Theory and Application of Broadband Frequency Invariant Beamforming

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of the Australian National University

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July 1996

Declaration

The contents of this thesis are the result of original research and have not been submitted for a higher degree to any other university or institution.

Much of the work presented in this thesis has been published or will be submitted for publication as journal or conference papers. Following is a list of these papers. In some cases the conference papers contain material overlapping with the journal papers.

Journal Papers

- D.B. Ward, R.A. Kennedy and R.C. Williamson, "Theory and design of broadband sensor arrays with frequency invariant far-field beam patterns", *J. Acoustical Society of America*, vol. 97, no. 2, pp. 1023-1034, Feb. 1995.

- D.B. Ward, R.A. Kennedy, and R.C. Williamson, "FIR filter design for frequency invariant beamformers", *IEEE Signal Processing Letters*, vol. 3, no. 3, pp. 69-71, Mar. 1996.

- D.B. Ward, Z. Ding, and R.A. Kennedy, "Direction of arrival estimation for wide-band signals using frequency invariant beamspace processing", *IEEE Trans. Signal Processing*, (to be submitted).

- R.A. Kennedy, T. Abhayapala, and D.B. Ward, "Broadband nearfield beamforming using a radial beampattern transformation", *IEEE Trans. Signal Processing*, (to be submitted).

- P.J. Kootsookos, D.B. Ward and R.C. Williamson, "Imposing pattern nulls in broad-band array responses", *J. Acoustical Society of America*, (to be submitted).

Conference Papers

- D.B. Ward, R.A. Kennedy and R.C. Williamson, "Design of frequency-invariant broadband far-field sensor arrays", in *1994 IEEE Antennas and Propagation Society Int. Symp. Digest*, vol. 2, pp. 1274-1277, Seattle, USA, June 1994.

- D.B. Ward, R.A. Kennedy, and R.C. Williamson, "Broadband beamforming with a single set of filter coefficients", in *Proc. 1995 IEEE Singapore Int. Conf. on Signal Processing, Circuits and Systems*, pp. 88-93, Singapore, July 1995.

- D.B. Ward, Z. Ding, and R.A. Kennedy, "Broadband direction of arrival estimation using frequency-invariant beam-space processing", in *Proc. IEEE Int. Conf. on Acoust., Speech, and Signal Processing, (ICASSP'96)*, pp. 2892-2895, Atlanta, USA, May 1996.

- R.A. Kennedy, T. Abhayapala, D.B. Ward, and R.C. Williamson, "Near-field broadband frequency invariant beamforming", in *Proc. IEEE Int. Conf. on Acoust., Speech, and Signal Processing, (ICASSP'96)*, pp. 905-908, Atlanta, USA, May 1996.

- P.J. Kootsookos, D.B. Ward, and R.C. Williamson, "Frequency invariant beamforming with exact null design", in *Proc. IEEE Workshop on Statistical Signal and Array Processing, (SSAP'96)*, pp. 105–108, Corfu, Greece, June 1996.

The research presented in this thesis has been performed jointly with Dr Rodney A. Kennedy, Dr Robert C. Williamson, Dr Peter J. Kootsookos, Prof. Zhi Ding (Auburn University, USA) and Mr Thushara Abhayapala. The majority, approximately 70%, of this work was my own.

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July 1996

Abstract

In many engineering applications, including radar, sonar, communications and seismology, the direction of impinging signal wavefronts can be used to discriminate between competing sources. Often these source signals cover a wide bandwidth and conventional narrowband beamforming techniques are ineffective, since spatial resolution varies significantly across the band. In this thesis we consider the problem of beamforming for broadband signals, primarily when the spatial response remains constant as a function of frequency. This is called a *frequency invariant beamformer* (FIB).

Rather than applying the numerical technique of multi-parameter optimisation to solve for the beamformer parameters, we attempt to address the fundamental nature of the FIB problem. The general philosophy is to use a theoretical continuous sensor to derive relationships between a desired FI beampattern and the required signal processing structure. Beamforming using an array of discrete sensors can then be formulated as an approximation problem. This approach reveals a natural structure to the FIB which is otherwise buried in a numerical optimisation procedure.

Measured results from a microphone array are presented to verify that the simple FIB structure can be successfully implemented. We then consider imposing broadband pattern nulls in the FI beampattern, and show that (i) it is possible to impose an exact null which is present over all frequencies, and (ii) it is possible to calculate *a priori* how many constraints are required to achieve a null of a given depth in a FIB. We also show that the FIB can be applied to the problem of broadband direction of arrival (DOA) estimation and provides computational advantages over other broadband DOA estimators.

Through the theoretical continuous sensor approach, we show that the FIB theory can be generalised to the problem of designing a *general broadband beamformer* (GBB) which realizes a broadband angle-versus-frequency beampattern specification. Coupled with a technique for radial beampattern transformation, the GBB can be applied to a wide class of problems covering both *nearfield* beamforming (in which the shape of the impinging wavefront must be considered) and *farfield* beamforming (which is simplified by the assumption of planar wavefronts) for a broadband beampattern specified over both angle and frequency.

Acknowledgements

The work presented in this thesis would not have been possible without the support of the following people and organisations, and they are gratefully acknowledged below:

My supervisors Dr Rod Kennedy and Dr Bob Williamson, for their guidance, insight, and enthusiasm, and individually for their unique wit.

Dr Peter Kootsookos, for his many suggestions on this work, and for helping me make annoying noises in public places in order to test the microphone array.

My advisor Dr Iven Mareels, for his encouragement and advice, and for his valuable comments at my mid-term review.

The Cooperative Research Centre for Robust and Adaptive Systems for funding a Graduate Student Assistantship; the Commonwealth Government for an Australian Postgraduate Award; and the Australian Research Council for financial support for laboratory equipment.

The Head of Department, Prof. Darrell Williamson, for the use of departmental facilities in the production of this thesis.

My fellow students in the Graduate Program in Engineering, for their friendship and for providing me with a library.

My family, for all they have given me in terms of education, encouragement, and moral and financial support.

And finally Belle, for her companionship, her unwaivering support, for understanding when I had to spend so much time working and so little time with her, and for her cute smile.

Contents

Glossary of Definitions

Notation

$\mathbb{C}$	complex plain
$\mathbb{R}$	real numbers
$\mathbb{R}^+$	non-zero real numbers
$\mathbb{Z}$	integers
$\mathbb{N}$	natural numbers
a^*	complex conjugate of scalar a
$\mathbf{a}^T$	transpose of matrix or vector $\mathbf{a}$
$\mathbf{a}^H$	conjugate transpose of matrix or vector $\mathbf{a}$
$\mathbf{A}^\dagger$	matrix pseudo-inverse: $\mathbf{A}^\dagger \triangleq [\mathbf{A}^H\mathbf{A}]^{-1}\mathbf{A}^H$
$\otimes$	Kronecker product: $\mathbf{a} \otimes \mathbf{b} \triangleq [a_1\mathbf{b} \cdots a_N\mathbf{b}]$, where N is length of $\mathbf{a}$
$\star$	convolution: if $z[k] = x[k] \star y[k]$, then $z[k] \triangleq \sum_{l=0}^{k} x[k-l]y[l]$
$\delta[\cdot]$	Kronecker delta: $\delta[k] \triangleq 1, k = 0$, and $\delta[k] \triangleq 0, k \neq 0$
$E\{\cdot\}$	expectation operator
$\mathcal{F}\{\cdot\}$	Fourier transform operator
$\mathrm{Re}\{\cdot\}$	real part
$\mathrm{Im}\{\cdot\}$	imaginary part

Abbreviations

CFVB	Controlled Frequency Variant Beamformer
CSS	Coherent Signal Subspace
DOA	Direction of Arrival
FI	Frequency Invariant
FIB	Frequency Invariant Beamformer
GBB	General Broadband Beamformer
LCMV	Linear Constrained Minimum Variance