

Reasoning with Inconsistent Information

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Except where otherwise indicated, this thesis is my own original work.

Paul Wong

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To Fiona, Little Woks, Smallness, Tickers, Tigger, Gollie and Mother Superior.

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...[M]ysticism might be characterized as the study of those propositions which are equivalent to their own negations. The Western point of view is that the class of all such propositions is empty. The Eastern point of view is that this class is empty if and only if it isn't.

Raymond Smullyan [174]

Abstract

In this thesis we are concerned with developing formal and representational mechanisms for reasoning with inconsistent information. Strictly speaking there are two conceptually distinct senses in which we are interested in reasoning with inconsistent information. In one sense, we are interested in using logical deduction to draw inferences in a symbolic system. More specifically, we are interested in mechanisms that can continue to perform deduction in a *reasonable* manner despite the threat of inconsistencies as a direct result of errors or misrepresentations. So in this sense we are interested in inconsistency-tolerant or paraconsistent deduction.

However, not every case of inconsistent description is a case of misrepresentation. In many practical situations, logically inconsistent descriptions may be deployed as representations for problems that are inherently conflicting. The issue of error or misrepresentation is irrelevant in these cases. Rather the main concern in these cases is to provide meaningful analyses of the underlying structure and properties of our logical representation which in turn informs us about the salient features of the problem under consideration. So in this second sense, we are interested in deploying logic as a *representation* to model situations involving conflict.

In this thesis we adopt a novel framework to unify both logic-as-deduction and logic-as-representation approaches to reasoning with inconsistent information. From a *preservational* view point, we take deduction as a process by which metalogical properties are preserved from premises to conclusions. Thus methodologically we may begin by identifying inconsistency-tolerant deduction mechanisms and then investigate what additional properties of inconsistent premises are preserved by these mechanisms; or alternatively we may begin by identifying properties of inconsistent logical descriptions and investigate which deductive mechanisms can preserve these properties. We view these as two aspects of the same investigation. A key assumption in this work is that adequate analyses of inconsistencies require provisions to quantitatively measure and compare inconsistent logical representations. While paraconsistent logics have enjoyed considerable success in recent years, proper quantitative analysis of inconsistencies seems to have lapsed behind to some extent. In this thesis we'll explore different ways in which we can compare and measure inconsistencies. We hope

to show that both inference and analysis can fruitfully be brought to bear on the issue of inconsistency handling under the same methodological scheme.

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