and the forecasters' knowledge of the current economic situation, fairly accurate estimates of  $Z_{1Ft+1}$  and  $S(Z_{1Ft+1})$  can be made. The elements of the  $S$  matrix in this case should be relatively very small.

(f) Forecast of the Endogenous Economic Variables. When the econometric model is linear the forecast values of the effect variables are obtained by substituting all of the forecast numerical values of the causal variables in the structural reduced form (3), followed by computation of the  $Y$  vector. Thus

$$(3a) \quad Y'_{Ft+1} = -B'_{Ft+1} C'_{Ft+1} Z'_{Ft+1} + B'_{Ft+1} u'_{pFt+1}.$$

Almost every element on the RHS of (3a) is a random variable with a probability distribution. The values substituted in (3a) are the expected values of these distributions. It follows that  $Y'_{Ft+1}$  is a vector of random variables, with each element a probability distribution. The values computed from (3a) are the expected values of these distributions. In the next chapter we shall develop a formula for the standard deviations of each of these distributions - the standard error of forecast - which will give us a measure of the dispersion or spread of each of them.

In case the model is nonlinear, its solution for a forecast of  $Y'_{Ft+1}$  will follow the same principles and steps as above, but now the matrix and vector solution (3a) cannot be used. Instead a process of continuous substitution until a single equation in one variable is reached, may provide a solution. This final equation

may be polynomial, the appropriate root of which can be found by Horner's method. If this procedure is not appropriate, iterative methods from a point in the neighbourhood of the solution will yield the desired solution.

5. Problems Related to the Conversion of the Forecast to a Prediction of Economic Movements.

In applied economics percentage movements from a selected base tend to be used more often than absolute values. In tabulating a forecast for presentation it is customary to show the best available estimates of the observed value  $Y_{i0t}$ , forecast value  $Y_{iFt+1}$ , and the percentage change  $(Y_{iFt+1} - Y_{i0t}) / Y_{i0t} \times 100 = \Delta Y_{iFt+1}$ . Complications arise whenever the total disturbances  $u_{sit}$  and  $u_{sit+1}$  are of the same order of magnitude as the change  $Y_{iFt+1} - Y_{i0t}$ . For now it is possible that the movement  $\Delta Y_{iFt+1}$  may be anomalous -- different from what economic theory would suggest. Thus in Fig. 1

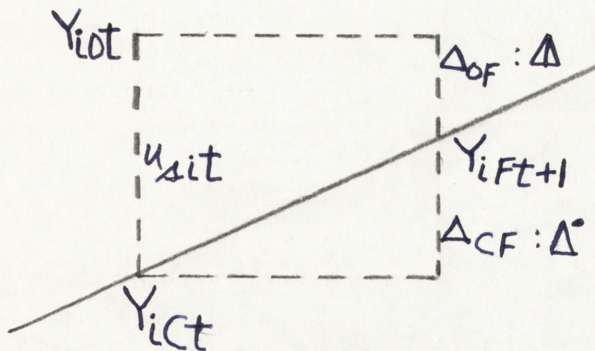


Fig. 1.



$Y_{iCt}$  is the model solution value for  $Y_{it}$  and  $Y_{iFt+1}$  is the corresponding model solution value for  $t+1$ .  $(Y_{iFt+1} - Y_{iCt}) / Y_{iCt} \times 100 = \Delta^{\circ} Y_{iFt+1}$  will be in accordance with the economic theory of the model and will appear rational. But  $\Delta Y_{iFt+1}$  (Fig. 1) will be opposite to the economic theory expressed by the model, and is anomalous. Yet  $\Delta$  may be closer to the true movement ultimately observed than will be  $\Delta^{\circ}$ , if we happen to be in an area of relatively large disturbances.

Are there any ways out of this dilemma? One is to show only  $\Delta^{\circ}$  as the forecast of economic movements, with the warning that disturbances in both base period  $t$  and forecast period  $t+1$  can contribute to error in this forecast. In effect this procedure assumes perfect serial correlation of disturbances from  $t$  to  $t+1$ .<sup>1</sup> Another is to present  $\Delta$ , explaining that any anomalous results are due to disturbances in the base period. A third way is to show  $\Delta^{\circ}$ ,  $u_s^{\circ} = u_{sit} / Y_{iCt} \times 100$ , and  $\Delta^{\circ} - u_s^{\circ} = \Delta^{\circ}$ . In this way the purely economic movements predicted are shown separately from the disturbances. A fourth alternative is to study all disturbances  $u_{sit}$  and  $u_{pit}$  and from them revise the structure B,C, so that the model is 'tuned' to explain  $t$  almost perfectly. This is followed by using the new and revised  $\Delta^{\circ}$  as the forecast. There is much to recommend this procedure, but it too assumes that structural disturbances for  $t$  before 'tuning' will persist into  $t+1$ , and that residual disturbances after adjustment or 'tuning' will also have perfect serial correlation

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1. I am indebted to Dr. L. R. Klein for pointing this out.

from  $t$  into  $t+1$ . A fifth alternative is to compute  $\Delta^*$ ,  $S(\Delta^*)$  and  $\Delta^* \pm S(\Delta^*)$ , where  $S(\Delta^*)$  is the standard error of  $\Delta^*$  ( see next chapter).  $\Delta^*$  can be tabulated as the forecast of purely economic movements. Then  $\Delta^* \pm S(\Delta^*)$  can be shown as the range within which the true value  $\Delta_{0^{y_{it+1}}} = (y_{i0t+1} - y_{i0t}) / y_{i0t} \times 100$  will fall approximately 70 percent of the time. The possible 'distortions' within this range can be explained as due to possible aberrations from normal behaviour throughout the model and in both periods  $t$  and  $t+1$ .

#### 6. The Time Span of a Forecast.

The time span may be defined as the time duration from the date when the forecast is completed to the date at the end of time period  $t+1$ . The shorter the time span the easier and more accurate the forecast will be. This is because existing trends in exogenous data will be preserved by inertia in the short run, but can change considerably in unforeseeable ways in the longer run. At the same time a greater number of unforeseeable random disturbances will be likely in a longer time period than in a short run. Structural change is also likely to occur in ways not foreseen in the longer time span. This means that forecast error will increase rapidly as the time span increases, until a point is reached where a forecast of a sufficiently distant period is of no value. There is probably little to be gained in making forecasts with a time span much greater than one year.

## 7. Projection.

Projection may be defined as the calculation of the implications of carrying forward into the future, for as large a time span as may be felt useful, the endogenous and exogenous trends and assumptions which appear currently to be appropriate to the present and future of the economy. The range of uncertainty surrounding projections is necessarily large because of unforeseen changes in exogenous data economic structure and disturbances. Hence they should not be called forecasts because no strong credence can be given to their detailed results. But they are useful, and indeed necessary as a guide to long range investment, both in the public and the private sectors. Middle run projections of from two to five year time spans may also prove useful in that they can show more clearly the nature of the endogenous trend-cycle in the economy, and hence place the shorter-run forecast within a broader context and background.<sup>1</sup> The shorter run forecasts can give better guidance for policy, when given within such a context.

## 8. Separation of the Forecast of Economic Variables into Components by Causes.

While the calculation of  $Y_{Ft+1}$  can be completed in one operation it is likely to be useful to separate the forecast movements into components, according to basic causes. Suppose for example that we first compute  $Y_{Ct}$  and  $Y_{F1t+1}$ , where  $B, C, Z_e$ , and  $u_p$

1. This suggestion arises from remarks on forecasting made by Dr. D.J. Daly.



are held constant at  $t$  values for both sets of computations. Only lags  $Z_1$  ( $Y_1$ ) are allowed to vary. Then  $Y_{F1t+1} - Y_{Ct}$  will reveal the effects of the endogenous trends and cycles inherent in the model as a result of stocks and lags. Next we change  $Z_{et}$  to  $Z_{eFt+1}$  and compute  $Y_{F2t+1}$ . Now  $Y_{F2t+1} - Y_{F1t+1}$  reveals the component of the forecast that is due to expected changes in the exogenous data. Changing the order in which we perform these two steps will give slightly different results, and the order must be selected on the basis of particular circumstances. Next  $u_{pt}$  are changed to  $u_{pFt+1}$  and the comparison  $Y_{F3t+1} - Y_{F2t+1}$  made to obtain the effect of the disturbances. Finally  $B_t C_t = A_t$  is changed to  $A_{Ft+1}$  and the comparison  $Y_{F4t+1} - Y_{F3t+1}$  made to obtain the separate effects of any structural changes. The sum of these four separate effects will of course add up to the total forecast change  $Y_{Ft+1} - Y_{Ct}$ .

Through a knowledge of the relative magnitudes of the effects of the four separate types of causes more appropriate policies can be selected.

Chapter 26.STANDARD ERROR OF FORECAST1. Preliminary Concepts.

Almost every element in a complete economic forecast is a random variable with a probability distribution. Up until now we have discussed methods of obtaining the expected values of these distributions, for both cause and effect variables, and variances and covariances of the errors in the causal groups. It remains for us to explore the computation of estimates of variances and standard deviations for the error distributions of the endogenous effect variables  $Y_F$ . The widths or dispersions of these distributions are of great importance when the forecast is to be used as a guide to economic policy. The standard deviation of a distribution is a good measure of its spread; and if the distribution is normal its standard deviation and expected value are sufficient to define it completely.

In this part of the research we consider the model to be fully linearized. We consider the complete forecast to consist of the matrices of economic structure  $B_{Ft+1}$ ,  $C_{Ft+1}$ , the vectors of exogenous variables and disturbances  $Z_{Ft+1}$ ,  $u_{pFt+1}$ , and finally the vector of endogenous economic variables  $Y_{Ft+1}$ . These are all expected values, and the corresponding VC matrices for all but  $Y$  are:  $S(a_F)$ ,  $S(Z_F)$  and  $S(u_{pF})$ .

Let the complete forecast of  $B, C, Z, u_p, Y$  be converted to a vector  $X_F$ . What do we mean by the forecast error of  $X_i$ , an element of this vector? The forecast value is  $X_{iFt+1}$ . The subsequent-

ly observed true value is a concept which we represent by  $X_{i0t+1}$ .

Then the forecast error is

(1)  $e_{iFt+1} = X_{iFt+1} - X_{i0t+1}$ . Henceforth we drop the subscript  $t+1$ , but assume it to be understood.  $e_{iF}$  is assumed to be a random variable,  $X_{i0}$  a constant, and  $X_{iF}$  a random variable with equivalent distribution to that of  $e_{iF}$ . Taking expectations on (1) we have,

(2)  $E(e_{iF}) = E(X_{iF}) - X_{i0}$ . But  $E(X_{iF}) = X_{i0}$  and  $E(e_{iF}) = 0$ , for

we assume all of our estimations of  $X_{iF}$  to be unbiased. Correspondingly the variance of the forecast  $X_{iF}$  equals the variance of  $e_{iF}$  from (2), since  $\sigma^2(X_{i0}) = 0$ . Thus  $S(X_{iF}) = \text{est. } \sigma(X_{iF}) = S(e_{iF}) = \underline{\text{standard error of forecast.}}$

## 2. Development of Formula for Computation of Standard Error of Forecast For the Endogenous Variables.

The ultimately causal and the effect variables of the forecast are connected by the structural equations, or basic economic laws

(3)  $B Y' + C Z' = u'_p$ , and by the structural reduced form equations

(4)  $Y'_F = -B_F^{-1} C_F Z'_F + B_F^{-1} u'_{pF}$ . The latter can be written as

(4a)  $Y'_F = F_F Z'_F + u'_{sF}$ .

Each individual equation out of (4a) explains an individual  $Y_F$ , say  $Y_{iF}$ , in terms of ultimately causal variables only. The interdependence among the  $Y$ 's is already taken care of, is in a sense 'ex post' in (4a), but is still evident from each  $Y_i$ 's dependence on the same  $B, C, Z, u_p$ , as any other  $Y_j$ . Hence we can work with each of these

equations separately. The formula for  $S(Y_{iF})$  will be of indential form as for the standard error of forecast of any other  $Y, Y_j$ . Hence we need only develop it for one,  $Y_i$ .

The  $i$ 'th equation from (4a) is

(5)  $Y_{iF} = f_{i1F}Z_1 + \dots + f_{ikF}Z_{kF} + u_{siF}$ , where  $Z_1 \equiv 1$ , and  $f_{i1F}Z_1$  is the constant term of the equation. If the true values for all of the causal variables for the forecast period were known, and the structure was truly linear, the true value  $Y_{i0t+1}$  could be forecast from the 'true' equation.

(5a)  $Y_{i0} = \phi_{i1}Z_{10} + \dots + \phi_{ik}Z_{k0} + \mu_{si0}$ .  $\phi_i$  is the vector of true parameters effective in  $t+1$  in the  $i$ 'th structural reduced form equation. The forecast error is

(6)  $Y_{iF} - Y_{i0} = f_{i1F}Z_1 - \phi_{i1}Z_1 + \dots + f_{ikF}Z_{kF} - \phi_{ik}Z_{k0} + u_{siF} - \mu_{si0} = e_{iF}$ . Let

(7)  $Y_{iF} - Y_{i0} = \Delta Y_{iF}$ ;  $f_i - \phi_i = \Delta f_i$ ;  $Z_F - Z_0 = \Delta Z_F$ ; and  $u_{siF} - \mu_{si0} = \Delta u_{siF}$ . Subscript F is temporarily dropped from  $f_{iF}$ , but

understood. Then

(8)  $f_{ij}Z_{jF} - \phi_{ij}Z_{j0} = f_{ij}Z_{jF} - (f_{ij} - \Delta f_{ij})(Z_{jF} - \Delta Z_{jF})$   
 $= f_{ij} \Delta Z_{jF} + (\Delta f_{ij})Z_{jF} - (\Delta f_{ij})(\Delta Z_{jF})$ .

Using (7), (8) transform (6) into vector form,

(9)  $\Delta Y_{iF} = f_i (\Delta Z_F)' + (\Delta f_i) Z_F' - (\Delta f_i) (\Delta Z_F)' = \Delta u_{siF}$ .

Let  $b^i$  be the  $i$ 'th row of  $B^{-1}$ . Then

(10)  $\Delta u_{siF} = b_F^i u_{pF}' - \beta^i \mu_{p0}' = b^i (\Delta u_{pF}') + (\Delta b^i) u_{pF}' - (\Delta b^i) (\Delta u_{pF}')$ .

In order to find the standard deviation of  $Y_{iF}$  we must first find its variance, which will be equal to the average or expected value of  $(\Delta Y_{iF})^2$  as  $\Delta Y_{iF}$  ranges over its complete universe or distribution. If the distribution function is  $g(\Delta Y_{iF})$ ,

$$(11) \quad \sigma^2(Y_{iF}) = E(\Delta Y_{iF})^2 = \int_{-\infty}^{+\infty} (\Delta Y_{iF})^2 g(\Delta Y_{iF}) d(\Delta Y_{iF}) .$$

$(\Delta Y_{iF})^2$  can be expressed in terms of already available estimates of causal factors and their errors, by substituting (10) into (9), and then multiplying the result by its transpose. The resulting rather bulky product simplifies considerably because of the independence of the forecast errors of the three vectors  $f$ ,  $Z$  and  $u_p$  with respect to each other. The following vector-matrix identities are also helpful. Let  $a$ ,  $b$ ,  $x$ ,  $y$  be four vectors, each containing  $n$  elements.

Then

$$(12) \quad \begin{aligned} (i) \quad (ax')(xa') &\equiv a [x'x] a' \\ (ii) \quad a [x'x] a' &\equiv x [a'a] x' \\ (iii) \quad (ax')(by') &\equiv a [x'y] b' \\ (iv) \quad a [x'x] a' &\equiv \text{vec}(a'a) \cdot \text{vec}'(x'x) \\ &\equiv \text{trace} [x'x] [a'a] . \end{aligned}$$

Also the expected value of terms containing an odd number of error elements will be zero.

Using the above method and tools, and temporarily dropping all subscripts, we obtain

$$(13) \quad \begin{aligned} \sigma^2(Y) &= f \sigma(Z) f' + Z \sigma(f) Z' + \text{vec} \sigma(f) \text{vec}' \sigma(Z) \\ &+ b^i \sigma(u_p) b^{i'} + u_p \sigma(b^i) u_p' + \text{vec} \sigma(b^i) \text{vec}' \sigma(u_p) \\ &+ 2 u_p \sigma(b^i f) Z' . \end{aligned}$$

The matrix  $\sigma(f)$  is the variance-covariance matrix of the elements of  $f_i$ . The method for computing it has been previously worked out by the writer in [10.7]. The matrix  $\sigma(b^i f)$  is made up of the covariances of the  $b_j^i$  and  $f_{ik}$  which come together as products in  $[b^i f]$ . These covariances can be picked out of the variance-covariance matrix  $\sigma(b^i f_i)$  of the vector  $(b^i f_i)$ , which can be computed by the same methods used to arrive at  $\sigma(f_i)$  in [10.7].

Expressed in symbols which imply estimated values derived from a limited sample of data, (13) becomes

$$(14) \quad S^2(Y_{iF}) = f_{iF} \left[ S(Z_F) \right] f_{iF}' + Z_F \left[ S(f_{iF}) \right] Z_F' + \text{vec } S(Z_F) \text{vec}' S(f_{iF}) \\ + b_F^i \left[ S(u_{pF}) \right] b_F^{i'} + u_{pF} \left[ S(b_F^i) \right] u_{pF}' + \text{vec } S(u_{pF}) \text{vec}' S(b_F^i) + \\ 2u_{pF} \left[ S(b_F^i f_{iF}) \right] Z_F'.$$

This is the desired formula for the variance of the forecast. Its square root  $S(Y_{iF})$  is the standard error of forecast for the endogenous variable  $Y_{iF}$ .

Discussion on the problem of adjustment of  $S(Y_{iF})$  to  $\bar{S}(Y_{iF})$  for small sample estimation of  $B, C, S_p$  and for loss of degrees of freedom, and on the presentation of the  $X_{iF}$  results in confidence or tolerance intervals, is also found in Brown [10.7]. If results are presented in the form of a range of values  $Y_{iF} \pm k \bar{S}(Y_{iF})$ , and if the distribution of  $Y_{iF}$  is approximately normal, it is possible to state the probability that  $Y_{i0}$  will fall within this range. This probability will be roughly .68 when  $k = 1$  and .95 when  $k = 2$ .

### 3. Separation of the Error Formula into Different Causal Components.

The endogenous variable forecast  $Y_{iF}$  is based on prior prediction of three groups of causal forces: economic structure, predetermined data, and disturbances. Formula (14) can be approximately separated into three components which correspond to these causal groups. Thus

$$(15) \quad S_s^2 (Y_{iF}) = Z_F S(f_{iF}) Z_F' + w_{s1} \text{vec } S(f_{iF}) \text{vec}' S(Z_F) + u_{pF} S(b_F^i) u_{pF}' \\ + w_{s2} \text{vec } S(u_{pF}) \text{vec}' S(b_F^i) + 2u_{pF} S(b_F^i f_{iF}) Z_F'$$

can be attributed to errors in forecasting structure.

$w_{s1}$  is an arbitrary weight equal to the ratio of the second term on RHS of (14) to the sum of the first two terms;  $w_{s2}$  equals the ratio of the fifth to the sum of the fourth and fifth terms. Note how in  $S_s^2$  the errors in the structure forecast increase quadratically with increases of  $Z_F$  and  $u_{pF}$ . Thus for forecasts beyond the ranges of  $Z$  and  $u_p$  in the estimation sample of data the errors in  $Y_{iF}$  will undergo accelerated expansion. This is another source of large error in long range forecasting.

Similarly the components of (14) attributable to errors in predicting predetermined data are summed in

$$(16) \quad S_p^2 (Y_{iF}) = f_{iF} S(Z_F) f_{iF}' + w_p \text{vec } S(f_{iF}) \text{vec}' S(Z_F); (w_p + w_{s1} = 1).$$

The remaining portion of  $S^2(Y_{iF})$  is due to errors in predicting disturbances - largely the occurrence of unpredictable random disturbances.

$$(17) \quad S_d^2 (Y_{iF}) = b_F^i S(u_{pF}) b_F^{i'} + w_d \text{vec } S(u_{pF}) \text{vec}' S(b_F^i); (w_d + w_{s2} = 1).$$

Structure and disturbance error have a special importance in that combined they reveal the variability, with respect to prediction, of the laws of aggregative economic behaviour. Thus we can abstract the economic structure from its exogenous links with other non-economic fields and forces. It is vitally important for both macroeconomic theory and policy that the magnitude of this variability be known. We may refer to it as model, economic or conditional error variance ( conditional on the exact knowledge of the predetermined data), and define it as

$$(18) \quad S_m^2 = \text{sum of terms 2,4,5,6,7 of (14) .}$$

The advantage of separating the relative contributions of structure, predetermined data and disturbances to the total variance of forecast is that this reveals to the forecast group the relatively strong and weak components in the forecast. A better allocation of resources for improving the forecast machinery and reducing the spread of forecast error distributions can be achieved with this knowledge.

#### 4. Total Forecast Error, 'A Posteriori'.

Formula (14) provides us with what is essentially an 'a priori' or deductive concept of forecast error. It was however based on certain simplifying assumptions. Hence a distribution of observed forecast error ( $Y_{iF} - Y_{iO}$ ) may be found to be 'wider' in terms of standard deviation than that given by (14). The main error



Sources not fully covered by the a priori analysis can be listed as follows:

- (a) Nonlinear Structure. Our linear structure actually includes many nonlinear situations by using linear arms joined at critical corners; and by using tangent and secant planes to curved surfaces, in the neighbourhood of current activity. But there will still be some residual error due to nonlinearities of the true structure.
- (b) Faulty Structural Estimation due to Evolution of Structure. The true economic structure presumably undergoes slow evolution. Time series estimation will accordingly produce an estimate of the average structure over the time period covered by the data.
- (c) Aggregation Error. This error was discussed in Chapter 5.
- (d) Observation Error. The estimation theory assumes that all variables are observed without error.
- (e) Degrees of Freedom and 'Asymptotic' Error. Maximum Likelihood methods of structure estimation are asymptotic and only reach their final 'true' values as the number of observations approaches infinity ( [10.21] p. 210 ). Also precise correction of  $S(f_i)$  and  $S(u_p)$  for loss of degrees of freedom in estimation has not as yet been worked out.
- (f) Variables Omitted from Structural Equations. All of the lesser omissions are assumed to be reflected in the random disturbances. But sometimes major variables may be omitted because of faulty theory, or because they have not been measured.
- (g) Errors in Estimates of  $S(Z_F)$ . These errors will gradually be reduced as a forecasting project gathers experience, and data on past errors.

We may define ex post or a posteriori observations and records of the form

(19)  $e_{TiF} = Y_{iF} - Y_{i0}$  as total forecast error. As data on this error is collected for the same variables, and for the same time span of forecast, a frequency distribution will begin to emerge. The variance of this distribution can be estimated by

$$(20) S_T^2(Y_{iF}) = \frac{1}{n} \sum_{t=1}^{t=n} e_{TiFt}^2 = \underline{\text{total forecast error variance.}}$$

Once such data as (20) become available it is possible to estimate how much of the errors of forecasting lie outside of the a priori analysis. Thus

$$(21) S_T^2(Y_{iF}) = S^2(Y_{iF}) \text{ ( from (14) )} + S_a^2(Y_{iF}),$$

where  $S_a^2$  represents error variance due to sources of error additional to those covered in the a priori analysis.  $S_a^2$  is a most important component of the total error variance for it reveals the magnitude of error in macroeconomic forecasting caused by the above sources of error (a) --- (g), which lie outside of the 'a priori' and linear theory developed here. This error can be estimated directly from (21), and it can also be estimated in the following way. Assume that observed data is available for the time periods 0, ---,  $t+n$ . Fit a model to the data for 0, ---,  $t$ . Use this model to 'predict' for the periods  $t+1$ , ---,  $t+n$ , using 'true' observed Z data. Then obtain from  $(Y_{iFt+r} - Y_{i0t+r})$  values

$$(22) \hat{S}_T^2 = S_m^2 + S_a^2 ; (S_p^2 = 0). \text{ Compute } S_m^2 \text{ independently from (18)}$$

for the average values of Z for the period  $t+1$ , ---,  $t+n$ , and thereby estimate  $S_a^2$  from (22).

### 5. Standard Error of Forecast of Economic Movements.

In Chapter 25 the forecast economic movement of  $Y_i$  was defined as

$$(23) \quad \Delta^{\circ} Y_{iFt+1} = (Y_{iFt+1} - Y_{iCt}) / Y_{iCt} \times 100.$$

Errors in  $\Delta^{\circ}$  occur as a result of disturbances, structure errors and predetermined data involved in the computation of both  $Y_{iCt}$  and  $Y_{iFt+1}$ .

The variance of  $\Delta^{\circ}$  is given approximately by

$$(24) \quad \frac{S^2 (\Delta^{\circ} Y_{iFt+1})}{10^4} = \frac{S^2 (Y_{iFt+1})}{Y_{iCt}^2} - 2 \left( \frac{Y_{iFt+1}}{Y_{iCt}} \right) Z_{Ft} S (f_{iF}) Z'_{Ft+1} + u_{PFt} S(b_{iF}^i) u_{PFt+1} + \frac{Y_{iFt+1}^2}{Y_{iCt}^4} S^2 (Y_{iCt}).$$

The development of this formula is omitted to save space. The middle term allows for the fact that in short-run forecasting almost the same structure will be used for  $t$  and  $t+1$ , and almost the same structure errors will occur in both periods.

### 6. Concluding Remarks.

Empirical research on all of the components of forecast error is vitally needed in order to resolve the many doubts about the possibility of obtaining a useful empirical system of macroeconomics for overall policy purposes. In particular the relative magnitudes of "model" and "additional" errors expressed as "coefficients of variations" -  $S_m (Y_{iF}) / Y_{iF} \times 100$  and  $S_a (Y_{iF}) / Y_{iF} \times 100$  - will reveal the possibility of useful empirical macroeconomic science. One can have faith that such a science is possible from the observation

that modern governments achieve a reasonable measure of good policy results using much weaker tools than those suggested here. If the random components of macroeconomic behaviour were excessively large this would scarcely be possible. Preliminary empirical research by the writer [10.6] gives support to this belief.

One can hope that with the more precise tools suggested here policy can become much more refined, accurate and certain than it is even now. For it is still subject to wide areas of uncertainty.

Part XIAPPLICATIONS, POLICY AND CONCLUSIONChapter 27.IMPACT ANALYSIS AND THE EXPLANATION OF CHANGE.1. General.

With a numerical economic model which fits the aggregate economy faithfully it is possible to deduce the impact of changes in some economic causes or variables on other economic variables. Such deduction can apply to the future, for which parts of a policy program are being worked out, or to the past where a separate piece of historical explanation is being constructed. The term cause is here being used in a wide sense to include one way influence on some variables from any of the variables of an economic system - structure ( B, C), exogenous (  $Z_e$ ), lagged (  $Z_l$ ), disturbances (  $u_p$ ), and endogenous Y. The explanation may be proximate, intermediate, or ultimate.

Where the explanation is proximate only the developments in a single structural equation are considered. Let such an equation be

$$(1) \quad Y_i = f_i(Y_c, Y_d ; Z_p, Z_q, Z_r) + u_{pi}.$$

Then changes in  $Y_i$  occur as the effects of changes in  $f_i$ ,  $Y_c$ , ---,  $u_{pi}$  = vector X. Let  $f_{i1}$  be the unchanged part of  $f_i$  such that

$$(2) \quad Y_i = f_{i1}(X) = f_{i1}(X_1, --- X_j, X_k, X_m, --- X_r ).$$

Suppose that changes occur only in  $X_j$ ,  $X_k$ ,  $X_m$ , producing a change in  $Y_i$ . Then  $X_j$ ,  $X_k$ ,  $X_m$  are the proximate causes of change in  $Y_i$ , producing

$$(3) Y_i + \Delta Y_i = f_{il}(X_1, \dots, X_j + \Delta X_j, X_k + \Delta X_k, X_m + \Delta X_m, \dots, X_r)$$

$\Delta Y_i$  is explained by the causes  $\Delta X_j$ ,  $\Delta X_k$ ,  $\Delta X_m$ , =  $\Delta X_{jkm}$ , and the separate and joint impacts of these on  $Y_i$ . To get at the separate impacts we use the Taylor Expansion

$$(4) \Delta Y_i = (\Delta X_j, \Delta X_k, \Delta X_m) \left( \frac{\partial f_{il}}{\partial X_j}, \frac{\partial f_{il}}{\partial X_k}, \frac{\partial f_{il}}{\partial X_m} \right) + 1/2 \Delta X_{jkm} \left[ \frac{\partial^2 f_{il}}{\partial X^j \partial X^k \partial X^m} \right]_{jkm} \Delta X'_{jkm} + \dots$$

To a first approximation, satisfactory for small changes, the separate effects of the three causes on  $Y_i$  are

$$(5) \frac{\partial f_{il}}{\partial X_j} \Delta X_j, \dots, \frac{\partial f_{il}}{\partial X_m} \Delta X_m .$$

For larger changes, if it is necessary to use the quadratic term of (4), composite influences can be approximately separated by some appropriately selected weighting system. In this way both historical changes, and the impact of future policies, can be given proximate explanation.

Explanation can be carried to an intermediate stage if two or more structural equations like (1) are involved, and if  $Y_i$  and any of  $X_j$ ,  $X_k$ ,  $X_m$  appear in the structural equations other than (1). Now both  $\Delta Y_i$  and  $\Delta X_{jkm}$  become proximate causes in these other equations which explain say  $Y_g$  and  $Y_h$ . An example of the chain of explanation now is

(6)  $(\Delta X_j, \Delta X_k, \Delta X_m) \rightarrow \Delta Y_i; (\Delta Y_i, \Delta X_k) \rightarrow \Delta Y_g; (\Delta Y_i, \Delta Y_g, \Delta X_j, \Delta X_m) \rightarrow \Delta Y_h$ . In each stage of (6) we have proximate causes explaining certain effects, with effects from prior stages becoming causes in subsequent stages.

Finally we can consider the ultimate explanation of certain effects. Here we are concerned only with primary or ultimate causes which enter the economic system in the realms of structure, exogenous, lagged, and disturbance variables; and with the final effects of these on the economic variables  $Y$ . In this kind of explanation all intermediate and proximate chains of cause and effect are assumed to have worked themselves out within the time period of the analysis, and are ignored. Explanation is now most conveniently cast in terms of the structural reduced form (see Part X). This is

(7)  $Y' = -B^{-1}CZ' + B^{-1}u_p' = FZ' + u_s'$ , in the linear case. In a nonlinear model consisting of all autonomous structural equations like (1), the structural reduced form can be represented by

(8)  $Y' = G(a)\{Z, u_p\}$ , where "a" is the vector of all parameters in the structural model, and  $G(a)$  is a vector of functions of "a".  $G(a)$  is derived from the solution of the structural model for all  $Y$ 's in terms of  $Z$ 's and  $u_p$ 's. Changes in each element of  $Y$  in (8) for any time period can now be explained separately in terms of changes in  $a, f(a), Z, u_p$ , using the method associated with (1) --- (5) above. Explanation is now in terms only of remote, ultimate or primary causes entering the economic system essentially from outside.

## 2. Multiplier or Repercussion Tables.

For both historical analysis, and the approximate and partial solution to current policy problems, it will be found useful to precompute many of the partial derivatives of the total economic system. Firstly we can readily compute proximate partial derivatives or multipliers from the complete structural model built from equations like (1). If X represents all variables and parameters in the complete model, the matrix of proximate multipliers will be of the form

$$(9) \left[ \frac{\partial Y'}{\partial X} \right]_p, \text{ where subscript p means "proximate" and computed}$$

from the basic structural model. Finally a system of ultimate partial derivatives or multipliers could be worked out from the structural reduced form(8). Let U represent only the primary causal variables "a", Z,  $u_p$  in the structural model. Then from (8) compute the matrix

$$(10) \left[ \frac{\partial Y'}{\partial U} \right]_u, \text{ where subscript u implies that the matrix gives}$$

the ultimate multiplier effects from primary causes. A simple example of such a matrix as (10), but excluding partial derivatives related to structure, is shown in Brown [10.5] p. 369. Both of these multiplier matrices should be evaluated for the values of X relevant to the time period for which they are to be used. (10) may be a system of either short-run or long-run multipliers, depending on the time period used in the analysis ( Cf. Brown [2.6] pp. 366-370 ). Each element of (10) will of course be a function of a large number, sometimes all, of the parameters from the original structural model.

Once these matrices are available they can be used



to give approximate causal analysis with great speed, using simple formulas like

$$(11) \Delta_p Y' = \left[ \frac{\partial Y'}{\partial X_1} \right]_p \cdot \Delta X_1 ;$$

$$(12) \Delta_{123u} Y' = \left[ \frac{\partial Y'}{\partial (U_1, U_2, U_3)} \right]_u (\Delta U_1, \Delta U_2, \Delta U_3)'$$

Chapter 28.ECONOMIC POLICY1. Purpose of Chapter.

It is not the purpose of this chapter to go into any detail on policy, or to attempt to cover the subject at all thoroughly. Such would be beyond the scope or space of the present study. Instead the purpose is mainly to show how an economic model, and the forecast and impact techniques presented above, can help to improve and refine policy, to make it more scientific, and more certain of success.

2. The Meaning of Economic Policy.

A society and its government evolve from experience desired economic states or goals. These can be thought of as very aggregative variables, in aggregative preference, utility or social welfare functions. As we discussed in Part VI on Government and in Part VIII on the Price System, assistance in the attainment of these goals has become, in modern societies, a major part of government agenda. In all aspects of the goals where the individual is powerless, government is increasingly expected to take over. The major economic goals could be listed as : full employment; economic development to attain higher living standards; stable value of money; an equitable distribution of income; the prevention of monopolistic restrictions on output; the attainment of an optimum allocation of resources; the pres-

ervation of adequate international reserves (IR), the preservation of the external value of domestic money ( $1/e_1$ ). Each society selects, through its political and social institutions, that set of economic goals which maximizes its social welfare function, subject to environmental constraints, and also subject to all of the noneconomic values which are also a part of the welfare function. Let us henceforth refer to this set of goals as the goal pattern, or simply as the economic goals of the society.

Economic policy in its most general sense represents the acts or steps planned and taken by a society, mainly through its government, to achieve its economic goals. It involves changing those parts of the ultimate causal system of the economy which are subject to the control of the society. Those changes will be sought which convert the economy from a given state to one which conforms to the goal pattern. The ultimate sources of economic change which are subject to domestic variation are known as instruments of policy (Tinbergen [11.14], [11.15]).

Government economic policy tends to fall into two broad categories: indirect controls and direct controls. The indirect controls are blanket, impersonal actions or legal rules which tend to affect all economic units involved in much the same way and in such a way that their maximization activities and free initiative are not interfered with. They find their own adaptation to the new situation. Thus indirect controls change the environment, within which economic units can operate freely. Examples of indirect controls are deliberate

changes in : government spending, taxation, transfer payments, net borrowing; the supply of money; the rate of exchange ( $e_1$ ); the level of tariffs and other import taxes.

Direct controls on the other hand involve an abrogation of the free play of markets, and of the free choices of firms and households to maximize within a given environment. Where the indirect controls changed the environment, the direct controls change behaviour directly. Examples of direct controls are: household rationing, price control, output regulation, import control, export control, wage rate regulation, manpower allocation.

In general indirect controls are preferred to direct controls in peacetime, since they leave greater freedom and initiative to the individual, and thereby promote better attitudes and higher productivity. In wartime or other national emergency however, direct controls are necessary to get fast specific results, and at such times these will usually have the support of public opinion.

The present study is mainly concerned with the techniques of using indirect policy, although the machinery developed will also reveal some of the occasions when direct controls become necessary to achieve the goals. For example it will reveal circumstances when import controls become necessary to conserve IR, where exchange depreciation ( increase of  $e_1$ ) would threaten the price level. The indirect controls tend to fall into three main areas - fiscal policy, monetary policy, and foreign trade, finance and exchange rate

policy.

### 3. The Use of an Economic Model as an Aid to Policy Formation.

Symbolize the economic model by

$$(1) \quad B Y' + C Z_1' + D Z_e' = u_p'$$

Let the totality of variables in (1)

be represented by vector  $X$ . The economic state in period  $t$  is  $X_t$ .

The goal for  $t+1$  is  $X_{t+1}^*$ . A forecast is made for  $t+1$ , assuming no change in policy, and revealing  $X_{Ft+1} = k S(X_{iFt+1})$  ( $i$  ranging over all elements of  $X$ ). Usually  $X_{Ft+1}$  will differ from  $X_{t+1}^*$ . Policy represents a vector of deliberate changes in controllable causal variables which will deflect the economy during  $t+1$  from  $X_{Ft+1}$  to  $X_{t+1}^*$ . This is represented diagrammatically in Fig. 1.

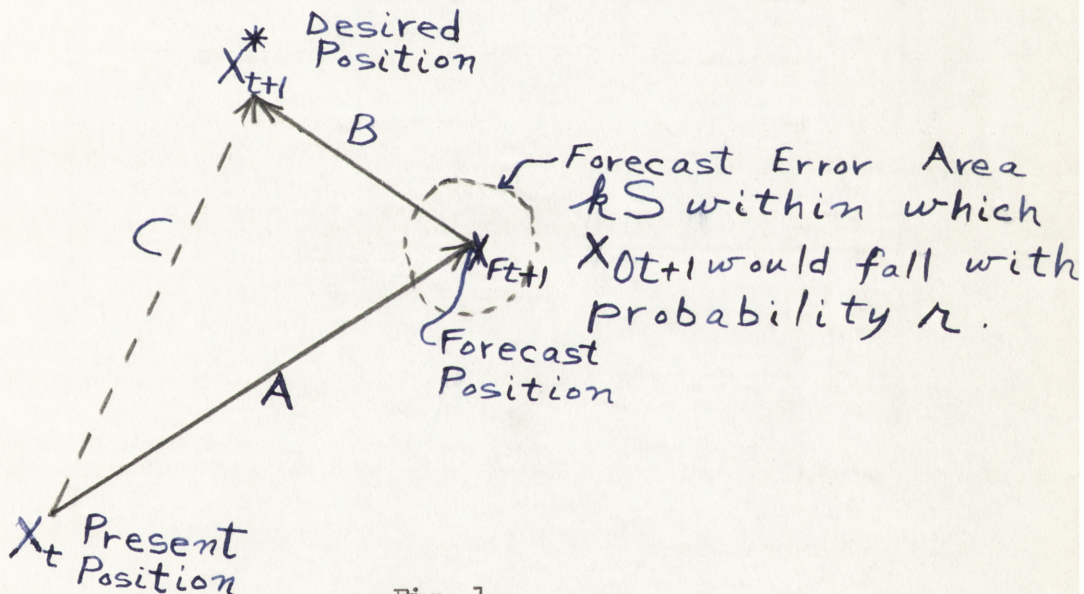


Fig. 1.

The vector A represents the direction in which the economy appears to be moving, subject to the forecast error represented by the area  $kS$ . The vector B represents the changes that policy must

make to  $X_F$  to bring it to  $X^*$ .  $C$  is the course on which the policy makers would like the economy to move.

The instruments of policy fall mainly within the  $Z_e$  vector of (1). Let us designate the elements of  $Z_e$  that are controllable domestically as  $Z_p$ . These are like the switches on a massive control panel. Which ones shall be pulled, and by how much, in order to bring the economy to  $X^*$ ?

The usual approach to this problem is one of trial and error. A policy that is acceptable politically and socially and which seems likely to succeed, on the basis of guess, hunch and experience, is put into operation. If it fails alterations will be made, some of them not until the next budget. This is a slow and uncertain approach, and may not serve in really difficult times. An approach something like this, but making use of the numerical economic model is to devise several alternative policies  $Z_p$ , with the help of the multiplier tables (9) and (10) and formulas (11) and (12) of Chap. 27. Each of these  $Z_p$  policy vectors is then tested in the model, (1), to see which will bring a solution closest to  $X^*$ . The policies may then be weighed with respect to risk or probability of error, and the costs of different kinds of error. The costs of the policies themselves will also be taken into account in making the choice. This may involve some revision of the constrained maximum welfare position  $X^*$ .

Professor Tinbergen( [11.14] , [11.15] ) has



shown how at times some of the guesswork can be taken out of policy selection. Suppose that the goal,  $X^*$  fixes only  $Y^*$  ( $m$  elements) of  $Y$  as targets (Tinbergen), leaving  $Y_f$  ( $n-m$  elements) still free to vary. Let  $Z_p$  contain  $p$  elements. The unknowns in the problem now become  $Y_f$  and  $Z_p$  ( $n - m + p$  elements). They become variables constrained within the original  $n$  equations. The system (1), can be rearranged, into the form

$$(2) \quad E(Z_p, Y_f) = F(Z_r', Y^*) + u_p'; \quad (Z_r, Z_p = Z = Z_1, Z_e).$$

A solution for  $Z_p$ ,  $Y_f$  and hence for the desired policy vector  $Z_p$  can be obtained directly if matrix  $E$  is square. But  $E$  is of order  $n \times (n - m + p)$ . Hence for direct solution  $m$ , the number of targets, must equal  $p$ , the number of policy instruments.

Where  $m \neq p$  the problem of finding  $Z_p$  is more difficult. Let us consider the two cases.

Case 1.  $m < p$ . There are more instruments than targets, and more unknowns in (2) than there are equations. There will now be an infinity of solutions. The attempt should be made to decrease the number of instruments and increase the number of targets, until equality is reached. If this is not possible, the next step is to apply the infinity of solutions to the social welfare or public opinion function, reaching a new, and perhaps slightly revised, constrained maximum. The constraints now include (2), boundary maxima on many variables, and the noneconomic components of total welfare, most of which would already have been taken into account in arriving at  $X^*$ . Boundary maxima or limitations include variables like the labour force, minimum IR,

the stock of producers' fixed capital,  $H_{\min}$ , and so on.

A simpler approach than this may be to leave  $X^*$  unrevised, let  $X_c$  represent the infinite set of solutions, and find  $\bar{X}_c$  which minimizes the distance  $(X^* - X_c)$ , subject to the many constraints of boundaries, costs and risks. In both cases the problem is now similar to the problems attacked by linear programming methods.

Case 2  $m > p$ . Now there are more equations than there are unknowns. If targets cannot be reduced and instruments increased, we are left with a finite number of solutions ( $n^{C_{n-m+p}}$ ). Out of these a 'best' solution for  $(Z_p, Y_f)$  must be selected. The approaches suggested for Case 1 seem to be equally applicable here.

#### 4. Errors, and Policy Execution.

Errors in the economic model and in the prediction of non-controllable exogenous data will produce a policy error. This error will be compounded from the forecast error, and the estimation of the policy vector ( apart from forecast error). These errors will prevent the economy from reaching the goal  $X^*$ . How can they be circumvented?

The problem is in many ways analagous to sea or air navigation. The first course set by a navigator will rarely take his craft straight to destination. This is because of errors in his instruments, errors in the prediction of wind and weather or atmospheric conditions ( exogenous data), and unexpected disturbances of many kinds. During a flight the navigator must check on his current pos-



ition, on atmospheric conditions and on the wind. On the basis of his findings he must forecast a future position, and on the strength of this alter course to destination. He repeats this process several times, until his goal is reached.

The economist likewise sets an initial course to his goal, with an initial policy (course) based on an initial forecast. Then he must maintain a continuous outlook for his current position and for changes in external, exogenous factors. Periodically he must revise his forecast of where the economy is going, and along with this compute a revised policy vector (course) to the desired goal. Continuing this process, with ever shrinking errors as the span of forecast decreases, the economist like the navigator should be able to guide the economy successfully to destination.

##### 5. Difficult Policy Problems.

There may be times when exogenous conditions or constraints of one kind or another render indirect policy solutions difficult to discover. A few such cases are mentioned here briefly to suggest situations which can be revealed, but not always solved, by the methods above.

Case 1. There is danger of domestic inflation in A arising from inflation in the rest of the world, especially in A's important customers and sources of supply. The foreign inflation is transmitted to A through import prices, export demand and prices, and foreign financial and direct investment in A. Given this situation, however well A's policies

may produce internal price stability and full employment, these external influences will cause  $P'$  to rise. Is there any way out of this situation?

It would seem that it is the natural function of the exchange rate  $e_1$  to insulate a country against such outside forces. An appropriate appreciation of A's currency ( reduction of  $e_1$  ) would reduce import prices  $P_1$ , export demand, and  $P_{2gs}$  sufficiently to check the pressure on  $P'$  from these sources. The appreciation can be achieved by letting market forces do most of the job, by direct control of  $e_1$ , or by a combination of these. If however A is committed for any reason to a fixed exchange rate, no indirect policy is likely to be found which will stabilize  $P'$ . The policy search using the model should reveal this. Then direct policies such as price control, combined with taxation of exporters and subsidization of importers seem to be the necessary alternative.

Even with exchange rate appreciation the capital inflow may still prove embarrassing. As domestic monetary restraint is applied foreigners can still obtain funds for domestic investment, while many residents cannot [7.2, Annual Report, 1956]. Direct controls may again become necessary to prevent too much foreign domination of the economy. The direct controls might take the form of allowing only portfolio and preventing direct investment from abroad, until the time when such control became unnecessary.

Case 2. Labour unions, trade and professional associations, and monopolistic businesses pursue a trenchant policy of restricting their

individual supply by pressing up their wages, fees and prices by amounts greater than any productivity gains they make. This causes the global supply function to shift backward,  $P'$  to rise, and A to suffer from inflation. What indirect policy can stop this inflation?

The cause of this inflation lies in the realm of costs, as distinct from demand. To apply a policy program of fiscal and monetary restraints on global demand would not reduce prices much if at all, at least in the short run. All that would be achieved by such policy would be unemployment. There is no indirect policy solution to this problem. If a solution is to be found at all it must lie in a combination of direct controls on wages and prices, coupled with an educational and persuasion program, beamed at the groups at fault. This program would need to teach the relationship between wage and profit increases, productivity, and the value of the country's money. (See eqns. (156) - (160) Part IX ).

Case 3. Country A pursues proper domestic full employment policies, but its exports fall off because of a depression in world economic conditions. Soon because of balance of payments difficulties it is forced to restrict imports, even of basic raw materials complementary to its domestic factors. A will suffer from unemployment both in its export industries and in its industries which require these raw materials. No indirect domestic fiscal and monetary policies can solve this unemployment problem. Either A must reorganize its whole pattern of industry, presumably falling to a lower productivity, or the world depression must be solved by a world economic organization, and a

world policy [11.10], [11.16].

These examples are mentioned merely to remind one that all economic goals cannot be reached by indirect, overall policy measures. But the economic model here developed will usually show when the fiscal and monetary methods will work, and when they will be defeated either by boundary limitation conditions, or by uncontrollable exogenous developments. When a solution by the overall, indirect methods will not suffice, a more direct approach to the problem must be sought; but even in this case the model is likely to be able to provide guidance.

Chapter 29.CONCLUSION

Much past human suffering can be traced to the ills of the economic system - poor economic development, unstable employment and prices, and a severe and unfair distribution of income -- to mention some of the main ones. The post World War II era has seen many countries improve their development, employment, and income distributions, though none have been able to prevent inflation. No country can take undue credit for solving its unemployment problems, for the backlog of depression and war created needs and wants, plus the liquidity carried out of the war, plus the cold war with its 'hot' periods, have provided an abundance of demand, ready made. Recessions have occurred but have been mild. Balance of payments problems have in many cases been troublesome. But the overall successes should not lull us to sleep. There may come times when unemployment becomes a serious problem again, and in the meantime the problems of inflation, and of foreign trade and finance, seem to be always with us.

The present study has attempted to lay a foundation for a more comprehensive and scientific approach to difficult economic problems or illnesses. The approach has been to attempt to see the detailed workings of the economy, and to see it also as a whole, the better to understand how to foresee and prevent its ills.

There has been a bias in favour of a considerable use of the quantitative mathematical approach along with literary and descriptive methods, since a large part of the total economy is quantitative and numerical in its nature. It is truly a giant aggregative structure which, in continuous interaction, and like a giant calculating machine, produces continuously changing economic data, which we can measure numerically. If we wish to control this giant for human welfare we must first infer its structure from the data it reveals to us. Then we must use this structure, as a system of natural law, to predict and to control. The recent expansion of economic measurement and data, and the advent of the electronic computer, make this task much more feasible than it ever could have been twenty five years ago.

This study is dedicated to the belief that we need not suffer from inflation or unemployment if we use tools sufficiently sharp and sturdy to prevent them, and if we recognize that few countries, if any, can "go it alone". Both domestic and world solutions are necessary. The work is also dedicated to the hope that there will be less and less, and finally no human lives wasted in unemployment.

Appendix ABIBLIOGRAPHYPart 1PRELIMINARY SURVEY OF THE GLOBAL ECONOMY CONSIDERED AS AN ORGANISM

- [1.1] BOULDING, K.E., Economic Analysis, Harper, New York, Second ed., 1948.
- [1.2] BROWN, H.P., "Statement to Commonwealth Court of Conciliation and Arbitration, Basic Wage Case 1952-3," May, 1953.
- [1.3] ELY, R.T., Studies in the Evolution of Industrial Society, Macmillan, New York, 1920.
- [1.4] HAAVELMO, Trygve, "The Probability Approach in Econometrics", Econometrica, Vol 12, Supplement, July, 1944.
- [1.5] HICKS, J.R., The Social Framework, Clarendon Press, Oxford, 1943.
- [1.6] KEYNES, J.M., The General Theory of Employment, Interest and Money, Macmillan, London, Reprint, 1947.
- [1.7] KING, H.W., The Pattern of Human Activities, Australasian Publishing Co., Sydney, Second ed., 1951.
- [1.8] KLEIN, Lawrence R., "Macroeconomics and the Theory of Rational Behaviour", Econometrica, Vol. 14 No. 2, April 1946.
- [1.9] KLEIN, Lawrence R., "Remarks on the Theory of Aggregation", Econometrica, Vol. 14 No. 14, October, 1946.
- [1.10] KLEIN, Lawrence R., The Keynesian Revolution, Macmillan, New York, 1947.
- [1.11] KLEIN, Lawrence R., Economic Fluctuations in the United States, 1921-1941, Wiley, New York, 1950. (Cowles Commission Monograph No. 11).
- [1.12] KLEIN, Lawrence R., A Textbook of Econometrics, Row, Peterson and Co., Evanston, Ill., 1953.
- [1.13] KNIGHT, F.H., Risk, Uncertainty and Profit, No. 16 in Series of Reprints of Scarce Tracts in Economics, London School of Economics, London, Sixth Impression, 1946.

- [1.14] LERNER, A.P., Economics of Employment, McGraw-Hill, New York, 1951.
- [1.15] MARSHALL, Alfred, Principles of Economics, Macmillan, London, Eighth ed., 1920.
- [1.16] MILLER, Austin, The Skin of the Earth, Methuen, London, 1953.
- [1.17] OZGA, S.A., "Measurable Utility and Probability: A Simplified Rendering", The Economic Journal, Vol. LXVI No. 263, September, 1956.
- [1.18] PIGOU, A.C., The Economics of Welfare, Macmillan, London, Fourth ed., 1932.
- [1.19] ROBERTSON, D.H., "Utility and All That", The Manchester School of Economic and Social Studies, Vol. XIX No. 2 May, 1951.
- [1.20] SAMUELSON, P.A., Economics, McGraw-Hill, New York, Second ed. 1951.
- [1.21] THEIL, H., Linear Aggregation of Economic Relations, North-Holland Publ. Co., Amsterdam, 1954.
- [1.22] TINBERGEN, Jan, Econometrics, Blakiston, Philadelphia, 1951.
- [1.23] WOLD, H., Discussion of paper by W.M. Gorman on Aggregation and Index Numbers, Econometrica, Vol. 22 No. 1, January 1954, p. 114.
- [1.24] ZIMMERMANN, E.W., World Resources and Industries, Harper, New York, 1933.

## Part II

### ANALYSIS OF BEHAVIOUR IN THE HOUSEHOLD SECTOR

- [2.1] AITCHESON, J. and BROWN, J.A.C., "A Synthesis of Engel Curve Theory" The Review of Economic Studies, Vol. XXII (1) No. 57, 1954-5
- [2.2] ALLEN, R.D.G., "The Work of Eugene Slutsky", Econometrica, Vol. 18 No. 3, June, 1950.
- [2.3] ATKINSON, L.J., "Consumer Markets for Durable Goods" Survey of Current Business, U.S. Dept. of Commerce, April, 1952.



- [2.4] BORCH, K., "Effects on Demand of Changes in the Distribution of Income", Econometrica, Vol. 21 No. 2, April, 1953.
- [2.5] BROWN, T. M., "A Version of the Consumption Function", abstract in Econometrica, Vol. 20 No. 1, pp 83-4, January 1952.
- [2.6] BROWN, T.M. "Habit Persistence and Lags in Consumer Behaviour", Econometrica, Vol. 20 No. 3, July 1952.
- [2.7] COHEN, Morris, "Liquid Assets and the Consumption Function", The Review of Economics and Statistics, Vol. XXXVI No. 2, May, 1954.
- [2.8] DAVIS, J.S. "Standards and Content of Living" The American Economic Review, Vol XXXV No. 3, March, 1945.
- [2.9] DUESENBERY, J.S., Income, Saving and the Theory of Consumer Behaviour, Harvard Univ. Press, Cambridge, 1950.
- [2.10] HAMBURGER, W., "Consumption and Wealth", Ph.D. Thesis No. 1320, University of Chicago, Microfilm, 1951.
- [2.11] HAMBURGER, W., "The Determinants of Aggregate Consumption", The Review of Economic Studies, Vol. XXII No. 57, October, 1954.
- [2.12] HICKS, J.R., Value and Capital, Clarendon Press, Oxford, Second ed. 1946, Reprint 1948.
- [2.13] JACOBS, W. and WILSON, C., "The Postwar Furniture Market and the Factors Determining Demand," Survey of Current Business, U.S. Dept. of Commerce, May 1950.
- [2.14] KEYNES [1.6]
- [2.15] KLEIN [1.11]
- [2.16] LESSER, C.E., "The Consumer's Demand for Money", Econometrica Vol. II, July-October, 1943.
- [2.17] MACK, Ruth P. "Economics of Consumption", Chapter 2 in [3.14].
- [2.18] MELVILLE, L.G., "Consumption, Income and Wealth", The Review of Economics and Statistics, Vol. XXXVI No. 2, May, 1954.

- [2.19] MODIGLIANI, Franco, "Fluctuations in the Savings-Income Ratio : A Problem in Economic Forecasting", Part V in Studies in Income and Wealth, Volume Eleven, National Bureau of Economic Research, New York, 1949.
- [2.20] NORRIS, Ruby T., The Theory of Consumer's Demand, Yale Univ. Press, New Haven, Revised ed., 1952.
- [2.21] PATINKIN, Don, "Price Flexibility and Full Employment", The American Economic Review, Vol. XXXVIII No. 4, September, 1948. Reprinted in [8.1] .
- [2.22] PIGOU, A.C., "Economic Progress in a Stable Environment", Economica Vol. 14 No. 3, August, 1947. Reprinted in [8.1] .
- [2.23] PRAIS, S. J., "Non-Linear Estimates of the Engel Curves", The Review of Economic Studies, Vol. XX (2) No. 52, February 1953.
- [2.24] RANDALL, R.J. "The 'Pigou Effect' and Other Aspects of Asset Holdings on Consumption", The South African Journal of Economics, Vol. 21 No. 1, March 1953.
- [2.25] ROOS, C.F. and von SZELISKI, V.S., "The Demand for Durable Goods", Econometrica, Vol. 11 No. 2, April, 1943.
- [2.26] ROOS, C.F. and von SZELISKI, V.S., in Dynamics of Automobile Demand, General Motors Corporation, New York, 1939.
- [2.27] SAMUELSON, P.A., Foundations of Economic Analysis, Harvard Univ. Press, Cambridge, 1948.
- [2.28] SLUTSKY, Eugene, "On the Theory of the Budget of the Consumer", Giornali degli Economisti, 1915, Reprinted in Readings in Price Theory [8.1] .
- [2.29] VEBLEN, Thorstein, The Theory of the Leisure Class, Allen and Unwin, London, 1924.
- [2.30] WOLD, H. Demand Analysis : A Study in Econometrics, Wiley, New York, 1953.

### Part III

#### THE FIRM, THE PRODUCTIVE SYSTEM, AND SUPPLY

- [3.1] ABRAMOVITZ, Moses, "Economics of Growth", in a A Survey of Contemporary Economics, Vol. II ( See [3.14])
- [3.2] ABRAMOVITZ, Moses, Inventories and Business Cycles, National Bureau of Economic Research, New York, 1950.
- [3.3] BURNS, A.F. The Frontiers of Economic Knowledge, Princeton Univ. Press, Princeton, 1954.
- [3.4] CASSELS, J.M., "On the Law of Variable Proportions" in Explorations in Economics, see [3.5]
- [3.5] CHAMBERLIN, E.H., "Monopolistic Competition and The Productivity Theory of Distribution" in Explorations in Economics, McGraw-Hill, New York, 1936. ( Reprinted in [4.1] ).
- [3.6] CHAMBERLIN, E.H., "Proportionality, Divisibility, and Economies of Scale", The Quarterly Journal of Economics, Vol. LXII, No. 1, February, 1948.
- [3.7] CHIPMAN, J.S., The Theory of Inter-Sectoral Money Flows and Income Formation, Johns Hopkins Press, Baltimore, 1950.
- [3.8] CLARK, J.M., "Business Acceleration and the Law of Demand: A Technical Factor in Economic Cycles", The Journal of Political Economy, 1917; reprinted in Readings in Business Cycle Theory [10.2].
- [3.9] DEAN, Joel, "Measuring the Productivity of Capital", Harvard Business Review, Vol 32 No. 1, February 1954.
- [3.10] Dominion Bureau of Statistics, "The Inter-Industry Flow of Goods and Services, Canada, 1949", Reference Paper No. 72, Ottawa, 1956.
- [3.11] DORFMAN, R., Application of Linear Programming to the Theory of the Firm, Univ. of California Press, Los Angeles, 1951.
- [3.12] DOUGLAS [4.6]
- [3.13] GOODWIN, R.M., "The Multiplier as Matrix ", The Economic Journal, Vol LIX No. 236, December, 1949.
- [3.14] HALEY, B.F. Ed., A Survey of Contemporary Economics, Vol. II, for the American Economic Association, R.D. Irwin, Homewood, Ill., 1952.

- [3.15] HICKS, J.R., A Contribution to the Theory of the Trade Cycle, Oxford Univ. Press, Oxford, 1950.
- [3.16] HICKS, [2.12] .
- [3.17] KEYNES [1.6] .
- [3.18] KLEIN [1.10] , [1.11] .
- [3.19] KNOX, A.D., "The Acceleration Principle and the Theory of Investment: A Survey", Economica, Vol. XIX No. 75, August, 1952.
- [3.20] KOOPMANS, T.C., Ed. Activity Analysis of Production and Allocation, Wiley, New York, 1951. (Cowles Commission Monograph No. 13. )
- [3.21] KUTISH, L.J., "A Theory of Production in the Short Run", The Journal of Political Economy, Vol. LXI No. 1, February, 1953.
- [3.22] KUZNETS, Simon, Economic Change, Norton, New York, 1953.
- [3.23] KUZNETS, S, and GOLDSMITH, R.W., Income and Wealth of the United States: Trends and Structure, Income and Wealth Series II, Cambridge, England, 1952.
- [3.24] LEONTIEF, W.W., The Structure of American Economy, 1919-1939, Oxford Univ. Press, New York, 1951.
- [3.25] LERNER, A.P. Essays in Economic Analysis, Macmillan, London, 1953.
- [3.26] MADDISON, A., "Productivity in an Expanding Economy", The Economic Journal, Vol. LXII No. 247, September, 1952.
- [3.27] MELMAN, S., "Cost in Relation to the Size of the Firm", Applied Statistics (Royal Statistical Society), Vol III No. 1, March 1954.
- [3.28] PAPANDREAU, A.G., "Some Basic Problems in the Theory of the Firm," Chapter 5 in [3.14] .
- [3.29] PHELPS-BROWN, E.H., and HANFIELD-JONES, S.J., "The Climacteric of the 1890's", Oxford Economic Papers, Vol. 4 No. 3, October, 1952.

- [3.30] ROBINSON, Joan, "The Production Function," The Economic Journal, Vol. LXV No. 257, March 1955.
- [3.31] SAMUELSON [1.20]
- [3.32] SAMUELSON [2.27]
- [3.33] SCHUMPETER, J.A., Business Cycles, Vols. I and II, McGraw-Hill, New York, 1939.
- [3.34] TRIFFIN, R., Monopolistic Competition and General Equilibrium Theory, Harvard Univ. Press, Cambridge, 1949.
- [3.35] WESTERFIELD, F.M., "Marginal Analysis, Multiplant Firms, and Business Practice : An Example", The Quarterly Journal of Economics, Vol. LXIX No. 2, May, 1955.
- [3.36] von NEUMAN, J. and O. MORGENSTERN, Theory of Games and Economic Behaviour, Princeton Univ. Press, Princeton, 1944.

#### Part IV

##### THE LABOUR MARKET

- [4.1] The American Economic Association, Readings in the Theory of Income Distribution, Blakiston, Philadelphia, 1946.
- [4.2] BRONFENBRENNER, Martin, "The Incidence of Collective Bargaining", The American Economic Review, Vol. XLIV No. 2, May, 1954.
- [4.3] BROWN, H.P. [1.2]
- [4.4] COPLAND, Sir Douglas, "The Full Employment Economy, with Special Reference to Wages Policy", Oxford Economic Papers, Vol. 5 No. 3, October, 1953.
- [4.5] DOMINION BUREAU OF STATISTICS, The Labour Force, November 1945 - January 1955, Reference Paper No. 58, Ottawa, 1955.
- [4.6] DOUGLAS, Paul, H. The Theory of Wages, Macmillan, New York 1934.
- [4.7] HEATON, H., Economic History of Europe, Harper, New York, 1936, Rev. 1948.

- [4.8] HICKS, J.R., The Theory of Wages, Macmillan, London and New York, 1932. Reprint Peter Smith, New York 1948.
- [4.9] JOHNSON, D.G., "The Functional Distribution of Income in the United States, 1850-1952", The Review of Economics and Statistics, Vol. XXXVI No. 2, May 1954.
- [4.10] KLEIN [1.10] [1.11].
- [4.11] LESTER, R.A., Economics of Labour, Macmillan, New York, 1941, Repr. 1949.
- [4.12] LONG, Clarence D., "Impact of Effective Demand on the Labour Supply", The American Economic Review, Vol. XLIII No. 2, May, 1953.
- [4.13] LONG, Clarence D., "Labour Force in War and Transition : Four Countries", National Bureau of Economic Research Occasional Paper No. 36, 1952.
- [4.14] MARX, Karl, Capital, The Modern Library, New York, 1906.

#### Part V

##### THE FOREIGN TRADE AND FINANCE SECTOR

- [5.1] The American Economic Association, Readings in the Theory of International Trade, Blakiston, Philadelphia, 1949.
- [5.2] Commonwealth Bureau of Census and Statistics, The Australian Balance of Payments, 1928-9 to 1951-2, Canberra, 1954.
- [5.3] Dominion Bureau of Statistics, The Canadian Balance of International Payments. A Study of Methods and Results, Ottawa, 1939.
- [5.4] ELLSWORTH, P.T., The International Economy, Macmillan, New York, 1950.
- [5.5] HABERLER, Gotfried, "The Relevance of the Classical Theory under Modern Conditions", The American Economic Review, Vol. XLIV No. 2, May 1954.
- [5.6] HABERLER, Gotfried, The Theory of International Trade, Hodge, London, 1936.
- [5.7] International Monetary Fund, Balance of Payments Manual, Washington, 1950.

- [5.8] KATZ, S.I., "Sterling Instability and the Postwar Sterling System", The Review of Economics and Statistics, Vol. XXXVI No. 1, February, 1954.
- [5.9] MACHLUP, F., International Trade and the National Income Multiplier, Blakiston, Philadelphia 1943.
- [5.10] MALACH, V.W., International Cycles and Canada's Balance of Payments, 1921-33, University of Toronto Press, Toronto, 1954.
- [5.11] MARCUS, E., Canada and the International Business Cycle 1927-1939, Bookman Associates, New York, 1954.
- [5.12] MARSH, D.B., World Trade and Investment, Harcourt Brace, New York, 1951.
- [5.13] MOSAK, J.L. General Equilibrium Theory in International Trade, Principia, Bloomington Ill., 1944. (Cowles Commission Monograph No. 7).
- [5.14] ROBINSON, Ronney, "Factor Proportions and Comparative Advantage", Part I and II, The Quarterly Journal of Economics, Vol. LXX, No's 2 and 3, May and August, 1956.
- [5.15] SAMUELSON [1.20] .
- [5.16] VINER, Jacob, International Economics, Free Press, Glencoe Ill., 1951.
- [5.17] VINER, Jacob, International Trade and Economic Development, Free Press, Glencoe Ill., 1952.
- [5.18] WATTS, G.S., "The Canadian Balance of International Payments, 1950-2, and the Mechanism of Adjustment", The Canadian Journal of Economics and Political Science, Vol. 20, No. 1, February, 1954.
- [5.19] WYNHOLDS, H.W.J., "The Theory of International Trade", The South African Journal of Economics, Vol. 21 No. 3, September, 1953.

#### Part VI

#### GOVERNMENT AND ITS RELATION TO THE ECONOMIC SYSTEM

- [6.1] Canadian Tax Foundation, "The National Finances, 1955-56", Pamphlet, Toronto, 1955.

- [6.2] Dominion Bureau of Statistics, "National Accounts Income and Expenditure 1938-1945, 1926-1950", Ottawa, 1946, 1951.
- [6.3] FABRICANT, Solomon, The Trend of Government Activity in the United States Since 1900, National Bureau of Economic Research, New York, 1953.
- [6.4] HARRISS, C.L., "Public Finance", Chapter 7 in A Survey of Contemporary Economics, Vol. II ( see [3.14] ).
- [6.5] HICKS, Ursula K., Public Finance, University Press, Cambridge 1947 ( Repr. Jan. 1948 ).
- [6.6] FIGOU, A.C., Economics of Welfare, Macmillan, New York, 1932.
- [6.7] FIGOU, A.C., A Study in Public Finance, 3<sup>rd</sup> (rev.) ed., Macmillan, London, 1947.
- [6.8] SAMUELSON [1.20]
- [6.9] STEINER, G.A., Governments's Role in Economic Life, McGraw-Hill, New York, 1953.

### Part VII

#### THE MONEY AND FINANCE SYSTEM

- [7.1] The American Economic Association, Readings in Monetary Theory, Allen and Unwin, London, 1952.
- [7.2] The Bank of Canada, "Statistical Summary", monthly and yearly; 1956 and other "Annual Reports" of Governor.
- [7.3] BERNOULLI, Daniel, "Exposition of a New Theory on the Measurement of Risk", 1738, Tranlation in Econometrica, Vol. 22 No. 1, January, 1954.
- [7.4] CHANDLER, L.V., "Federal Reserve Policy and the Federal Debt", The American Economic Review, Vol. 39, No. 2, March 1949, Reprinted in [7.1] .
- [7.5A] Commonwealth Bank of Australia, "Statistical Bulletin, Banking Supplements", and annual "Reports" of Chairman.
- [7.5] Commonwealth Bureau of Census and Statistics, Year Book of the Commonwealth of Australia, No. 40, 1954, Canberra, 1955.
- [7.6] COPELAND, M.A., A Study of Moneyflows in the United States, National Bureau of Economic Research, New York, 1952.



- [7.7] Dominion Bureau of Statistics, The Canada Year Book 1955, Ottawa, 1955.
- [7.8] FROST, R., "Some Implications of the Growth of Investment by Institutions", Oxford Economic Papers, Vol. 7 No. 1, February, 1955.
- [7.9] GIFFORD, J.K., and WOOD, J.V., Australian Banking, Univ. of Queensland, Brisbane, 1947.
- [7.10] HAHN, T.H., "The Rate of Interest and General Equilibrium Analysis", The Economic Journal, Vol. LXV No. 257, March, 1955.
- [7.11] HICKS [2.12] .
- [7.12] International Monetary Fund, International Financial Statistics, Vol VIII No. 1, January, 1955.
- [7.13] KEYNES [1.6]
- [7.14] LUTZ, F.A., "The Outcome of the Saving-Investment Discussion", The Quarterly Journal of Economics, Vol. LII No. 3, August, 1938. (Reprinted in [10.2] ).
- [7.15] MODIGLIANI, Franco, "Liquidity Preference and the Theory of Interest and Money", Econometrica, Vol. 12, 1944. ( Reprinted in [7.1] ).
- [7.16] PATINKIN, Don, " A Reconsideration of the General Equilibrium Theory of Money"; and "Further Considerations of the General Equilibrium Theory of Money", The Review of Economic Studies, 1949-50 Vol. XVIII (1), No. 45; and 1952-53 Vol. XIX (3) No. 50.
- [7.10A] HAHN, F.H., " The General Equilibrium Theory of Money - A Comment", The Review of Economic Studies 1952-53, Vol. XIX (3), No. 50
- [7.17] PETERS, R.W., " The Creation of the Money Supply in Australia; 1932-1952", in University Studies in History and Economics, Ed. F.K. Crowley, Univ. of Western Australia, Perth, 1954.
- [7.18] ROBERTSON, D.H., " Saving and Hoarding", The Economic Journal, Vol. XLIII No. 3, September 1933.
- [7.19] ROBERTSON, D.H., Essays in Monetary Theory, King, London, 1940.

- [7.20] ROBINSON, Joan, The Rate of Interest and Other Essays, Macmillan, London, 1950.
- [7.21] SAYERS, R.S., Ed., Banking in the British Commonwealth, Clarendon, Oxford, 1952.
- [7.22] TEW, Brian, "The Finance of Investment", Oxford Economic Papers, Vol. 4 No. 2, July, 1952.
- [7.23] TOBIN, James, "Liquidity Preference and Monetary Policy", The Review of Economics and Statistics, Vol. XXIX No. 2, May 1947.
- [7.24] TOBIN, James, "A Rejoinder", The Review of Economics and Statistics, Vol. XXX No. 4, November, 1948.

### Part VIII

#### THE PRICE SYSTEM AND THE PRICE LEVEL

- [8.1] The American Economic Association, Readings in Price Theory, Blakiston, Philadelphia, 1953.
- [8.2] BAUMOL, W.J. Economic Dynamics, Macmillan, New York, 1951.
- [8.3] BRUCE, C.M.F., "Some Comments on the Current State of Price Theory", The South African Journal of Economics, Vol. 21 No. 4, December, 1953.
- [8.4] CHAMBERLIN, E.H., The Theory of Monopolistic Competition, Harvard Univ. Press, Cambridge, 1933, Sixth ed. 1950.
- [8.5] CLOWER, R.W., and BUSHAW, D.W., "Price Determination in a Stock-Flow Economy", Econometrica, Vol. 22 No. 3, July, 1954.
- [8.6] GABOR, A., and PEARCE, I.F., "A New Approach to the Theory of the Firm", Oxford Economic Papers, Vol. 4 No. 3, October, 1952.
- [8.7] HANSEN, Bent, A Study in The Theory of Inflation, Allen & Unwin, London, 1951.
- [8.8] HOUGH, Louis, "The Price Level in Macroeconomics", The American Economic Review, Vol. XLIV No. 3, June, 1954.
- [8.9] KEYNES [1.6] .
- [8.10] KLEIN [1.11]

- [8.11] PATINKIN , Don, "Dichotomies of the Pricing Process in Economic Theory", Economica, Vol. 21 No. 2, May, 1954.
- [8.12] PEARCE, I.F., " A Study in Price Policy" Economica, Vol. XXIII No. 90, May 1956.
- [8.13] PEARCE, I.F., and L.R. AMEY , " Price Policy with a Branded Product", Review of Economic Studies, 1956-7 Vol. XXIV (1), No. 63.
- [8.14] SAMUELSON, P.A., " The Stability of Equilibrium: Comparative Statics and Dynamics", Econometrica, Vol. 9 No. 2, April, 1941. ( Also included in [2.27] ).

### Part IX

#### SUMMARY OF ECONOMIC MODELS

- [9.1] BROWN, T.M., "Economic Research and Forecasting, Some Canadian Experience", unpublished paper given at the meetings of the Econometric Society, December, 1951, Boston.
- [9.2] BROWN [2.6]
- [9.3] KLEIN [1.10]
- [9.4] KLEIN [1.11]
- [9.5] KLEIN, L.R., " The Empirical Foundations of Keynesian Economics", Chapter 11 in KENNETH K. KURIHARA, Post Keynesian Economics, Rutgers Univ. Press, New Brunswick, N.J., 1954.
- [9.6] KLEIN, Lawrence R., and A.S. GOLDBERGER, An Econometric Model of the United States 1929-1952, North-Holland Publ. Co., Amsterdam, 1955.
- [9.7] MAY, Sydney J., An Econometric Model of the Canadian Economy, Thesis for M. Comm. Degree, McGill University, Montreal, Que., July 1, 1953.
- [9.8] MAY, Sydney J., "Estimates of the Structure of The Canadian Economy", an unpublished paper presented at the Kingston Meeting of the Econometric Society, Sept. 1, 1953. Ref. Econometrica, Vol. 22 No. 4, Oct. 1954, p. 511.

- [9.9] TINBERGEN, Jan, Statistical Testing of Business Cycle Theories, Vols. I and II, League of Nations, Geneva, 1939.

Part X

ECONOMIC CHANGE AND FORECASTING

- [10.1] ALLEN, R.D.G., Mathematical Analysis for Economists, Macmillan, London, 1938, Repr. 1947.
- [10.2] The American Economic Association, Readings in Business Cycle Theory, Blakiston, Philadelphia, 1944.
- [10.3] BATES, Stewart, "Government Forecasting in Canada", The Canadian Journal of Economics and Political Science, Vol. 12 No. 3, August, 1946.
- [10.4] BRATT, E.C., Business Cycles and Forecasting, Irwin, Chicago, Third Ed., 1949.
- [10.5] BROWN [2.6]
- [10.6] BROWN, T.M., "Theory and Practice in the Use of Econometric Models as Aids to Forecasting", unpublished paper presented at the meetings of the Econometric Society, Kingston, Ont., Sept. 1, 1953. Ref. Econometrica, Vol. 22 No. 4, October, 1954, p. 511.
- [10.7] BROWN, T.M., "Standard Errors of Forecast of a Complete Econometric Model", Econometrica, Vol. 22 No. 2, April, 1954.
- [10.8] BURNS [3.3]
- [10.9] CHRIST, Carl, "A Test of an Econometric Model for the United States, 1921-1947" in Conference on Business Cycles, National Bureau of Economic Research Inc., New York, 1951.
- [10.10] DALY, D.J. "The Christ and Canadian Econometric Models - Some Comparisons in Approach", Cowles Commission Discussion Paper: Statistics No. 374, May 12, 1952.
- [10.11] EZEKIEL, Mordecai, Methods of Correlation Analysis, Wiley, New York, Second Ed., 1945.
- [10.12] FISHER, R.A., Statistical Methods for Research Workers, Oliver and Boyd, Edinburgh, Ninth Ed., 1944.

- [10.13] GORDON, R.A., "Business Cycles in the Interwar Period: The "Quantitative - Historical" Approach", American Economic Review, Vol. XXXIX No. 3, May, 1949.
- [10.14] HAAVELMO [1.4]
- [10.15] HOOD, Wm. C., and Tjalling C. KOOPMANS, Studies in Econometric Method, Wiley, New York, 1953. (Cowles Commission Monograph No. 14. )
- [10.16] HOTELLING, Harold, "Problems of Prediction", The American Journal of Sociology , Vol. 48, July, 1942.
- [10.17] HURWICZ, L., "Prediction and Least Squares", Chapter VI in [10.21] .
- [10.18] KENDALL, M.G., The Advanced Theory of Statistics, Vols. I and II, Griffen, London, 1947.
- [10.19] KLEIN [1.12] [9.6] .
- [10.20] KOOPMANS, T.C., " The Econometric Approach to Business Fluctuations", The American Economic Review, Vol. XXXIX No. 3, May, 1949.
- [10.21] KOOPMANS, T.C., Ed., Statistical Inference in Dynamic Economic Models, Wiley, New York, 1950. (Cowles Commission Monograph No. 10).
- [10.22] KOOPMANS, T.C., " When is an Equation System Complete for Statistical Purposes?", Chapter XVII in [10.21] .
- [10.23] MARRIS, R.L., " The Position of Economics and Economists in the Government Machine", The Economic Journal, Vol. LXIV No. 256, December 1954.
- [10.24] MARSCHAK , J. " Economic Structure, Path, Policy, and Prediction", The American Economic Review, Vol. XXXVII No. 2, May, 1947.
- [10.25] MARSCHAK, J., " Statistical Inference in Economics: An Introduction", Chapter I of [10.21] .
- [10.26] National Bureau of Economic Research, Long-Range Economic Projection, Studies in Income and Wealth, Vol. 16, Princeton Univ. Press, Princeton, 1954.
- [10.27] National Bureau of Economic Research, Short-Term Economic Forecasting, Studies in Income and Wealth, Vol. 17,

Princeton Univ. Press, Princeton, 1955.

- [10.28] SHULTZ, Henry, The Theory and Measurement of Demand,  
The Univ. of Chicago Press, Chicago, 1938.
- [10.29] SCHUMPETER [3.33]
- [10.30] SIMON, Herbert A., "On the Definition of the Causal  
Relation", The Journal of Philosophy, Vol. XLIX No.  
16, July, 1952. (Reprinted in Cowles Commission  
Papers, New Series, No. 70).
- [10.31] SIMON, Herbert A., "Causal Ordering and Identifiability",  
Chapter III of [10.15] .
- [10.32] WALLIS, W. A., "Tolerance Intervals for Linear Regression",  
pp 43-51 in Proceedings off the Second Berkeley Symp-  
osium on Mathematical Statistics and Probability,  
ed. Jerzy Neyman, Univ. of California Press, Los  
Angeles 1951.
- [10.33] WOLD, H., "Causality and Econometrics, Econometrica Vol.  
22, No. 2, April 1954.
- [10.34] WORKING, Holbrook, and Harold HOTELLING, "Applications of  
the Theory of Error to the Interpretation of Trends",  
Journal of the American Statistical Association, Vol.  
24, 1929, Proceedings Supplement.

### Part XI

#### APPLICATIONS, POLICY AND CONCLUSION

- [11.1] ARISTOTLE., Politics, Transl. by Benjamin Jowett, The  
Modern Library, Random House, New York, 1943.
- [11.2] BRATT [10.4]
- [11.3] BROWN, H.P. [1.2] .
- [11.4] DOWNING, R.I., "Housing and Public Policy", The Economic  
Record, Vol. XXIV, No. 46, June, 1948.
- [11.5] HAAVELMO, Trygve, "Multiplier Effects of a Balanced Budget",  
Econometrica, Vol. 13, No. 4, October, 1945.

- [11.6] HANSEN, A. H., Fiscal Policies and Business Cycles, Norton, New York, 1941.
- [11.7] HICKS, Ursula, Public Finance, Cambridge Univ. Press, Cambridge, 1947, Repr. 1948.
- [11.8] KLEIN [1.10] , [9.6] .
- [11.9] KLEIN, Lawrence, R., " The Use of Econometric Models as a Guide to Economic Policy", Econometrica, Vol. 15 No. 2, April, 1947.
- [11.10] League of Nations, Economic Stability in the Post-War World, Geneva, 1945, (Repr. UN 1947).
- [11.11] FIGOU, A.C., The Economics of Welfare, Macmillan, London, 1920.
- [11.12] SAMUELSON, P.A. [2.27] .
- [11.13] SWAN, T.W., " The Problem of Wage Policy for 1955-56", unpublished paper, Australia and New Zealand Association for the Advancement of Science, August, 1955.
- [11.14] TINBERGEN, Jan, On the Theory of Economic Policy, North-Holland Publishing Co., Amsterdam, 1952.
- [11.15] TINBERGEN, Jan, Economic Policy: Principles and Design, North-Holland Publishing Co., Amsterdam, 1956.
- [11.16] The United Nations, National and International Measures for Full Employment, Dept. of Economic Affairs, Lake Success, 1949.

## APPENDIX B

### GLOSSARY OF SYMBOLS

Notes: (1) Symbols for variables which represent stocks or flows of real goods will be assumed to express the variable in real (deflated) terms. This symbol can then be converted to represent current market value or money terms through the addition of a subscript "m", or by multiplication by an appropriate price variable. Symbols for financial variables (money, securities, international reserves) will be assumed to express the variable in current market value or money terms. Such symbols can be converted to represent real purchasing power by the addition of a subscript "r", or by division by an appropriate price variable.

(2) The symbols are listed alphabetically.

(3) The main variables which are exogenous, or expediently treated as exogenous, will have their descriptions followed by a "★" in the glossary.

(4) Superscripts "d" and "s" are used to designate demand and supply functions.

(5)  $\cdot X^d, \cdot X^s$  represent ex post solution values of demand and supply variables  $X^d$  and  $X^s$ .

(6)  $X$  is the ex post observed value corresponding to all of  $X^d, X^s, \cdot X^d, \cdot X^s$ ,

#### A

$a$  = vector of age - sex distribution of  $N$ , such that  
 $\sum a = N. \star$

$A_{sd}, A_{hd}, A_a, A_{d+1}$  = household aspirational levels or standard of ownership of semi-durables, household durables, automobiles, and dwellings plus land.  $\star$



(2)

B

- b = vector of labour force participation rates, corresponding to a.
- $b_1$  = proportion of balance of payments surplus which is sold for CB deposits. ★
- $B_{di}$  = total long-term debt of government and other bodies in domestic economy to RW, where the promise to pay is written in different form from conventional securities. ★
- $B_{id}$  = obverse of  $B_{df}$ . ★
- $B_m(i,d,t)$  = an index number of the availability of long-term house mortgage loans, in terms of the rate of interest charged  $i_m$ , the proportion "d" of the purchase price of the house which the mortgage loan will cover, and the time "t" from the date of the loan to the date when it must be completely paid up.
- $B_{ms}(i,d,t)$  = mortgage conditions in the second hand or existing house markets.
- $B_s(i,d,t)$  = an index number of the availability of short-term loans to provide consumer credit for the purchase of household durables and automobiles, with same definitions as for  $B_m$ , but with  $i = i_{sh}$ .
- BF = value of defaulted liabilities of current business failures. ★
- BP = balance of international payments.

C

- c = critical value of ratio  $IR/\bar{F}_1$ , at which value exchange and import controls by central government can be expected.
- cr = cash reserve ratio of MCB =  $CR/M_b$ . ★

(3)

$cr_1 = \text{operating cr} = \frac{CR - M_{dcbs}}{M_b}$  (Australia). ★

$C = C_p + C_s + C_r + C_{sd} + C_{hd} + C_a$ .

$C^a = \text{total domestic consumption in all industries and households of agricultural products. ★}$

$C_d = GI_d$

$C_p, C_s, C_r, C_{sd}, C_{hd}, C_a, C_{d+1} = \text{aggregate consumer purchases of new perishable goods, services, house rent (paid + imputed) semi-durables, household durables, automobiles and dwellings plus land.}$

$C_r = \text{current consumption of the shelter services of the stock of dwellings.}$

$C_t = \text{aggregate consumer spending during time period } t \text{ on consumer goods } X_1, \dots, X_{n-1}, \text{ excluding saving, and valued at the price level of the base period } 0 \text{ of } P_C. \text{ Thus}$

$$C = \frac{P_1 X_1 + \dots + P_{n-1} X_{n-1}}{P_G} \text{ and } C_m = P_C C.$$

$C$  is thus valued in fixed base period valuation units - constant pounds or dollars.

$CB = \text{central bank.}$

$CG = \text{central government.}$

$CR = \text{cash reserves of MCB} = M_{mb} + M_{nb} + M_{cb}$

D

$d = \text{Proportion of cost of imports added by customs duties, on f.o.b. value. ★}$

$D = D_{f1} + D_{f2}$ .

$D_{eh} = \text{personal or household debt} = D_{eshf} + D_{emh}$

(4)

- $D_{emh}$  = mortgage debt of households to other households and to firms =  $S_{emhh} + S_{emhf} = S_{emh}$
- $D_{emhf}$  = mortgage debt of households to firms =  $S_{emhf}$ .
- $D_{esf}$  = short-term debt of firms =  $S_{esf}(hfg)$ .
- $D_{eshf}$  = short-term debt owed by households to firms, or consumer credit outstanding =  $S_{eshf}$ .
- $D_f$  = real depreciation or capital consumption of firms =  $D_{pcm}$ .
- $D_{f2}$  =  $T_{rf2} + T_{rf3}$  \*
- $D_{flm}$  = transfers by firms during time period to reserves for depreciation or capital consumption, in money terms. \*
- $D_{f2m}$  = bad debt losses of firms, money terms. \*
- $D_M$  = real depreciation (including wear, obsolescence, and accidental destruction) of producers' stocks of machinery and equipment during time period.
- $D_{PG}$  = real depreciation of the stock of producers' plant and construction.
- $D_{FCMg}$  = real depreciation of the government stock of capital goods =  $D_g$ .
- $D_{sdt}, D_{hdt}, D_{at}, D_{(d+1)t}$  = decline in real value of household stocks of semi-durables, household durables, automobiles and dwellings plus land, due to normal wear and deterioration with time, obsolescence, and destruction by accident or scrapping, during time period  $t$ ; referred to as depreciation for short.

E

- $e_1$  = rate of exchange factor or vector which converts international moneys into domestic money.
- $e_{10}$  = base period value of  $e_1$ . \*

(5)

- $e_2$  = rate of exchange conversion factor vector which converts RW prices into international money values. \*
- $E_2$  = index number of  $e_2$ . \*
- $E_{dr}$  =  $K^s_{(d+1)r} - K^d_{(d+1)r}$  = excess supply of dwelling units for rent, observable as empty houses for rent. \*
- $E_f$  = special expectations of firms. \*
- $E_h$  = special expectations of households. \*
- EFA = Exchange Fund Account (Canada).

F

- $F_1$  = imports of all goods and services from abroad, excluding income payments for domestic factors of production owned by foreigners =  $F_{1g} + F_{1sg} + F_{1sv}$
- $\bar{F}_1$  = average of the last three periods of normal imports, without any import restrictions or depression.
- $F_1^*$  = imports of all goods and services from abroad, including payments for factor shares owned abroad =  $F_1 + \overline{\Pi}_{di} = F_{1g} + F_{1s}$
- $F_{1C}$  = consumer good component of  $F_{1g}$ .
- $F_{1g}$  = imports of merchandise f.o.b. =  $F_{1rm} + F_{1GI} + F_{1C}$ .
- $F_{1GI}$  = producers durable capital component of  $F_{1g}$ .
- $F_{1gm} = F_{1g} \cdot P_{1g}$  ;  $F_{1sm} = F_{1s} \cdot P_{1s}$  .
- $F_{1rm}$  = raw material or industrial material and fuel component of  $F_{1g}$ .
- $F_{1s}$  = imports of services (freight, insurance, expenditure of domestic ships and business people abroad, tourist travel abroad, interest and dividends paid abroad, retained earnings on foreign direct investment in A) =  $F_{1sg} + F_{1sv} + \overline{\Pi}_{df}$ .

(6)

- $F_{1sg}$  = payments for freight and insurance on imports plus expenditure of domestic ships and businessmen abroad. (services largely related to goods imports).
- $F_{1sv}$  = domestic payments to foreign countries for tourist travel. \*
- $F_{1u}$  = total unilateral payments (no quid pro quo), resulting from gifts or emigration. \*
- $F_2$  = exports of all goods and services to foreign countries, excluding receipts or income from foreign factors of production owned by domestic residents =  $F_{2g} + F_{2sg} + F_{2sv}$
- $F_2^*$  = exports of all goods and services including income from foreign factors of production owned by domestic residents =  $F_2 + \Pi_{id} = F_{2g} + F_{2s}$
- $F_{2g}$  = exports of merchandise f.o.b.
- $F_{2gm} = F_{2g} \cdot P_{2g}$  ;  $F_{2sm} = F_{2s} \cdot P_{2s}$  .
- $F_{2go}$  = gold production available for export. \*
- $F_{2s}$  = exports of services (see  $F_{1s}$ )
- $F_{2sg}$  = receipts from freight and insurance exports, and from expenditure of foreign ships and businessmen in domestic economy.
- $F_{2sv}$  = receipts from tourist travel. \*
- $F_{2u}$  = total unilateral receipts (no quid pro quo), resulting from gifts and immigration. \*
- $F_c$  = current account balance.
- $F_{kl}$  = net long term capital inflow from abroad.
- $F_{ks}$  = net short-term capital inflow.
- FEP = foreign exchange pool.
- FML = full maximum likelihood.

(7)

G

- $G$  = gross cost to economy of government non-business services =  $G_c + G_k + D_g$ .
- $G^*$  = total government spending =  $G_c + G_k + \Delta K_{pg}$ .
- $G^{**}$  = total government spending on new goods and services =  $G_c + G_k$ .
- $G_1$  = government output or value added =  $(w_g N_g + w_m N_m) / P + D_g$ .
- $G_c$  = government spending on current (short use) goods, including its own hired workers =  $G_{ms} + w_g N_g + w_m N_m$ .
- $G_i$  =  $G_1' - D_g$  = transfer portion of  $G_1'$ .
- $G_i'$  = total government interest bill on public debt =  $D_g + G_{ip} + G_{imp} + G_{ig}$ .
- $G_{ic}$  = private corporation component of  $G_{imp}$ .
- $G_{ig}$  = portion of  $G_i'$  received as revenue by government departments, and assumed to fall completely within  $G_i$ .
- $G_{imp}$  = portion of  $G_1$  paid to non-personal sector.
- $G_{ip}$  = portion of  $G_1$  paid to personal sector.
- $G_k$  = government spending on new capital (long use) goods.
- $G_{ms}$  = current goods and services purchased by government from firms.
- $G_n$  = net cost of government services =  $G_c + D_g$ .
- $GDF$  = total flow of new goods through domestic economy during a time period =  $GDS + \Delta H$  = gross domestic supply flow.
- $GDP$  = gross domestic production during a time period =  $GDS + \Delta H - F_1$  = total value added by factors located within geographical boundaries of domestic economy.
- $GDP_f$  = gross domestic product of firms, and hence of private sector and excluding imputations for example of rent earned by owner - occupiers of houses; excluding also institutional output.
- $GDP^o$  = highest previous level of GDP.

(8)

GDS = aggregate sales of final goods in the domestic economy during a time period =  $C + GI_d + G + GI + F_2$ .

GI = total investment spending by firms on durable capital goods =  $GI_{PG} + GI_M$

$GI_d = C_d$

$GI_M$  = investment by firms in machinery and equipment.

$GI_{PG}$  = investment by firms in plant and construction projects.

GNP = gross national product =  $GDP + \pi_{fd} - \pi_{df}$ .

H

h = average hours worked per employed paid worker during time period.

$h_s$  = normal or standard hours of work when firms are at full capacity operations with no overtime or short-time work.

$H_{fg}$  = inventories of finished goods.

$H_{gp}$  = inventories of goods in process.

$H_m$  = book value of inventory stocks.

$\Delta H_m = P_{\Delta H} \Delta H + (H + \Delta H) \Delta P_H = P_{\Delta H} \Delta H + J_m$ .

$H_{rm}$  = inventories of raw materials

$H_t$  = total stocks of goods held in inventories at end of time period t.  $H = H_{rm} + H_{gp} + H_{fg}$ .

I

i = an appropriate average of the components of  $i_h$  and  $i_f$ .

(9)

- $i_b$  = average interest rate charged by MCB on its short-term loans to firms and households.
- $i_b'$  = average yield on all MCB security holdings.
- $i_c$  = Bank Rate at which CB lends deposits to MCB. ★
- $i_e$  = yield on equity securities.
- $i_f$  = average of  $i_s, i_l, i_b, i_m, i_e$  appropriate to firms.
- $i_h$  = average money yield on all savings or wealth goods held by households.
- $i_l$  = yield on long-term bonds and debentures. = f ( $i_{lg}, i_{lf}$ )
- $i_{lf}$  = yield on high grade corporate and government, other than CG, bonds and debentures.
- $i_{lg}$  = yield on CG bonds.
- $i_m$  = yield on mortgage loans.
- $i_s$  = yield on short-term securities, other than for consumer credit.
- $i_{sh}$  = effective rate of interest on short-term consumer credit loans.
- $\overline{if}$  = proportion of f.o.b. cost of imports added by insurance and freight. ★
- $I$  = estimate of production of institutions in national accounts. ★
- $IM$  = international money holdings of a country = national holdings of gold plus pounds sterling (£ S) plus U.S. dollars (US \$) plus any other moneys of convertible value for foreign trade.
- $IR$  = international reserves of a country = national holdings of gold plus money and short-term securities of foreign exchange centers (London and New York), as well as of some other countries =  $IR_g + IR_p = \bar{e}_1 IM + S_{esid}$ .
- $IR_1$  =  $IR / \overline{F}_1$
- $IR_2$  =  $(c - IR / \overline{F}_1)$
- $IR_g$  = central government holdings of IR. ★



(10)

$IR_p$  = private sector (individuals, firms, institutions) holdings of IR.

J

$J_m$  = capital gains or losses of firms on inventories due to price increases or decreases, in money terms and included in

$$\pi_{im} = \Delta H_m - (\Delta H) P_{\Delta H} = J_m.$$

K

$k_1$  = a critical rate of unemployment in terms of man-hours, such that  $w_{ph}$  moves up rapidly once the rate falls below  $k_1$ .

$k_2$  = ratio of net worth in government business to net worth in other corporations in private sector. \*

$K_{di}$  = direct investment of foreigners in the domestic economy [5.7 p. 57]: branches of foreign firms in domestic economy; subsidiaries of foreign firms in domestic economy; commercial real estate in domestic economy owned by foreigners.

$K_{(d+1)r}$  = stock of dwelling units for rent.

$K_g$  =  $K_{PCMLg}$  = stock of government fixed capital in plant, construction, machinery, equipment and land. \*

$K_h$  =  $K_{sd} + K_{hd} + K_a + K_{d+1}$ .

$K_{hds}, K_{as}, K_{(d+1)s}$  = stocks of household durables, automobiles, and dwellings plus land, for sale in second hand markets. \*

$K_{id}$  = direct investment of residents of domestic economy in RW.

$K_l$  = stock of site land owned by firms. \*

$K_M$  = stock of machinery and equipment owned by firms, at end of time period.

$K_{PC}$  = stock of plant and construction works owned by firms.

(11)

$K_{PCML}$  = stock of producers fixed capital owned by firms =  $K_f$ .

$K_s$  = saturation level of  $K_f$ , when  $r \leq i_{1f}$

$K_{sdhda} = K_{sd} + K_{hd} + K_a$ .

$K_{sdt}, K_{hdt}, K_{at}, K_{(d+1)t}$  = household real stocks of semi-durables, household durables, automobiles and dwellings plus land, at end of time period  $t$ .

$K_{ub}$  = real net worth of unincorporated business. ★

$\Delta K_{pg}$  = net government acquisition of second hand capital assets from the private sector. ★

$\Delta_1 K_{ub}$  = net investment or withdrawal of capital from unincorporated business. ★

L

$L_f$  = liquid asset holdings of firms =  $M_f + S_{ef} - S_{emf}$ .

$L_h$  = liquid asset holdings of households =  $M_h + S_{eh}$ .

M

$m_1$  = proportion of its money the public chooses to hold as currency =  $M_{cp} / M$ . ★

$m_2$  = ratio of MCB to public holdings of currency =  $M_{cb} / M_{cp}$ . ★

$M$  = total money =  $M_{cp} + M_b$ .

$M_a$  = active money in the productive system.

$M_b$  = bank deposit money created by MCB.

$M_c$  = currency =  $M_m + M_n$ .

$M_{cb}$  = MCB currency holdings.

(12)

- $M_{cp}$  = public currency holdings.
- $M_{dcb}$  = deposit money with CB, held by MCB.
- $M_{dcbs}$  = Special Accounts of MCB with the CB in Australia. \*
- $M_{dcbg}$  = government deposits with CB. \*
- $M_{di}$  = domestic money held by foreigners. \*
- $M_f$  = money holdings of firms.
- $M_g^i$  = government holdings of money. \*
- $M_h$  = money holdings of households.
- $M_{id}$  = internal money of foreigners held by domestic residents. \*
- $M_m$  = coin or metallic money, produced in mint.
- $M_n$  = paper note money issued by CB.
- $M_P$  = private sector (households, firms, institutions, excluding MCB) holdings of money.
- $M_s$  = total money in SP =  $M_{s1} + M_{s2}$ .
- $M_{s1}$  = active money in the SP.
- $M_{s2}$  = inactive money in the SP.
- $\Delta M_p^g$  = change in private sector holdings of money due to government fiscal activities.
- MCB = commercial, or trading, or money creating banks.

N

- $N$  = total population resident in the economy. \*
- $N_{enp}$  = number of entrepreneurs (employers, and own account or self employed workers), and unpaid family workers engaged in production in firms. \*

(13)

- $N_f$  = number of families in the economy. \*
- $N_g$  = number of civilian employees in all levels of government. \*
- $N_h$  = the number of household units in the economy. \*
- $N_i$  = employees of non-commercial institutions, other than government. \*
- $N_l$  = average number of workers in labour force or labour supply during time period =  $N_p + N_{enp} + N_i + N_g + N_m + N_u$ .
- $N_{lf}$  = effective labour force available for hire to firms =  $N_l - N_m - N_g - N_i - N_{enp}$ .
- $N_m$  = number of personnel in the military, or armed forces. \*
- $N_p$  = number of paid workers or employees hired by firms.
- $N_{pe}$  = total employment in firms =  $N_p + N_{enp}$ .
- $N_u$  = number of unemployed workers in labour force; persons without jobs, and seeking work or who would have been seeking work but believed no work available.
- NDP = net domestic product =  $GDP - D_{fl} = NDE$ .
- $Nh$  = total man-hours worked in firms during time period =  $N_{pe}h$ .
- $Nh_u$  = number of unemployed man-hours.
- $NL_f$  = net liquid wealth of firms =  $L_f - D_{esf}$ .
- NDE = net domestic expenditure.
- NNIFC = net national income at factor cost =  $W_a + \pi$ .
- $NS_f$  = net conventional savings of firms =  $S_f - D_{f1} - D_{f2}$ .
- $NS'_f$  = true saving of firms =  $S_f - D_f$ .
- $NW_f$  = net worth of firms =  $W_f - S_{eifor}$ .
- $NW_h$  = net real wealth of households =  $W_h - D_{eshfr} - D_{emhr}$ .

(14)

P

$P_K$  =  $GDP_f / (H_f + K_f)$  = average productivity in firms per unit of real capital.

$P_L$  = average productivity in firms per man-hour =  $GDP_f / Nh$

$P_T$  = total productivity.

$P$  = price level of net domestic product, NDP.

$\rightarrow P$  = the trend in  $P = f(P_{-2}, P_{-1}, P)$ .

$P'$  =  $P_{GDF}$  = price level of gross domestic supply flow.

$P^*$  =  $P_{GDP}$  = price level of gross domestic product.

$P_1$  =  $P_{1g} (1 + if + d + s + ds) / (1 + if + d + s + ds)_0$  = price level of foreign goods confronting domestic importer. \*

$P_{1g}(P_{1s})$  = price level of import goods (services) converted to domestic money =  $e_1 P_{1gi} (e_1 P_{1si})$ ; deflators of  $F_{1gm}$ ,  $F_{1s}$ .

$P_{1gi} (P_{1si})$  = price index of goods (services) produced in  $RW$  and of interest to A as imports, in terms of some international currency, and built from  $e_{2i} P_{1gii}$ . \*

$P_{1gs}$  = price deflator of  $F_{1m}$ .

$P_{1gsi} = f(P_{1rmi}, P_{1GIi}, P_{1Ci}, P_{1sgi})$ . \*

$P_{1rm}, P_{1GI}, P_{1C}, P_{1sg}$  = appropriate deflators of  $F_{1rmm}, F_{1GI}, F_{1Cm}, F_{1sgm} = e_1(P_{1rmi}, P_{1GIi}, P_{1Ci}, P_{1sgi})$

$P_{1rmi}, P_{1GIi}, P_{1Ci}, P_{1sgi}$  = price level of imports of  $rm, GI, C, sg$  in terms of some international currency. \*

$P_{1sg}$  = price index which converts  $F_{1sg}$  to  $F_{1sgm} = P_{F1}$ .

$P_{1C}$  = consumer good component of  $P_1$ .

$P_{1GI}$  = producers' durable capital component of  $P_1$ .

$P_{1rm}$  = raw material or industrial material component of  $P_1$ .

$P_2 = \frac{P_{2g}}{E_2} \cdot \frac{1}{e_1/e_{10}} \cdot \left[ \frac{1 + if + d + s + ds}{(1 + if + d + s + ds)_0} \right]_{RW2}$   
= price level of A goods confronting  $RW2$  importers.

(15)

- $P_{2g}$  = price index of A's exports of goods in A currency.
- $P_{2gi}$  = price level of A goods exports in an international currency.
- $P_{2gs}$  = domestic price level of export goods and services =  $P_{F2}$  = price deflator of  $F_{2m}$ .
- $P_{2sg}$  = price index which converts  $F_{2sg}$  to  $F_{2sgm}$ .
- $P_{2si}$  = price level of A service exports in an international currency.
- $P_{2sii}$  = price level of A services confronting foreign importers.  
=  $P_{2si} / E_2$ .
- $P_C$  = price index aggregating prices  $p_1, \dots, p_{n-1}$  of quantities of current consumer goods  $X_1, \dots, X_{n-1}$ , excluding savings goods. Then  $\frac{p_{1t}X_{1t} + \dots + p_{n-1t}X_{n-1t}}{P_{Ct}} = C_t$ .
- $P_e$  = price index of equity share stocks.
- $P_f$  = price level of  $GDP_f$ .
- $P_{fg}$  = price index of finished goods inventory.
- $P_G$  = price index of G.
- $P_{GI}$  =  $f(P_{PC}, P_M)$ .
- $P_{gp}$  = price index of goods in process inventory.
- $P_h$  = price index appropriate for deflating household income to constant value units =  $\frac{P_G (C_m + S_{hm})}{(C_m + P_{ih} S_{hm})}$ .
- $P_H$  = price level of H.
- $P_{\Delta H}$  = price level of  $\Delta H$ .
- $P_{hds}, P_{as}, P_{(d+1)s}$  = price indexes of consumer durables for sale in second hand markets. ★
- $P_{ih}$  = index number of change in the accumulation factor  $(1+i_h)$  for household savings goods; derived from formula  
 $P_{ihot} = \frac{(1+i_{ht})}{(1+i_{ho})}$ .

- $P_K$  = price index of new fixed capital.  
 $P_{la}$  = price of land complementary to dwellings. ★  
 $P_M$  = price index of machinery and equipment.  
 $P_m$  = price index of intermediate industrial materials.  
 $P_{mK}$  = price index of industrial materials and fixed capital.  
 $P_p, P_s, P_r, P_{sd}, P_{hd}, P_a, P(d+1)$  = price indexes of household purchases of perishables, services, house rent, semi-durables, household durables, automobiles, dwellings plus land.  
 $P_{PC}$  = price index of new plant and construction.  
 $P_r$  = price index of gross house rents paid and imputed.  
 $P_{rm}$  = price index of raw material inventory, including storage costs.  
 $P_{RW2}$  = index of domestic price levels in RW2. ★  
 $P_{Sh}$  = 'price' index number of outlay on savings goods which accumulates to one constant value unit in one time period =  $P_C / P_{ih}$ .  
 $P_{slf}$  = price index of long term bonds and debentures of firms.  
 $P_{slg}$  = price index of long term government securities.  
 $P_w$  = price index of hourly wage rates.

Q

$q_c \cdot \frac{P_C}{P}$  =  $\frac{C}{GDF}$  (similarly for  $q_d, q_g, q_{gi}, q_h, q_f$  in relation to  $GI_d, G, GI, \Delta H, F_2$ ) and represents the rate at which the proportion of C to GDF changes as the ratio of  $P_C$  to  $P$  changes.

(17)

R

- $r$  = marginal efficiency or yield of new capital goods.
- $r'$  =  $\frac{\pi_f}{H_f + K_f}$  = an approximation of  $r$ .
- $r_d$  = profit yield on rented dwellings.
- $r_{di}$  = average rate of interest and dividends or yield paid to RW on foreign owned direct investment and securities of A (domestic economy).
- $r_{ub}$  = profit yield on unincorporated business.
- RW = rest of world.
- RW2 = the domestic economy A's best customers in RW.

S

- $s$  = proportion of cost of imports added by sales, excise and other import taxes, on duty paid value.
- $S_f$  = gross saving of firms.
- $S_{g1}$  = current account surplus of government.
- $S_{g2}$  = cash surplus of government.
- $S_{g3}$  = national income surplus of government.
- $S_{ht}$  = conventional savings of households out of income during time period  $t = Y_{ht} - C_t$ .
- $S'_h$  = true savings of households for future use =  $S_h + \Delta K_h$ .
- $S_L$  = labour share of output of firms.
- $S_{\pi}$  = capital share of output of firms.
- $S_{eab}$  = securities, other than government, held by MCB in exchange for  $M_b$ .
- $S_{ee(hfgi)}$  = equity securities issued by firms and held by households firms, government and RW =  $S_{eeo}$ .



- $S_{ef}$  = security holdings of firms, including accounts receivable and bills of exchange.
- $S_{egb}$  =  $S_{esgb} + S_{elgb}$ .
- $S_{egh}$  = securities of the central government held by households.
- $S_{eh}$  = household or personal holdings of securities.
- $S_{eif(hfgi)} = S_{eifo}$  = fixed interest (non-equity) securities issued by firms and held by households, firms, government and RW (including accounts and bills of exchange payable).
- $S_{eil}$  = long term fixed interest securities.
- $S_{eil}^S(fi)$  = domestic market supply of existing issues of long-term fixed interest securities by domestic firms, and foreign firms and governments.
- $S_{eldi}$  = total long-term securities (bonds, debentures, mortgages, equity shares) issued by domestic economy and payable to foreigners.
- $S_{elfo}$  = long-term securities issued by firms and held by others (h,f,g,i) =  $S_{eeo} + S_{eilfo}$ .
- $S_{elgp}$  = long-term securities of government held by private sector (households, firms, institutions, but excluding MCB and RW).
- $S_{elid}$  = total long-term securities issued by RW and payable to residents, firms, and governments within domestic economy.
- $S_{emf}$  = mortgages held by firms.
- $S_{emf(hf)}$  = mortgage debt of firms.
- $S_{emh} = S_{emhh} + S_{emhf} = S_{emh(hf)}$ .
- $S_{emhf}$  = mortgage paper representing mortgage debt, issued by households (mortgagors) and held by firms (mortgagees).
- $S_{emhh}$  = mortgage paper issued by households and held by households.
- $S_{epg}$  = direct loans by government to the private sector.



$S_{esdi}$  = total short-term 'securities' (currency and bank accounts, accounts payable, bills of exchange, bank drafts, treasury bills, promissory notes, call loans) payable by domestic economy to foreigners.

$S_{esf(hfgi)}$  = short-term securities issued by firms (mainly accounts and bills of exchange payable) =  $S_{esfo}$  =  $Desf$ .

$S_{esgb}$ ,  $S_{elgb}$  = short and long-term securities issued by government and held by the money creating banks (MCB).

$S_{esgc}$ ,  $S_{elgc}$  = short and long-term securities issued by the government and held by the CB.

$S_{esgp}$  = short-term securities issued by government and held by private sector.

$S_{eshf}$  = short-term securities representing consumer debt owed to firms.

$S_{esid}$  = total short-term securities issued by foreigners but held by domestic economy, and payable to domestic economy.

$\Delta_1 S_{eeco}$  = net issue and sale of new equity securities by corporations.

$\Delta_1 S_{emho}$  = sale of new mortgages during time period.

$\Delta_2 S_{emho}$  = repayments of mortgage principal during time period.

SCR = secondary cash reserves of MCB, consisting of very short-term securities - day-to-day loans, Treasury Bills, and bills of exchange.

SP = The Savings Pool.

T

$t_c$  = average rate of corporation profits tax. \*

$t_{di}$  = average rate of withholding tax. \*

$t_{gb}$  = average rate of transfer of government business profits to consolidated revenue. \*

- $T = T_w + T_{\pi p} + T_{\pi np} + T_{i-s} + T_o =$  total government revenue.
- $T', T'_d = T - T_o + T'_o, T_d - T_o + T'_o.$
- $T_d = T - T_r =$  government disposable income.
- $T_{di} =$  withholding tax related to interest and dividends paid to non-residents,  $\pi_{df}$ .
- $T_{i-s} =$  total indirect taxes less subsidies.
- $T_o =$  all other government revenue; including  $G_{ig}$ . \*
- $T'_o =$  portion of  $T_o$  produced in current time period by domestic factors.
- $T_r = T_{rw} + T_{r\pi p} + T_{r\pi np} =$  total government transfer payments.
- $T_{rf1} =$  charitable contributions of firms. \*
- $T_{rf2} =$  bad debt losses of firms to wage income earners. \*
- $T_{rf2} + T_{rf3} = D_{f2}.$  \*
- $T_{rf3} =$  bad debt losses of firms to other firms. \*
- $T_{rw} =$  transfer payments to wage - salary incomes.
- $T_{r\pi np} =$  transfer payments to nonwage non-personal incomes.
- $T_{r\pi p} =$  transfer payments to nonwage personal incomes.
- $T_w =$  direct taxes on wage-salary incomes.
- $T_{\pi c} =$  taxes on corporation profits,  $\pi_c$ .
- $T_{\pi gb} =$  portion of profits of government business (net of losses) transferred to consolidated revenue of government.
- $T_{\pi np} = T_{\pi c} + T_{df} + t_{gb} \pi_{gb} =$  direct taxes on corporation profits, plus withholding tax, plus portion of trading profits of government business transferred to consolidated revenue of government.
- $T_{\pi p} =$  direct taxes on nonwage income of persons plus succession duties on personal wealth.

(21)

U

- $u_{fg}$  = undesired inventories (plus or minus) of finished goods =  $H_{fg} - \cdot H_{fg}^d$ .
- $u_h$  = total undesired inventories =  $H - \cdot H^d$ .
- $U_{id}$  = international surplus of assets over liabilities of country A with RW.

V

- VC = variance - covariance (matrix).

W

- $w_g$  = average wage-salary per time period of civilian government employees. ★
- $w_i$  = average wage-salary per time period of employees in non-commercial institutions, other than government. ★
- $w_m$  = average pay and allowances per time period of military or armed forces. ★
- $w_{ph}$  = average hourly earnings of paid workers hired by firms.
- $W_a$  = real wage bill =  $(w_{ph} N_{ph} + w_i N_i + w_g N_g + w_m N_m) / P$ .
- $W_a^a$  = real wage bill in agriculture =  $(w_{ph}^a N_p^a h^a) / P$ . ★
- $W_f$  = total wealth holdings of firms =  $M_{fr} + S_{efr} + H_f + K_{PCML} + K_{d+1,f}$ .
- $W_{ht}$  = wealth holdings of households at end of period t, consisting of real goods, securities and money, with securities and money representing claims to future real goods =  $M_{hr} + S_{ehr} + K_{sd} + K_{hd} + K_a + K_{d+1,h} + K_{ub}$ .

(22)

X

- X = balancing item in consolidated balance sheets of CB and MCB between money of public and loans made in creating this money, plus IR plus  $M_{cp}$ .
- $X^a$  = inputs into agriculture from all other industries and RW.

Y

- $Y_h$  = disposable income of all households and persons in the economy =  $Y_w + Y_{\pi p}$ .
- $Y_w$  = disposable wage-salary income.
- $Y_{\pi}$  =  $Y_{\pi p} + Y_{\pi np}$  =  $Y_{\pi a} + Y_{\pi na}$ .
- $Y_{\pi a}$  = disposable nonwage income of the agricultural or farm sector. \*
- $Y_{\pi na}$  = disposable nonwage nonfarm income.
- $Y_{\pi np}$  = disposable nonwage income of non-household nonpersonal domestic private sector; that is remaining in firms and private institutions.
- $Y_{\pi p}$  = disposable nonwage or property - enterprise income flowing into household and personal sector.

Z

- Z =  $Z_1 + Z_2$  = excess demand.
- $Z_1$  = unfilled orders. \*
- $Z_2$  = latent unsatisfied demand.

GREEK

$\pi$  = total property enterprise income of residents =  $\pi_f + \pi_{io} + \pi_{ri} + \pi_{go} + \pi_{id} - \pi_{di} = \pi_p + \pi_{np}$ .

$\pi'$  =  $\pi + \pi_{di} - \pi_{id}$  = nonwage income of domestic capital located within the geographical boundaries of the nation.

$\pi^a$  = nonwage income originating in the agricultural (including pastoral) industry. \*

$\pi_c$  = corporation profits excluding any  $G_{ic}$ , and not including  $\pi_{gb}$ .

$\pi_{dfo}$  = dividend payments by firms.

$\pi_{di}$  = payment of nonwage income mainly interest and dividends to foreigners, on account of the domestic real and financial capital owned by them.

$\pi_{dub}$  = net drawings by owners of unincorporated business.

$\pi_f$  = property and enterprise income originating in firms = net profits of firms plus interest and rent paid by them, less interest, dividends and rent received by them.

$\pi_{fn}$  = net profit of firms.

$\pi_{gb}$  = profits of government business.

$\pi_{go}$  = interest payments on public debt associated with government fixed capital, and defined equal to  $D_g$ . \*

$\pi_{id}$  = receipts of nonwage income from foreign countries on account of real and financial capital owned in them by domestic residents. \*

$\pi_{ifo}$  = interest payments by firms.

$\pi_{io}$  = interest payments by non-commercial institutions on debt associated with their capital. \*

$\pi_{np}$  = nonwage income not going to households and persons.

$\pi_p$  = nonwage income of household and personal sector.

$\pi_{rfo}$  = rent payments by firms.

(24)

$\pi_{ri}$  = imputed nonwage income in the national accounts  
(e.g. imputed rents). \*

$\pi_{ub}$  = net profits of unincorporated business.