A SPATIAL MODEL ANALYSING FIRMS' DECISIONS ON ACCESSIBILITY IMPROVEMENT

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A thesis submitted in partial fulfilment of the requirements for graduation of

APPLIED ECONOMICS HONOURS

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THE AUSTRALIAN NATIONAL UNIVERSITY

31 October 2014

Acknowlegements

Firstly, I would like to thank my supervisor, Prof. Kieron Meagher. If not for your patience and guidance throughout his year, I would not be able to finish my thesis with current standard. Your knowledge and ideas provided me with great support for the construction of my thesis. Secondly, I would like to thank my friend Shauna Ng, who helped edit my thesis. I am very grateful for the amount of time and effort you put into my thesis. It would not be as readable as it is now without your help. Next, I would like to give my special thanks to a particular group of friends (Shauna Ng, Han Wu, Junyi Ye, Shichao Wang and Jiadong Qian). This year would be so much more stressful if not for your company and moral support. Last but not least, I would like to thank my parents for providing me with the opportunity to study abroad. I really appreciate the faith you have in me and the amount of investment you have put into my life.

Contents

Abstract

Firms can differentiate their products through improving their accessibilities so that it costs less for the consumers to collect information on firms' products. In this way, firms' products become more attractive to the consumers and thus more demand will be generated. However, this improvement requires a fair amount of investment from firms. This essay will use the spatial model, where two identical firms with fixed locations engage in a symmetric and simultaneous game, to analyse whether, and if so, when firms should invest to improve their accessibilities. Two cases where firms charge mill pricing and discriminatory pricing will be looked into as well. We conclude that firms should and will make the investment when, first, they have perfect information on consumers and charge discriminatory prices; second, when the cost investment incurred is sufficiently low. If, on the other hand, cost of investment is sufficiently large, then it is most optimal, economically and socially, for both firms to stay out of making such an investment.

1 Introduction

Regardless of whether a firm is large or small, competition is inevitable. One of the most common strategies employed by firms in competition is product differentiation. There are several ways in which product differentiation influences competition. First, this method allows firms to improve the desirability of their products, which leads to an increase in demand. Second, such a move will increase price competition because firms that do not differentiate their products will inevitably be forced to lower their prices in the hope of maintaining their market share. When a firm tries to improve the desirability of its product, they may either (1) differentiate their products through product modification to meet the needs of various consumers, or (2) compete for sales by making strategic decisions when choosing the ideal location for their business. However, these methods of product differentiation are short-sighted. Therefore, this study will seek to propose that firms should start looking at ways of improving the more intangible, value-adding aspects of the purchase experience.

As consumers become increasingly spoilt for choice, it is not so much on what the product is, or where the firm is located, but the experience of using the product - i.e. from pre-purchase right up to maintenance of the product's usability. We will identify the various optimal situation for firms to improve their accessibilities so that consumers can enjoy lower "accessibility costs". For the purposes of this article, "accessibility" is defined as the ease at which consumers are able to interact with the firm - e.g. via improved customer care and product support.

When firms look to improving their accessibility, they hope that this investment will make their products stand out from the many other options available in the market. If the investment is made by only one firm, for example, enhancing customer support for the blind and deaf, then such an investment is said to target a niche market. However, if the investment is made by multiple firms, this becomes a norm. The result obtained in this study shows that when there are two firms in the market, it is both optimal and socially desirable when they either invest together or maintain status quo. The reason for this is that if only one firm invests, this leading firm will incur high research and development cost, while the follower achieves the same result by simply copying. This would deter firms from exploring potential niche markets. Furthermore, if only one firm manages to capture the niche market, the price charged to this particular consumer base will be high. Having two investing firms, per contra, would allow them to keep each other's price discrimination in check.

In the *literature review* section, we will look into various models in order to select the most optimal model for the purpose of this article. Following that, in the *model* section, we explain the set-up and derivation of the spatial model. After the model has been established, we will find Nash Equilibrium and social optimal outcome for the cases where firms charge mill prices and discriminatory prices separately. We will then discuss and make generalization on the results obtained. Finally, in the *conclusion*, limitations and corresponding future research directions will be stated.

2 Literature Review

The historical background of this study can be traced back to Hotelling (1929), who started the earliest discussion on spatial competition. He set up the fundamental framework for spatial competition, and nearly all the later discussions and extensions in this field were inspired by his work. However, in an article by d'Aspremont, Gabszewicz and Thisse (1979), Hotelling's principle was proven to be flawed as no equilibrium in price would exist if firms are located too close to each other. Albeit, many important concepts such as the linear city model, linear feature of the transportation cost, and the uniform distribution of consumers, remained of great value to the later exploration of spatial competition and product differentiation. These concepts were fairly comprehensive and had allowed people to tackle the issues in spatial competition in a relatively simple way. Furthermore, these concepts played an important role in facilitating the mathematical working process as well as the generalization of the issues. These features were proved to be beneficial to many recent extended works, such as the introduction of cooperation among firms through information exchange (Mai & Peng 1999), the study of production localization (Heikinnen 2013), the research on the effect of price-matching policy (Zhang 1995), etc.

Half a century later, Salop's circular city model (Salop 1979) started to gain popularity. Unlike the linear city model, this model allowed for the study of spatial competition without the need to worry about entry problem, i.e. a firm will face the same situation no matter where it enters the market. This model was one step closer to approximating the layout of a city in reality, and it provided academics with more inspirations to develope new models that can more closely replicate different types of layout in real world cities, e.g. Braid's (1989) intersecting roadway model was used to approximate the layout of a small city. More importantly, Salop model eliminated the end-points issue from Hotelling model. 'Polar' consumers were no longer confined to purchasing from the firm closest to him, but experience equal opportunity to choose from at least two firms. Therefore, we will be using Salop model in this study.

Product differentiation is another important feature introduced in spatial competition. As mentioned by Chamberlin (1933), besides the characteristics of the product itself, such as quality, design, etc., external factors such as location can also make a product more attractive to potential consumers. Hotelling (1929, p. 54) also implied earlier that "distance, as we have used it for illustration purposes, is only a figurative term for a greater congeries of qualities". Apart from location benefits, transportation costs also gives firms market power, since consumers will be more willing to shop with the firm that cost them less to travel to (Anderson, De Palma & Thisse 1992). This concept might also be applied to accessibility costs borne by the consumers. As mentioned above, consumers are more likely to purchase products from the firm that is more accessible and provides better purchase experiences. Hence, given the high similarly between these product differentiation concepts and the lack of research on accessibility costs, we will use the spatial competition model to study the accessibility issue and approximate the accessibility costs with transportation costs.

When considering which type of transportation cost should be used for the approximation of accessibility costs, it is important to note that different aspects of transportation cost influence spatial competition in different ways. The main differences between d'Aspremont, Gabszewicz and Thisse (1979) and Hotelling (1929) was the way in which the form of transportation cost was introduced. d'Aspremont, Gabszewicz and Thisse (1979) introduced maximum differentiation by using the quadratic transportation cost function (which can be generalized as convex transportation cost (Anderson 1988)) rather than linear transportation cost. Other forms of the transportation cost were also developed for tackling spatial problems, such as the Iceberg transportation cost, which was developed by Samuelson (1954) and extended by Martinez-Giralt and Usategui (2009); and the concave transportation cost mentioned by De Frutos, Hamoudi and Jarque (2001). These models were designed to analyse issues faced when engaging in product differentiation by looking into the various perspectives on location choices, as well as ensuring close approximation of reality. As mentioned by Heikkinen (2013, p.2), the reason that some papers do not follow linear transportation costs model is due to the 'tractability problem of the linear cost model', and this problem is insignificant when demand is discontinued. In our model, the demand is unitary, which means the linear cost model is appropriate. Plus, as Hotelling's linear transportation cost is comprehensive and easy to apply, we will use linear transportation cost model to approximate accessibility cost.

We also want to see whether the pricing strategy applied by firms also affects their decisions on improving the accessibilities. Thus, we introduced first degree (perfect) price discrimination. As mentioned by Phlips (1983, p.12), perfect price discrimination occurs when a firm charges a different price for every unit consumed. In this way, firms will be able to charge the maximum possible price so that the entire consumer surplus will be captured. The discussion on spatial price discrimination was started by Hoover (1937), who argued that sellers have the potential to practice price discrimination. This means that, without regulations, firms will choose to set discriminatory prices for the purposes of maximizing their profits. Empirical evidence has been provided by Greenhut (1981) that price discrimination is a common phenomenon in developed economies such as West Germany, the United States and Japan. This indicates that the firms highly favour this pricing strategy, and we should take this issue into consideration in our study. This is because allowing firms to implement discriminatory price policy can potentially influence the decisions of the firms in a different way as mill pricing. Lastly, it has been illustrated by Lederer and Hurter (1986) that, with discriminatory pricing strategy, identical firms tend not to locate on the same spot. Therefore, it is reasonable for us to continue fixing the firms locations at two ends of an arbitrary diameter of a model with perfect information.

3 Model

The basic model set up follows von Ungern-Sternberg's (1988) article. Salop's circular city model was used for the basic layout of the game and consumers are uniformly distributed along the circle. Firms are located equidistantly on the circle with their locations fixed. The difference between von Ungern-Sternberg's model and the one proposed here is that we are only looking at two firms in the market[1]. This model assumes that the two firms are identical to each other and the products produced are homogeneous. Since we mentioned that the distances between firms are equal, it is expected that the two firms will be located on two ends across the diameter of the circular city. For clarity, we are fixing firm 1 at zero and firm 2 at $\frac{1}{2}$ (see Figure 1). According to Gupta et al. (2004),this allocation of the locations was proved to be sustainable as Nash Equilibrium. In order to facilitate the calculation, the circumference of the circle will be set as 1 and both firms will have zero marginal cost and zero fixed cost. Furthermore, following Hotelling (1929) and d'Aspremont, Gabszewicz and Thisse (1979), each of the consumers will have discrete unitary demand for the good from either firms. Finally, the reservation value of every customer is assumed to be high enough to ensure full participation.

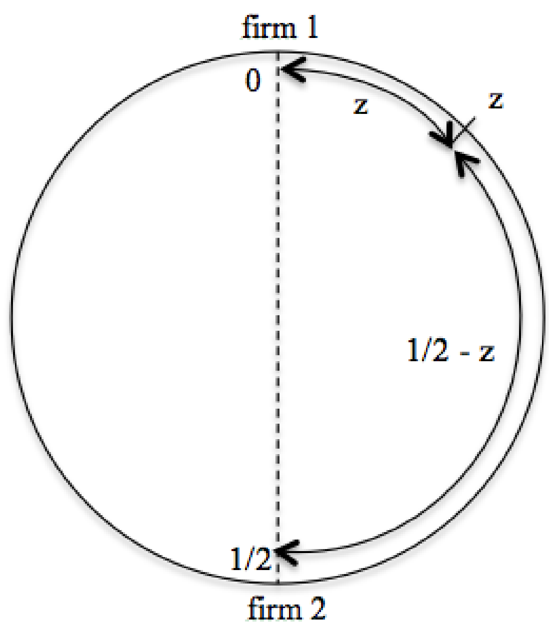

Figure 1: Landscape of the Model

[1]von Ungern-Sternberg (1988) generalized his model to N firms.

The consumers are assumed to be rational, and given the level of access[2] to the firms, they only shop with the firm that costs them the least. In order to access the firm, each consumer originally bears a per unit of accessibility cost of t . Here, t is assumed to be constant with respect to the level of access for different consumers. Thus, our accessibility cost will be linear.

Since firms need to make their decisions on the level of investment before they set prices for their products, they will be playing a simultaneous strategic game with two stages. In the first stage, they will make decisions on how much they will like to invest on improving their accessibilities. In the second stage, they will set their own prices given the investment decisions in the first stage. If a firm chooses to invest in improving the accessibility, it will incur firm i a marginal cost of investment of c_i and reduce each potential consumer t_i accessibility cost per unit. Again, we assume that the c_i and t_i are constant with respect to the level of access for all consumers. Therefore, the per unit accessibility cost after the investment will be reduced to $t - t_i$. After establishing that, we can come up with an equation (see equation 1) describing the equality of the disutilities of the consumer who is indifferent about buying from firm 1 or firm 2. We set level of access of this indifferent consumer as z, and $z \in (0, \frac{1}{2})$.

$$(t - t_1) * z + p_1 = (t - t_2) * (\frac{1}{2} - z) + p_2 \tag{1}$$

If we rearrange the equation, we get,

$$z = \frac{\frac{1}{2}(t - t_2) + (p_2 - p_1)}{2t - t_1 - t_2} \tag{2}$$

Since we have assumed zero marginal production cost and zero fixed cost for both firms, the only cost that affects the profit is the marginal cost of investment. Thus, the profit function for each firm should be,

$$\pi_1 = (p_1 - c_1) * 2z \tag{3}$$

$$\pi_2 = (p_2 - c_2) * (1 - 2z) \tag{4}$$

Since this is a two-stage game, we can solve it through backward induction. First, we assume the marginal cost of investment is given and work out the prices that the firms will

[2] This is represented by the concept of 'distance' in the model. We assume that the closer the consumer locates to the firm, the higher the level of access he has and the lower the total accessibility cost he bears.

set in the second stage. By plugging in the formula for z, taking the first derivative of the profit functions, and setting them as zero, we get the best response prices.

$$p_i = \frac{t - t_j}{4} + \frac{p_j + c_i}{2} \tag{5}$$

Once we work out best response prices, we can easily get the Nash Equilibrium prices that firms will set in the second stage by substituting one of the best response price function into the other.

$$p_i^* = \frac{1}{6}(3t - t_i - 2t_j + 2c_j + 4c_i) \tag{6}$$

If we plug p_1^* and p_2^* back into (2), we get a function of z that only depends on t_i and c_i. Subsequently, we substitute the new z function and the two optimal prices into (3) and (4) and obtain the profit functions represented by t, t_i and t_j, and c_i and c_j.

$$\pi_i = \frac{(3t - 2t_j - t_i + 2c_j - 2c_i)^2}{18(2t - t_i - t_j)} \tag{7}$$

We assume that firms have discrete choices and that each has two strategies: they can either make the investment or stay out. If firm i chooses to make the investment, it will incur the firm a marginal investment cost of $c_i = \hat{c_i}$ and reduce $t_i = \hat{t_i}$ per unit accessibility cost. We will assume that both $\hat{c_i}$ and $\hat{t_i}$ lie between zero and total per unit accessibility cost t and that both are parametric terms, i.e. they can take any positive value within the assumption range. If firm i chooses to stay out, it will incur zero cost, and the consumers will bear all the accessibility costs.

Given the optimal price and payoff functions, we will construct a two-by-two simultaneous game in the first stage. In this model, firms are assumed to not have perfect information on consumers and thus will be charging mill prices, i.e. each firm will charge one price for all the consumers. Given that each firm has two strategies ($t_i \in \{0, \hat{t_i}\}$), we will end up having four strategy profiles. We will derive the payoffs for two firms in each strategy profile through plugging in the investment decision on t_i. Following that, we will tabulate the payoffs in an attempt to find the Nash Equilibrium for the first stage of the simultaneous game through comparing the payoffs of one firm if the other firm's strategy is fixed. The amount of welfare for each strategy profile will also be looked at in order to decide which action profile is the most desirable for the economy.

The most important advantage of this model is that, by setting up simultaneous game with discrete choices and parametric cost terms for firms, the Nash Equilibrium we find is more helpful in gaining insight on whether and when firms will invest to improve their accessibilities in the competition. Furthermore, the fact that we do not presume any relationship on c_i and t_i makes the model more flexible in addressing the investment issue. Since it does not make sense for a firm to spend more than the maximum amount that consumers can save, then we will assume that the marginal investment cost $\hat{c_i}$ lies between zero and total per unit accessibility cost t prevents us from being distracted by the trivial part of the issue. Similarly, $\hat{t_i}$ should also be smaller than t due to the fact that the amount that consumers save should not exceed the total cost they bear.

After exploring the mill pricing case, we will look at the situation where firms have perfect information on consumers and charge perfectly discriminatory prices. The game with price discrimination will be set up in a similar way as the mill pricing case. Nash Equilibrium and the corresponding welfare figures will be worked out for the purpose of comparing and contrasting price discrimination and mill pricing.

4 Imperfect Information with Mill Pricing

A payoff matrix will be constructed to denote the simultaneous game in the first stage. The action set for firm i is $t_i = \{0, \hat{t_i}\}$. As firms do not have perfect information on the consumers, each will charge a universal price for all the consumers. We will assume that the model is symmetric. This means that if both firms decide to make the investment, they will choose to invest the same amount at the margin, and since they are identical, the per unit accessibility cost saved for consumers will be the same.

4.1 Equilibrium

To start, we assume that all parameters measuring identical constructs are equalled to each other, that is, $\hat{t_1} = \hat{t_2} = \hat{t}$ and $\hat{c_1} = \hat{c_2} = \hat{c}$. This means that the two firms have exactly the same strategy set $t_i = \{0, \hat{t}\}$. To get the payoffs, we simply plug in the corresponding marginal investment cost and consumer's saving in per unit accessibility cost into profit function (7) for each of the four strategy profiles (which are $(0,0)$, $(0,\hat{t})$, $(\hat{t},0)$ and $(\hat{t},\hat{t})$). Next, we tabulate the results to make the game clear (see Table 1).

To check whether any Nash Equilibrium exists in this game, we need to compare the cor-

		Firm 2	
		0	$\hat{t}$
Firm 1	0	$\frac{t}{4}, \frac{t}{4}$	$\frac{(3t-2\hat{t}+2\hat{c})^2}{18(2t-\hat{t})}, \frac{(3t-\hat{t}-2\hat{c})^2}{18(2t-\hat{t})}$
	$\hat{t}$	$\frac{(3t-\hat{t}-2\hat{c})^2}{18(2t-\hat{t})}, \frac{(3t-2\hat{t}+2\hat{c})^2}{18(2t-\hat{t})}$	$\frac{t-\hat{t}}{4}, \frac{t-\hat{t}}{4}$

Table 1: Payoff table for the symmetric game with mill pricing

responding payoffs for each action profile. We will also check what conditions need to be applied for Nash Equilibrium to occur.

The process of attaining equilibrium does not contain any difficult mathematical techniques. We simply keep one firm's strategy fixed and compare the payoffs of different strategies for the other firm.

PROPOSITION 1: *Within the optimal range of $\hat{c}$, which is $\hat{c} \in (0, t)$, staying out is the dominant strategy for both firms; thus, that neither of the firms invests is the only Nash Equilibrium.*

PROOF: See Appendix

In the case where firms do not have perfect information on consumers and charge mill prices, we can see that, within the assumption range of the marginal investment cost, Nash Equilibrium is always the status quo, i.e. both firms choose not to invest. This is because the dominant strategy for each firm is to not invest, which means that a firm is always better off choosing status quo no matter what the other firm chooses to do.

There are three possible explanations for this equilibrium outcome. Firstly, when the two firms invest simultaneously, the market price is,

$$p_i^* = \frac{1}{6}(3t - 3\hat{t} + 6\hat{c}) = \frac{t}{2} + \hat{c} - \frac{\hat{t}}{2}$$

This indicates that when the marginal investment cost is relatively low ($\hat{c} < \frac{\hat{t}}{2}$), firms' prices are lower than that in the case where no investment is made ($p_i^* = \frac{t}{2}$). Thus, given that the total demand remains unchanged, the profits are lower. The explanation for this is that when both firms are investing, the level of differentiation decreases due to the increasing competition, and in order to maintain their market share, they need to under cut each other on price.

Secondly, even though firms might be able to increase their prices (when the marginal investment cost is comparatively large, i.e. $\hat{c} > \frac{\hat{t}}{2}$), the increment in price is not enough to cover the cost incurred from trying to improve accessibility.

Lastly, there are two impacts associated with such an investment when only one firm invests. The first impact is that the investment results in a higher price for investing firm's product. This will either increase or decrease the demand and thus affecting profit: the improved accessibility will definitely attract more demand while the higher price might discourage some consumers. The second impact is that the cost of investment decreases firm's profit. Since (28) has been proven to be true for $\hat{c} \in (0, t)$ (see Appendix), we know that, overall, the investment reduces the profit of the investing firm. This could mean two things: (1) the cost impact is more significant, or (2) the improved accessibility does not attract enough demand to increase the overall profit. These reasons can also be applied in the real world to explain why most firms choose to remain passive despite complaints about their accessibility.

4.2 Welfare

Economic welfare is also a very important component of this study because it is able to advice us on what should be done to make the whole economy better off. An action or strategy will be socially desirable if it enhances the total welfare of the economy. With imperfect information, the total welfare change for each scenario is simply the overall change in consumer surplus and producer surplus; given that there are only two entities, consumer and firm, in the market. The rise in market price and the decrease of the accessibility cost are the main factors that contribute to the change in consumer surplus, while the marginal investment cost incurred and the profit change due to the change in price are the triggers for the change of producer's welfare. However, as the change of market price has exactly the same (in size) but contrary impact on consumers and producers, the total effect of this change is then zero. Thus, we only need to be concerned about the benefit that consumers receive from the reduction in accessibility cost and the corresponding investment cost incurred by the firms in order to calculate the total welfare change.

We treat the situation where neither of the firms takes action as the benchmark, against which we will measure the changes in welfare for the rest of the strategy profiles. Firstly, we start our evaluation from the simpler case where both firms choose to invest, and we denote this situation as 'state 1'. If both firms choose to invest by lowering $\hat{t}$ amount of per unit accessibility cost for each consumer, the location of the consumer who is indifferent about purchasing from firm 1 or firm 2 will not be affected. Thus, the total investment cost for the firms is simply $\hat{c}(2z) + \hat{c}(1 - 2z) = \hat{c}$ where z is 1/4, and the total welfare change for the firms is,

$$\Delta PS_1 = -\hat{c} + \Delta W_p \tag{8}$$

where ΔW_p is the welfare change for firms due to the price change. Hence, we can treat its opposite, $-\Delta W_p$, as the welfare change for consumers due to the price change.

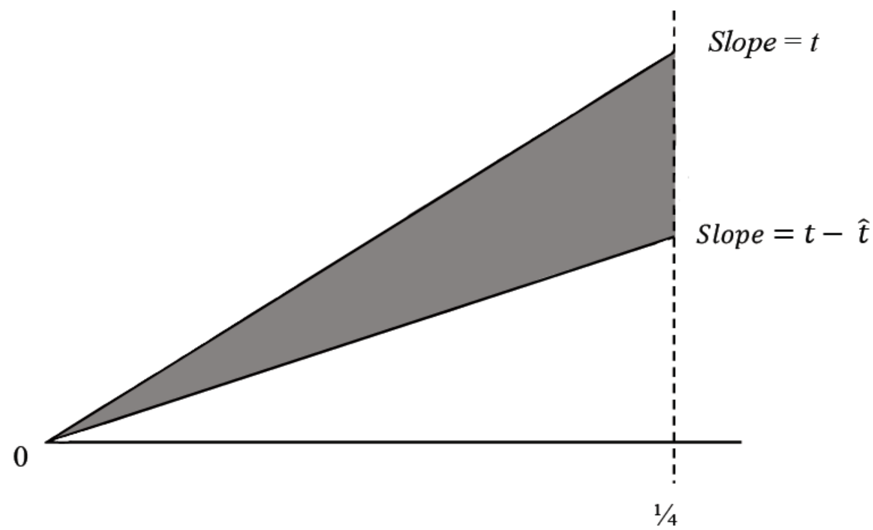

Figure 2: Disutility curves for one quarter of the consumers when both firms are investing

For consumers, the per unit accessibility cost decreases from t to $t - \hat{t}$. Since the market demand on each side of the two firms is a half and each firm gets half of the market, we only need to work out the benefit that consumers acquire in one quarter of the market and multiply it by four to get the total benefit. This is shown in Figure 2. The shaded area is the increase in consumer surplus for one quarter of the market, which is $\frac{\hat{t}}{32}$[3] And if we

[3]We can calculate the shaded area in Figure 1 using the formula,

$$Area = \frac{1}{2} * base * height$$

The base is simply $\frac{1}{4}\hat{t}$ and the height is $\frac{1}{4}$. Thus, the size of the shaded area is $\frac{\hat{t}}{32}$.

multiply it by four, we get the total increase in consumer surplus, which is $\frac{\hat{t}}{8}$. This indicates that the change for consumer surplus is,

$$\Delta CS_1 = \frac{\hat{t}}{8} - \Delta W_p \tag{9}$$

Therefore, the total welfare change for the economy in the case where both firms are making the investment is,

$$\Delta W_1 = \Delta PS_1 + \Delta CS_1 = \frac{\hat{t}}{8} - \hat{c} \tag{10}$$

Next, as the payoffs for the firms are symmetric, we only need to analyse one more scenario, in which there is only one firm making the investment. We denote this situation 'state 2'. In this case, we will simply assume that firm 1 invests and firm 2 does not. This means $t_1 = \hat{t}$ and $c_1 = \hat{c}$, and $t_2 = c_2 = 0$, and we can obtain the change of producer surplus.

$$\Delta PS_2 = -\hat{c}(2\hat{z}) + \Delta W_p \tag{11}$$

where

$$\hat{z} = \frac{3t - \hat{t} - 2\hat{c}}{6(2t - \hat{t})} \tag{12}$$

Following this, we need to come up with a method for calculating the change of consumer surplus. It is shown in Figure 3(a) that when $\hat{z}$ is less than 1/4, the change of consumer surplus is calculated as deducting area B from area A. Similarly, when $\hat{z}$ exceeds 1/4 (see Figure 3(b)), the change can be represented by area C, but will be calculated as (C+D)-D. Thus, it is not hard to see that the actual methods used for getting the expression of the change in consumers surplus are the same regardless of the relative size of $\hat{z}$, thereby leading to an unique expression for consumer surplus change.

$$\Delta CS_2 = 2[\frac{1}{2}\hat{t}\hat{z}^2 - t(\frac{1}{4} - \hat{z})^2] - \Delta W_p \tag{13}$$

Therefore, the total welfare change in state 2 is,

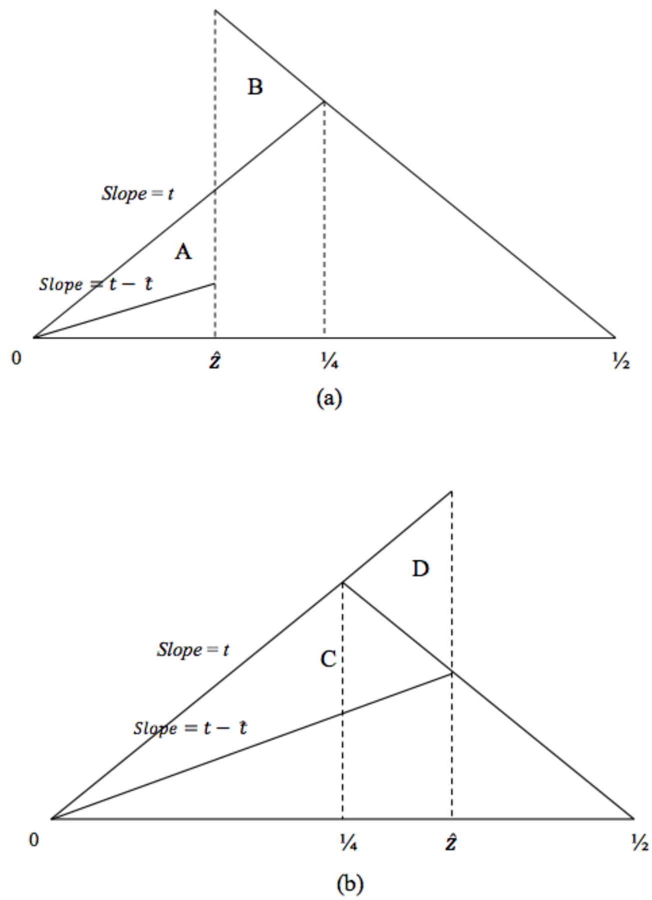

Figure 3: Disutility curves for half of the consumers when only one firm is investing

$$\Delta W_2 = \Delta PS_2 + \Delta CS_2 = \hat{t}\hat{z}^2 - 2t(\frac{1}{4} - \hat{z})^2 - 2\hat{z}\hat{c} \tag{14}$$

Since the game is symmetric, we can get the same results for the case where firm 2 invests and firm 1 does not.

PROPOSITION 2: *There are three different situations within the assumption range for $\hat{c}$, which is $\hat{c} \in (0, t)$:*
1. When $\hat{c} \in (0, 0.1t)$, the size of $\hat{t}$ will decide which welfare change is more positive than the other. 2. When $\hat{c} \in (0.1t, 0.138t)$, ΔW_2 will be positive and greater than ΔW_1.
3. When $\hat{c} \in (0.138t, t)$, both welfare changes will be negative.
PROOF: See Appendix.

Here, we see that things become more complicated for welfare change. Three plausible outcomes have been listed as marginal investment cost increases in the optimal range. The first two outcomes indicates that when the marginal investment cost is smaller than 13.8% of the total per unit accessibility cost, at least one firm should choose to improve its own system to reduce the accessibility cost for consumers if only for the sake of improving overall social welfare. However, as the equilibrium is always status quo where no firm will want to invest, the economy will end up having insufficient investment. Furthermore, it will not be socially desirable for firm/firms to make the investment of reducing the accessibility cost for customers if the marginal investment cost incurred exceeds 13.8% of the total per unit accessibility cost. The investment cost would be considered too high for the mere improvement of the social welfare. Therefore, status quo is both optimal and socially desirable, i.e. zero investment is the best for the economy.

5 Perfect Information with Perfect Price Discrimination

As the pricing strategy can affect firm's behaviour in the strategy, we introduce perfect price discrimination in this section. In this scenario, firms have perfect information of all the consumers and can charge each consumer what he is willing to pay. We will, again, find the Nash Equilibrium and socially desirable strategy profile, to see whether pricing strategy makes a difference in our model.

5.1 Equilibrium

With perfect information, firms can practice perfect price discrimination, i.e. both firms will charge the consumers the net marginal benefits they get from purchasing the product. Here, we set the reservation value of consumers as r and the initial price that both firms charge as $\hat{p}$ ($p_1 = p_2 = \hat{p}$). Thus, the marginal benefits for consumer j ($j = 1, 2, 3 \ldots$) purchasing from firm 1 and firm 2 is denoted by (15) and (16), respectively.

$$MB_j^1 = r - tz_j - \hat{p} \tag{15}$$

$$MB_j^2 = r - t(\frac{1}{2} - z_j) - \hat{p} \tag{16}$$

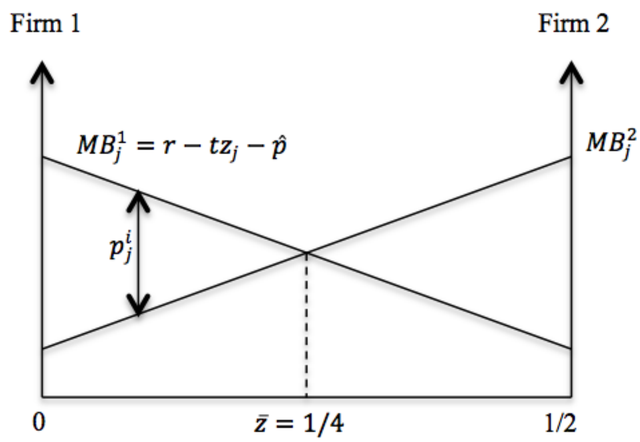

Figure 4: Marginal benefit curves and price discrimination for half of the market when neither firm is investing

If neither firm makes the investment, the marginal benefit curve for consumers purchasing from firm 1 is simply the mirror image of purchasing from firm 2 (see Figure 4). Thus, the consumer who is indifferent about buying from firm 1 or firm 2 will have an access level of $\bar{z} = 1/4$. This is also the case when both firms are investing. Consumer j (any consumer) who is located on the left of $\bar{z}$ will then purchase from firm 1, since the marginal benefit he gets from buying from firm 1 is higher than buying from firm 2. Knowing the marginal benefit difference of consumer j, firm 1 can maximize its profit by charging him the entire difference, which is the vertical distance between MB_j^1 and MB_j^2. Here, we use $p_j^i(s_1, s_2)$ to denote the price firm i charges for individual consumer j given both firms' strategies. Therefore, when both firms are doing nothing, the maximum price that firm 1 can set for consumer j is,

$$p_j^1(0,0) = MB_j^1 - MB_j^2 = \frac{t}{2} - 2tz_j \tag{17}$$

To derive the gross profit for firm 1, we simply integrate (30) with respect to z for $z \in (0, \frac{1}{4})$ and multiply it by 2. If we use $\pi_i(s_1, s_2)$ to denote the gross profit for firm i, we can get,

$$\pi_1(0,0) = 2\int_0^{\frac{1}{4}} (\frac{t}{2} - 2tz_j)\,\mathrm{d}z_j = \frac{t}{8} \tag{18}$$

Since the game is symmetric and firm 2 is identical to firm 1, firm 2 will have the same price schedule.

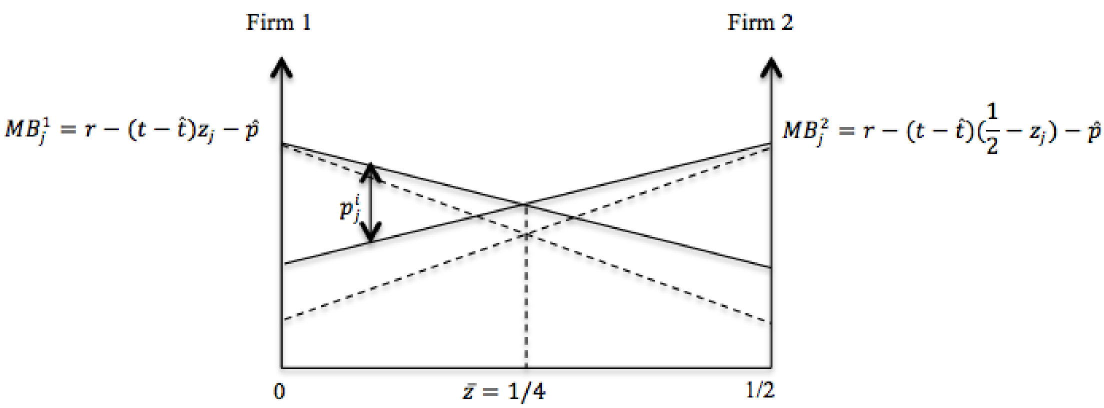

Figure 5: Marginal benefit curves and price discrimination for half of the market when both firms invest

When both firms reduce consumer j's per unit accessibility cost by $\hat{t}$ (see Figure 5), they will increase the price by the total amount of accessibility cost (which is $\hat{t}z_j$) and incur a marginal investment cost of $\hat{c}$. Thus, the new price firm 1 charges for consumer j is,

$$\begin{aligned} p_j^1(\hat{t},\hat{t}) &= [r - tz_j - (\hat{p} + \hat{t}z_j)] - [r - t(\frac{1}{2} - z_j) - (\hat{p} + \hat{t}(\frac{1}{2} - z_j))] - \hat{c} \\ &= -2(t - \hat{t})z_j + \frac{t - \hat{t}}{2} - \hat{c} \end{aligned} \tag{19}$$

Thus, the gross profit for firm 1 is,

$$
\begin{aligned}
\pi_1(\hat{t},\hat{t}) &= 2\int_0^{\frac{1}{4}} (-2(t-\hat{t})z_j + \frac{t-\hat{t}}{2} - \hat{c})\,\mathrm{d}z_j \\
&= \frac{t-\hat{t}-4\hat{c}}{8} \\
&= \pi_2(\hat{t},\hat{t})
\end{aligned}
\tag{20}
$$

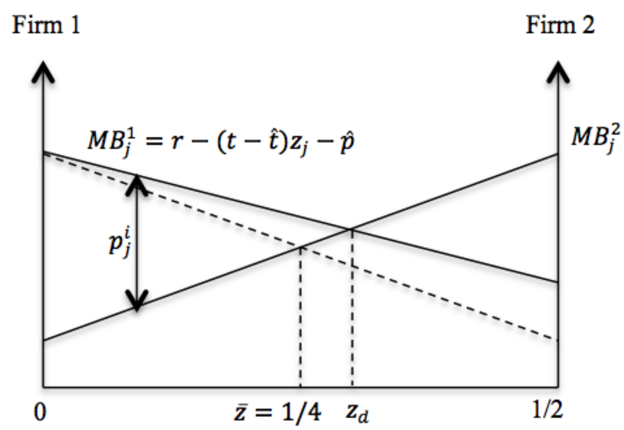

Figure 6: Marginal benefit curves and price discrimination for one quarter of the consumers when both firms are investing

When firm 2 invests and firm 1 does not, MB_j^1 will be flatter than MB_j^2 (see Figure 6), and the new intersection of these two curves can be derived by equating the two marginal benefit functions. If we use z_d to denote the location of the indifferent consumer, we can get,

$$
\begin{aligned}
r - (t-\hat{t})z_j - \hat{p} &= r - t(\frac{1}{2} - z_j) - \hat{p} \\
z_d &= \frac{t}{2(2t-\hat{t})}
\end{aligned}
\tag{21}
$$

Then the gross profit for firm 1 and firm 2 will be denoted by (22) and (23), respectively.

$$\begin{aligned}
\pi_1(\hat{t},0) &= 2\int_0^{z_d}([r-(t-\hat{t})z_j-\hat{p}]-[r-t(\frac{1}{2}-z_j)-\hat{p}]-\hat{c})\,\mathrm{d}z_j \\
&= 2\int_0^{\frac{t}{2(2t-\hat{t})}}(-(t-\hat{t})z_j+t(\frac{1}{2}-z_j)-\hat{c})\,\mathrm{d}z_j \\
&= \frac{t(t-4\hat{c})}{4(2t-\hat{t})} \qquad (22)
\end{aligned}$$

$$\begin{aligned}
\pi_2(\hat{t},0) &= 2\int_{z_d}^{1/2}([r-t(\frac{1}{2}-z_j)-\hat{p}]-[r-(t-\hat{t})z_j-\hat{p}]-\hat{c})\,\mathrm{d}z_j \\
&= \frac{(t-\hat{t})^2}{4(2t-\hat{t})} \qquad (23)
\end{aligned}$$

Since the game is symmetric, we will have $\pi_1(0,\hat{t})=\pi_2(\hat{t},0)$ and $\pi_2(0,\hat{t})=\pi_1(\hat{t},0)$ when firm 2 invests and firm 1 does not. Thus, we can, again, tabulate the results (Table 2).

		Firm 2: 0	Firm 2: $\hat{t}$
Firm 1	0	$\frac{t}{8},\frac{t}{8}$	$\frac{(t-\hat{t})^2}{4(2t-\hat{t})},\frac{t(t-4\hat{c})}{4(2t-\hat{t})}$
	$\hat{t}$	$\frac{t(t-4\hat{c})}{4(2t-\hat{t})},\frac{(t-\hat{t})^2}{4(2t-\hat{t})}$	$\frac{t-\hat{t}-4\hat{c}}{8},\frac{t-\hat{t}-4\hat{c}}{8}$

Table 2: Payoff table for symmetric game with discriminatory pricing

Proposition 3: *When firms have perfect information on consumers and charge perfectly discriminatory prices, the optimal range for marginal investment cost becomes $\hat{c}$.*
1. When $\hat{c}\in(0,min(0.043t,0.125\hat{t}))$, Nash Equilibrium will be obtained if both firms invest.
2. When $\hat{c}\in(max(0.043t,0.125\hat{t}),0.25t)$, Nash Equilibrium will be obtained if neither firms invest.

PROOF: To work out the possible Nash Equilibrium situation in this game, we simply need to compare $\frac{t}{8}$ and $\frac{t(t-4\hat{c})}{4(2t-\hat{t})}$ as well as $\frac{(t-\hat{t})^2}{4(2t-\hat{t})}$ and $\frac{t-\hat{t}-4\hat{c}}{8}$. In general, firms will not commit to invest if its gross profit is negative, which indicates that $\pi_1(\hat{t},0)$ needs to be greater than zero. Hence the game only makes sense when $\hat{c} \in (0, \frac{t}{4})$. Within this interval of $\hat{c}$, we can get the following relationships for the payoffs.

1. When $\hat{c} \in (0, 0.125\hat{t})$, $\frac{t}{8}$ is smaller than $\frac{t(t-4\hat{c})}{4(2t-\hat{t})}$, and the opposite is true when $\hat{c} \in (0.125\hat{t}, 0.25t)$.
2. When $\hat{c} \in (0, 0.043t)$, $\frac{t-\hat{t}-4\hat{c}}{8}$ is greater than $\frac{(t-\hat{t})^2}{4(2t-\hat{t})}$, and the opposite is true when $\hat{c} \in (0.043t, 0.25t)$[4].

Gathering all the information, we can see that the Nash Equilibrium varies when $\hat{c}$ takes different values.

1. When $\hat{c} \in (0, min(0.043t, 0.125\hat{t}))$, $(\hat{t}, \hat{t})$ will be the only Nash Equilibrium.
2. When $\hat{c} \in (max(0.043t, 0.125\hat{t}), 0.25t)$, $(0, 0)$ will be the only Nash Equilibrium.
3. When $\hat{c} \in (min(0.043t, 0.125\hat{t}), max(0.043t, 0.125\hat{t}))$,
   (a) $(0, 0)$ and $(\hat{t}, \hat{t})$ will be the Nash Equilibria if $0.043t > 0.125\hat{t}$;
   (b) $(\hat{t}, 0)$ and $(0, \hat{t})$ will be the Nash Equilibria if $0.043t < 0.125\hat{t}$.

Q.E.D.

When firms practice perfect price discrimination, the change in pricing caused by different behaviours seems to be fairly similar to that of mill pricing. If both firms invest, two

[4]When we compare these two payoffs, we can simply set one to be greater than the other. Assume we have,

$$\frac{t-\hat{t}-4\hat{c}}{8} < \frac{(t-\hat{t})^2}{4(2t-\hat{t})} \tag{24}$$

Then, if we rearrange it, we can get,

$$\hat{t}^2 - (t+4\hat{c})\hat{t} + 8t\hat{c} < 0$$

Solving this inequality, we can get,

$$\frac{1}{2}(t+4\hat{c}-\sqrt{16\hat{c}^2-24t\hat{c}+t^2}) < \hat{t} < \frac{1}{2}(t+4\hat{c}+\sqrt{16\hat{c}^2-24t\hat{c}+t^2})$$

It is obvious that, as long as the term in the square root is positive, (24) hold. Thus, we get,

$$\begin{aligned} \hat{c} &< \frac{3-2\sqrt{2}}{4}t \approx 0.043t, or \\ \hat{c} &> \frac{3+2\sqrt{2}}{4}t \approx 1.46t(invalid) \end{aligned}$$

Thus, when $\hat{c}$ lies between $0.043t$ and $0.25t$, the opposite of (24) hold.

marginal benefit curves both pivot upwards and become flatter. With this change, the consumer at z_j in Figure 5 will experience an the increment of marginal benefit received from firm 2's product, which is more than that from firm 1's product. As a result, the price firm 1 can charge for consumer j becomes less, leading to a reduction in firm 1's gross profit. Since the game is symmetric, same thing will happen to firm 2. Again, this situation can be explained by the increasing competition between firms. However, should one firm decide to invest in the market, that firm will be able to raise its price for each of its consumers. As the marginal benefit curve facing this firm pivots upward, the demand for its product will also increase. For the other firm, as the value of its product to the consumers drops, it will have to charge a lower price while experiencing a drop in its market share. Therefore, despite the profit gains made by the investing firm, the one that remains inactive will suffer a loss.

What makes discriminatory pricing different from milling pricing is multiple Nash Equilibria. Rather than just one, multiple Nash Equilibria exist for the game depending on the size of the marginal investment cost. When the marginal investment cost incurred for a firm is smaller than the lower of 0.125 times of consumer's saving on per unit accessibility cost and 4.3% of the total per unit accessibility cost, both firms will find it in their best interest to invest, even though the gross profit for each firm is smaller than that in status quo. The reason is that, both firms will find it profitable to deviate from the status quo, and if one firm deviates, the other also has the incentive to deviate. Contrary to that, if the cost incurred is sufficiently large (larger than the higher of those two values), neither will be interested in making the investment. Costs aside, firms might want to consider looking at per unit saving for the consumer. If it is comparatively large ($\hat{t} > \frac{0.043}{0.125}t = 0.334t$), making the investment will be one firm's best response. If, however, the per unit saving for consumer is comparatively small ($\hat{t} < 0.334t$), then Nash Equilibria will be obtained only if both firms invest or status quo is kept.

5.2 Welfare

The total welfare for the economy under each strategy profile is simply the sum of two firm's net profits and the potential benefits that all the consumers receive from purchasing the product. This can be measured through calculating the area under the marginal benefit curves in each graph in the previous section (Figure 4, 5 & 6) and deducting the total cost incurred by the corresponding firm(s). In this section, we will use W_s, W_1 and W_2 to denote the total welfare of the strategy profile $(0, 0)$ (status quo), $(\hat{t}, \hat{t})$ (state 1) and $(\hat{t}, 0)$ (which has the same welfare as $(0, \hat{t})$, and we call them state 2).

PROPOSITION 4: *Within the optimal range of marginal investment cost, which is when $\hat{c} \in (0, 0.25t)$, state 1 has the largest total welfare when $\hat{c} \in (0, 0.125\hat{t})$, and the status quo gives the largest social welfare when $\hat{c} \in (0.125\hat{t}, 0.25t)$.*

PROOF: We can use integration to calculate the total areas under the marginal benefit curves in each of the scenarios. The calculations are as follows.

$$\begin{aligned} W_s &= 2 * 2 \int_0^{\frac{1}{4}} (r - tz_j - \hat{p}) \, \mathrm{d}z_j \\ &= r - \hat{p} - \frac{t}{8} \qquad (25) \\ W_1 &= 2 * 2 \int_0^{\frac{1}{4}} (r - (t - \hat{t})z_j - \hat{p} - \hat{c}) \, \mathrm{d}z_j \\ &= (r - \hat{p} - \frac{t}{8}) + (\frac{\hat{t}}{8} - \hat{c}) \qquad (26) \\ W_2 &= 2[\int_0^{z_d} (r - (t - \hat{t})z_j - \hat{p} - \hat{c}) \, \mathrm{d}z_j + \int_{z_d}^{1/2} (r - tz_j - \hat{p}) \, \mathrm{d}z_j] \\ &= (r - \hat{p} - \frac{t}{8}) + (\frac{t}{8} - \frac{(t - \hat{t})z_d}{2} - 2\hat{c}z_d) \qquad (27) \end{aligned}$$

It is obvious that W_1 is greater than W_s if $\hat{c} < \frac{\hat{t}}{8}$. What is not obvious is the size of W_2. If we substitute z_d with what we have achieved in (34), we can derive that, for $W_2 > W_s$ to be true, $\hat{c}$ will also have to be smaller than $\frac{\hat{t}}{8}$ [5]. Thus, if the marginal investment cost lies between $\frac{\hat{t}}{8}$ and $\frac{t}{8}$, W_2 will be the largest. In addition, we can compare W_1 and W_2 for

[5] If we want W_2 to be greater than W_s, we will need,

$$\frac{t}{8} - \frac{(t - \hat{t})z_d}{2} - 2\hat{c}z_d > 0$$

If we plug in z_d and solve this inequality, we will get,

$$\begin{aligned} \frac{t}{8} - \frac{t(t - \hat{t})}{4(2t - \hat{t})} - \frac{t\hat{c}}{2t - \hat{t}} &> 0 \\ 2t - 2\hat{t} + 8\hat{c} &< 2t - \hat{t} \\ 8\hat{c} &< \hat{t} \\ \hat{c} &< \frac{\hat{t}}{8} \end{aligned}$$

$\hat{c} \in (0, \frac{\hat{t}}{8})$ through checking whether $\Delta W = W_1 - W_2$ is positive or negative. From the calculations done, we found that ΔW will always be positive for $\hat{c} \in (0, \frac{\hat{t}}{8})$ and $\hat{t} \in (0, t)$. Hence we can conclude that W_1 is always greater than W_2 when it is positive. *Q.E.D.*

The situation for welfare change is more straightforward with price discrimination. With a low marginal investment cost (smaller than 0.125 times of the per unit saving on accessibility cost), it is socially desirable for both firms to invest. This is because the additional revenue earned by the firms through extracting entire benefits received by consumers from the improved accessibility exceeds the cost of investment. Furthermore, the total profit earned when both firms invest is higher than that earned should only one firm make the investment. However, when the marginal investment cost goes beyond the threshold $0.125\hat{t}$, it would be socially desirable if neither firms invest. At this point, cost incurred for the investment will be so high that the additional profit earned will be insignificant.

Lastly, as mentioned earlier in the equilibrium section (Section 5.1), the size of savings in accessibility cost for consumers also makes a difference here. If the per unit accessibility cost saved by the consumer is relatively small ($\hat{t} < 0.334t$), Nash Equilibrium is only obtained if both firms invest for $\hat{c} \in (0.125\hat{t}, 0.043t)$ —when in fact, both firms staying out is the only socially desirable outcome. Thus, we may potentially have both firms making the investment when it is not efficient to do so. That is, the economy will experience over investment. On the other hand, if the saving on accessibility cost is comparatively large ($\hat{t} > 0.334t$), only one firm chooses to invest when both should be investing. Then, the economy will have too little investment.

6 General Discussion

Figure 7 is a summary of the equilibrium and welfare conditions for the two symmetric games. It is obvious that when firms do not have sufficient information on their consumers and only charge mill prices, it will never be worth their while improving their accessibilities (e.g. providing product support to the consumers). Firms will not know what the consumers want and by extension, how to support them. However, the welfare analysis for mill pricing indicates that firms should make the improvements, regardless, if the costs incurred is sufficiently low. This is because the increase in consumer surplus gained through higher satisfaction on the product and potential lowered prices will exceed the losses faced by the firms. Under such situations, an external entity, such as the government, will need to step

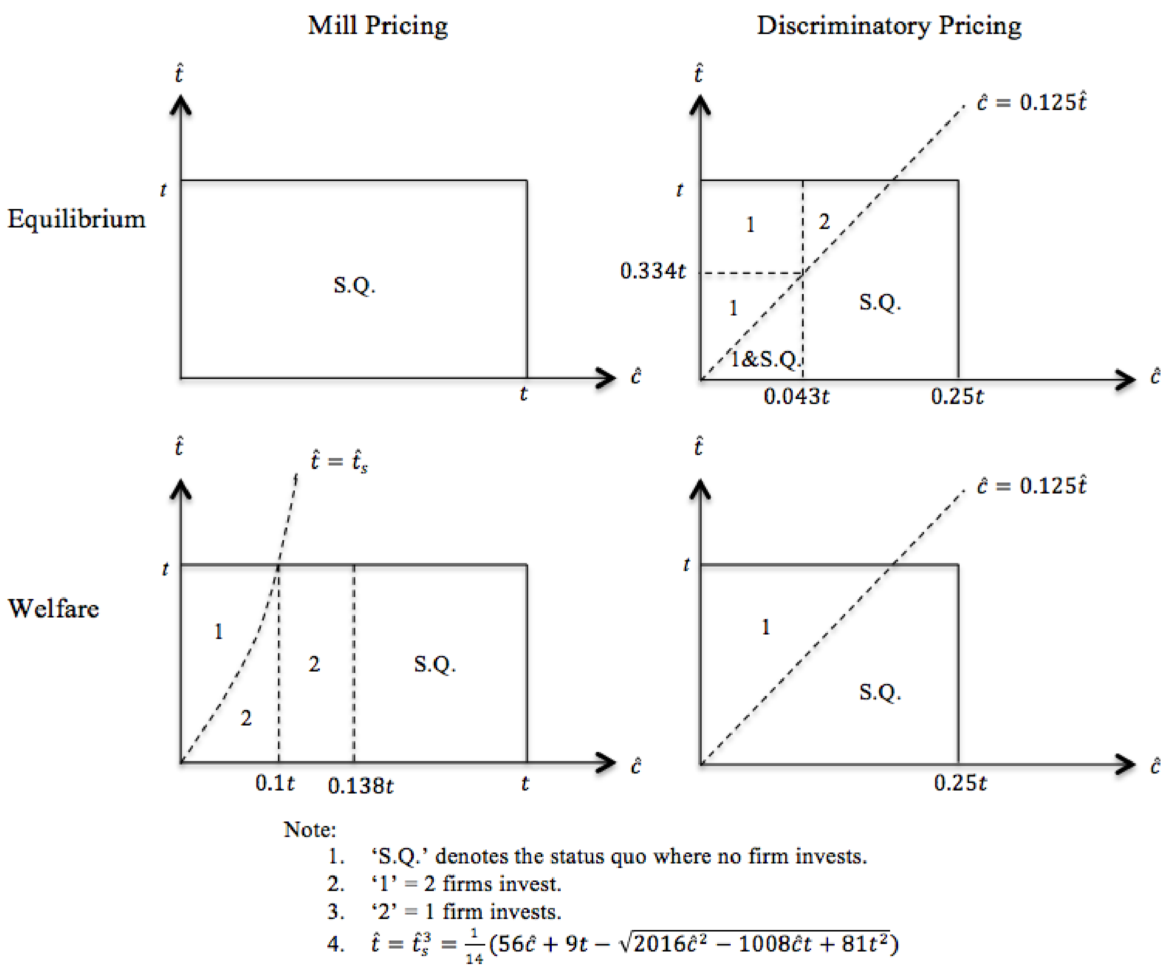

Figure 7: Summary of equilibria and welfare conditions for mill pricing and discriminatory pricing

in in order to ensure that the improvement takes place. Like most of its policies, government can either impose regulations on the firms or provide subsidies to make accessibility investments more attractive. In doing so, social welfare will be enhanced. Nevertheless, as suggest by Pigou (1924), welfare issues are hard to correct. Governments need to acquire sufficiently large amounts of information on the market before they can make decisions.

If we compare the equilibria conditions for the two games, we can easily see that, with sufficiently low costs of investing, firms will be more proactive when they have enough information and are able to practice price discrimination. The most common way of obtaining such information is through direct customer feedback. A feedback system will help firms gather the information they need on their consumers, such as consumers' preferences and willingness to pay for the products, access levels to the firm, knowledge levels on their products, etc. If the feedback system is well received and consumers bother to voice their concerns, firms would be able to differentiate their consumers and charge discriminatory prices. This encourages firms to provide the support that their consumers may need. Subsequently, with adequate and suitable support, consumers' satisfaction level will be significantly enhanced, which, in turn, improves social welfare.

Knowing the difference between the results of the two symmetric games, we can conclude that, in the real world, information imperfection is an important factor that prevents firms from improving their accessibility. Acquiring information will incur extra cost, which might also be a deterrence for firms to carry out the improvement. In addition to a lack of information, high marginal investment cost may also explain why firms are not making the move and why government are hesitant to step in.

Things will get better if information is easier to collect and if the cost of improvement is low. This is, indeed, what is happening in reality. As the operations go on and the technology develops, firms have easy access to larger amounts of information, while protecting the privacy of their consumers. Customized support is one such method as many companies make themselves more accessible by providing both online and telephone support. Some companies such as Vodafone made changes to their telephone support line by arranging to call consumers back rather than making them hold onto the call, thus reducing waiting time (The Vodafone Network Guarantee 2014). Another example is seen in IT as software developers start making their products easier to use by putting in more figures and less words in their guides. Software developers such as Microsoft also make their software compatible in more than one computer system (Watt 2002), so that consumers are able to benefit from the software regardless of which operating system they have. All these modifications on the service and support made by the firms make them more accessible and their products more attractive. Firms have the potential to make more profit by doing so since the overall demand of their products, unlike the setting in our model, will increase. As people are able to buy the products with better service and support, they will definitely be better off. Therefore, the welfare level of the whole society will rise.

We posit that this model is fully applicable to infrastructure or public transportation investments made by the government. As mentioned by Ghosh and Meagher (2011), the investment on infrastructure will benefit the consumers by lowering their transportation cost and mitigating the market power of firms. If it is not profitable for firms to make investment to lower consumers' transportation cost but 'profitable' for the economy, governments should step in to invest by improving infrastructure quality. However, investments will not be worthwhile if the total surplus of the economy is going to decrease. In addition, the model is applicable to the customization problem. Customization allows the consumers to order the product according to their specific needs, and consumers benefit from the customized products. Nevertheless, Dewan, Jing and Seidmann (2003) argued that customization might reduce the level of differentiation and thus decreasing the profitability for firms, which, to some extent, is consistent with our model.

7 Conclusion

The article built a model of spatial competition and applied some basic set-ups to address the lack of incentive firms face regarding accessibility improvement. We found that if firms can only charge mill prices, they will not be willing to improve their accessibility, which will, sometimes, result in too little investment. On the contrary, when firms are able to practice price discrimination, they should and will choose to make the improvement if marginal investment cost incurred is sufficiently low. This is optimal for the firms and also efficient for the economy. With regard to what we have mentioned in the introduction, however, this result implies that firms will not try to explore niche markets, but have more incentive to produce general products.

This model has several limitations. One limitation is the construction of the accessibility cost function. Linear cost model is an approximation, which may be too simple and may not reflect the nature of accessibility cost. Another limitation is the unitary demand of consumers. For some products, consumers have elastic demand and may purchase more than one unit of product.

For the purposes of future research, studies may wish to apply this model on analysing other aspects of firms, which could affect the differentiation of firms' product. The construction of the cost function needs to be done with care according to the issue that is studied. For example, if issue requires the demand of each consumer to be elastic and continuous, quadratic cost function, rather than linear cost function, should be considered. Therefore, despite the generalizability of this model to many constructs, the variables within the model will need adjusting.

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9 Appendix

PROOF OF PROPOSITION 1:

First, we fix firm 2's strategy to 'not invest'. Under this situation, we need to compare firm 1's payoffs between investing and not investing. We simply presume the payoff for investing is greater than that for not investing. Thus, we have,

$$\frac{t}{4} > \frac{(3t - \hat{t} - 2\hat{c})^2}{18(2t - \hat{t})} \tag{28}$$

After rearranging this inequality, we can get a quadratic function of $\hat{t}$ that is smaller than zero.

$$2\hat{t}^2 + \hat{t}(8\hat{c} - 3t) + 8\hat{c}^2 - 24t\hat{c} < 0 \tag{29}$$

Now we use the formula for solving quadratic equations to solve $\hat{t}$ as if (29) holds for equality. We then get two roots for $\hat{t}$ in terms of $\hat{c}$ and t. We will use $\hat{t_s}^1$ and $\hat{t_l}^1$ to denote the small root and the large root of $\hat{t}$, respectively.

$$\begin{aligned}
\hat{t_s}^1 &= \frac{3t - 8\hat{c} - \sqrt{9t^2 + 144t\hat{c}}}{4} \\
\hat{t_l}^1 &= \frac{3t - 8\hat{c} + \sqrt{9t^2 + 144t\hat{c}}}{4}
\end{aligned}$$

For inequality (29) to hold, we need $\hat{t_s}^1 < \hat{t} < \hat{t_l}^1$. It is obvious that $\hat{t_s}^1$ is always smaller than zero given that t and $\hat{c}$ are both positive numbers. Then we check whether $\hat{t_l}^1$ is greater than t, so that $\hat{t}$ is able to take any value in $\hat{t} \in (0, t)$. The condition we get for $\hat{t_l}^1 > t$ is as follows.

$$\begin{aligned}
\hat{t_l}^1 &> t \\
\frac{3t - 8\hat{c} + \sqrt{9t^2 + 144t\hat{c}}}{4} &> t \\
64\hat{c}^2 - 128t\hat{c} - 8t^2 &< 0
\end{aligned}$$

Then we have,

$$
\begin{aligned}
\frac{128-\sqrt{128^2+4*8*64}}{2*64}t &< \hat{c} < \frac{128+\sqrt{128^2+4*8*64}}{2*64}t \\
-0.06t \approx \frac{128-96\sqrt{2}}{128}t &< \hat{c} < \frac{128+96\sqrt{2}}{128}t \approx 2.06t \\
0 &< \hat{c} < 2.06t
\end{aligned}
$$

Since we have assumed that $\hat{c}$ cannot exceed t, $\hat{t}_l^1$ is greater than t in the whole range of $\hat{c}$. This means that, for $\hat{c} \in (0, t)$, inequality (29) holds, and thus (28) holds.

Next, we suppose firm 2 decides to invest with $\hat{t}$. Now, in order to compare the payoffs of different strategies for firm 1, we can apply the same method used above by setting the payoff of investing greater than that for not investing.

$$\frac{(3t-2\hat{t}+2\hat{c})^2}{18(2t-\hat{t})} > \frac{t-\hat{t}}{4} \tag{30}$$

After rearranging the inequality, we can get

$$8\hat{c}(\hat{c}+3t-2\hat{t})+\hat{t}(3t-\hat{t}) > 0 \tag{31}$$

From (31), we are certain that, for $\hat{c}, \hat{t} \in (0, t)$, all the factors on the left hand side of the inequality will be positive, thereby making (31) hold.

Therefore, it is clear that for firm 1, not investing will be a dominant strategy for $\hat{c} \in (0, t)$. Moreover, since the game is symmetric, firm 2's dominant stratagy is also not investing. Therefore, the Nash Equilibrium is found with neither of the firms making the investment.

PROOF OF PROPOSITION 2:

To see which state is more socially desirable than the others, the first step is to work out the conditions that make ΔW_1 and ΔW_2 positive. We will automatically know that the status quo is the most socially desirable case if both welfare changes fall below zero.

For ΔW_1, it is only positive when the marginal investment cost incurred for the firm is very small, i.e.

$$0 < \hat{c} < \frac{\hat{t}}{8}$$

Therefore, when (10) is satisfied, it is more socially desirable for two firms to invest comparing to the status quo (where both do nothing).

For ΔW_2 to be positive, we need to plug the expression of $\hat{z}$ (which is (12)) into (14), and see what conditions need to be applied. Thus, we have,

$$\begin{aligned}
\Delta W_2 = \hat{t}\hat{z}^2 - 2t(\frac{1}{4} - \hat{z})^2 - 2\hat{z}\hat{c} &> 0 \\
\frac{\hat{t}(3t - \hat{t} - 2\hat{c})}{6(2t - \hat{t})} - 2t(\frac{1}{4} - \frac{3t - \hat{t} - 2\hat{c}}{6(2t - \hat{t})})^2 - \frac{2\hat{c}(3t - \hat{t} - 2\hat{c})}{6(2t - \hat{t})} &> 0 \\
\frac{-2\hat{t}^2 + 40\hat{c}^2 - 72t\hat{c} + 16\hat{c}\hat{t} + 9\hat{t}t}{72(2t - \hat{t})} &> 0
\end{aligned}$$

Since the denominator is positive, we only need the numerator to be positive. We can rewrite the left hand side of the inequality above as a function of $\hat{t}$.

$$2\hat{t}^2 - (16\hat{c} + 9t)\hat{t} + (72t\hat{c} - 40\hat{c}^2) < 0 \tag{32}$$

Then, we can apply the same mathematical method as has been used in the proof of proposition 1 by writing out another pair of roots for $\hat{t}$. We use $\hat{t_s}^2$ and $\hat{t_l}^2$ to denote the small root and the large root, respectively.

$$\begin{aligned}
\hat{t_s}^2 &= \frac{16\hat{c} + 9t - \sqrt{576\hat{c}^2 - 288\hat{c}t + 81t^2}}{4} \\
\hat{t_l}^2 &= \frac{16\hat{c} + 9t + \sqrt{576\hat{c}^2 - 288\hat{c}t + 81t^2}}{4}
\end{aligned}$$

If we want to find the conditions that makes (32) hold, we need $\hat{t_s}^2 < \hat{t} < \hat{t_l}^2$. Since $\hat{t_l}^2 > t$, we only need to focus on measurement of the size of $\hat{t_s}^2$. For $\hat{t}$ to exist, $\hat{t_s}^2$ need to be at most as much as t and cannot be equal to t. Therefore, we will have,

$$\begin{aligned}
\hat{t_s}^2 &< t \\
\frac{16\hat{c} + 9t + \sqrt{576\hat{c}^2 - 288\hat{c}t + 81t^2}}{4} &< t \\
320\hat{c}^2 - 448\hat{c}t + 56t^2 &> 0
\end{aligned} \tag{33}$$

If we solve inequality (33), we get,

$$\begin{aligned}\hat{c} &< \frac{448t - \sqrt{(448t)^2 - 4*320*56t^2}}{2*320} \approx 0.138t, or \\ \hat{c} &> \frac{448t + \sqrt{(448t)^2 - 4*320*56t^2}}{2*320} \approx 1.26t\end{aligned}$$

Since $\hat{c} \in (0, t)$, only $\hat{c} \in (0, 0.138t)$ is valid. Within this range of marginal investment cost, $\hat{t} \in (0, t)$. Thus, with these conditions hold, state 2 (where one firm invests) is more socially desirable than status quo.

Next, we compare the total surplus change for state 1 and state 2, and decide which state gains the most in welfare. We deduct ΔW_1 from ΔW_2 and work out the condition for the difference to be greater than zero.

$$\begin{aligned}\Delta W_2 - \Delta W_1 &> 0 \\ \hat{t}\hat{z}^2 - 2t(\frac{1}{4} - \hat{z})^2 - 2\hat{z}\hat{c} - (\frac{\hat{t}}{8} - \hat{c}) &> 0\end{aligned}$$

After we plug in the formula of $\hat{z}$ (see (12)) and rearrange the inequality, we can get,

$$\begin{aligned}\frac{7\hat{t}^2 + 40\hat{c}^2 + 72\hat{c}t - 56\hat{c}\hat{t} - 9\hat{t}t}{72(2t - \hat{t})} &> 0 \\ 7\hat{t}^2 - (56\hat{c} + 9t)\hat{t} + (40\hat{c}^2 + 72\hat{c}t) &> 0 \qquad (34)\end{aligned}$$

Once again, we apply the 'root' method and define $\hat{t_s}^3$ and $\hat{t_l}^3$ as follows.

$$\begin{aligned}\hat{t_s}^3 &= \frac{56\hat{c} + 9t - \sqrt{2016\hat{c}^2 - 1008\hat{c}t + 81t^2}}{14} \\ \hat{t_l}^3 &= \frac{56\hat{c} + 9t + \sqrt{2016\hat{c}^2 - 1008\hat{c}t + 81t^2}}{14}\end{aligned}$$

To make the inequality (34) hold, $\hat{t}$ needs to be either greater than $\hat{t_l}^3$ or smaller than $\hat{t_s}^3$. Here, If we want the two roots to be well defined, we need,

$$2016\hat{c}^2 - 1008\hat{c}t + 81t^2 > 0$$

Solving this inequality, we get,

$$\begin{aligned}\hat{c} &< \frac{1008t - \sqrt{(1008t)^2 - 4*2016*81t^2}}{2*2016} \approx 0.1t, or \\ \hat{c} &> \frac{1008t + \sqrt{(1008t)^2 - 4*2016*81t^2}}{2*2016} \approx 0.4t\end{aligned}$$

When $\hat{c} \in (0, 0.1t)$, $\hat{t_l}^3$ is greater than t while $\hat{t_s}^3$ is greater than zero and smaller than t. Hence, if $\hat{t} \in (0, \hat{t_s}^3)$, (34) holds and $\Delta W_2 > \Delta W_1$. Note that as ΔW_2 is also greater than zero for this range of $\hat{c}$, it is most beneficial strategy for the economy to have one firm investing. On the contrary, if $\hat{t} \in (\hat{t_s}^3, t)$, the opposite of (34) holds and the case where both firms invest is most beneficial. Besides, when the roots are not defined, i.e. $2016\hat{c} - 1008\hat{c}t + 81t^2 < 0$, inequality (34) holds. This means that for $\hat{c} \in (0.1t, 0.138t)$, ΔW_2 is positive and greater than ΔW_1. In this case, it is more socially desirable for only one firm to invest. Last but not least, since both ΔW_1 and ΔW_2 are negative for $\hat{c} \in (0.138t, t)$, status quo is the most socially desirable outcome for this range.