

Expressions of Inner Freedom

An Experimental Study
of the Scattering and Fusion of Nuclei
at Energies Spanning the Coulomb Barrier

by

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Meinen Eltern,

für ihre Liebe und Unterstützung

“It is probably true quite generally that in the history of human thinking, the most fruitful developments frequently take place at those points where two different lines of thought meet.”

Werner Heisenberg

Abstract

This study investigates the fusion and scattering of nuclei at energies spanning the Coulomb barrier. The coupling of the relative motion of the nuclei to internal degrees of freedom can be thought to give rise to a distribution of potential barriers.

Two new methods to extract representations of these potential barrier distributions are suggested using the eigen-channel model. The new techniques are based on measurements of quasi-elastic and elastic backscattering excitation functions, from which the representations are extracted by differentiation. A third method utilizing transfer excitation functions is introduced using qualitative arguments. The techniques are investigated experimentally for the reactions $^{16}\text{O} + ^{92}\text{Zr}$, $^{144,154}\text{Sm}$, ^{186}W and ^{208}Pb . The results are compared with barrier distribution representations obtained from fusion data. The methods are further explored using the systems $^{40}\text{Ca} + ^{90,96}\text{Zr}$ and $^{32}\text{S} + ^{208}\text{Pb}$, for which scattering and fusion excitation functions have been measured. The new barrier distribution representations are consistent with the one from fusion. They are direct evidence of the effects of the internal degrees of freedom on channels other than the fusion channel.

The new representations are, however, less sensitive to the barrier distribution compared to their fusion counterpart. This observation is investigated using coupled-channels calculations. They suggest that residual weak reaction channels, which are not included in the coupling matrix, are responsible for the reduction in sensitivity. In the case of quasi-elastic scattering a distortion of the barrier structure above the average barrier is observed. This effect appears to be due to the de-phasing of the scattering amplitudes contributing to each eigen-channel. Using the heaviest system, $^{32}\text{S} + ^{208}\text{Pb}$, it is demonstrated that there is no improvement in sensitivity to the barrier distribution for systems with large Sommerfeld parameters. This suggests that diffraction effects are not likely to be the cause of the sensitivity reduction.

The new techniques may be employed successfully in systems with pronounced

barrier structure below the average barrier. This is the case for the reactions $^{40}\text{Ca} + ^{90,96}\text{Zr}$. It is shown that for these systems the quasi-elastic scattering and the fusion representations of the barrier distribution contain the same information. The extracted barrier distributions for the two reactions are distinctively different. They are compared to assess the relative importance of collective excitations and neutron transfer in fusion. Exact coupled-channels calculations show that the distribution for $^{40}\text{Ca} + ^{90}\text{Zr}$ arises from coupling of the relative motion to double phonon excitations of ^{90}Zr . Further calculations suggest that the reaction $^{40}\text{Ca} + ^{96}\text{Zr}$ involves additional coupling to sequential neutron transfer, which is proposed to be a precursor of neutron-neck formation.

Double phonon excitations are also seen to be important in the system $^{32}\text{S} + ^{208}\text{Pb}$, for which the barrier distribution representations show in addition signatures of one and two neutron transfer.

A Work Justified by Curiosity

A project as elaborate as that described in this thesis requires a motivation and in these days, which are ruled by the shortage of both time and money, it is also expected to have a justification. Unfortunately, the argument that science is the educated expression of our curiosity about the world surrounding us, does not always find recognition anymore. Increasingly, the term *wealth creation* is used in connection with research as an allegedly new aspect of it. The advocates of this fashionable slogan seem to forget that the overwhelming part of the materialistic wealth in modern society is based on one or another scientific work of the past¹.

It appears necessary to recall that most of the revolutionary discoveries were not the result of a clever interplay between financial investment and application-oriented technology development. On the contrary, they arose often from an uncoordinated research community of little-recognised individuals, who were motivated by enthusiasm and curiosity. The lack of recognition and interest by the majority of their contemporaries was not necessarily a disadvantage, since it provided these few with the freedom to explore aspects of nature which were often obscure to nearly everyone else at the time². A second, no less important reason for their success was the free exchange of ideas within the research community which was rarely hindered by economical or military interests.

Confining modern scientists to a few research tracks chosen by the political process and transferring those to industry puts in jeopardy both intellectual freedom and the unrestricted exchange of ideas. This will eventually harm the scientific and technological advancement and as a consequence may even lead to the opposite of wealth creation, namely *wealth reduction*.

The work presented in this thesis is a contribution to our advancement in un-

¹Physics and Industry, Proc. Acad. Sess., XXI Ge. Ass. IUPAP, ed. E. Maruyama, H. Watanabe, Lec. Not. Phys. 435, Springer (1993).

²U. Wengenroth, *Historische Aspekte des Forschungs- und Innovationsprozesses*, in *Von der Hypothese zum Produkt*, Stifterverband für die Deutsche Wissenschaft (1994).

derstanding nature and in particular the atomic nucleus and its constituents. It was driven by enthusiasm and curiosity and explores new grounds which have never been sighted before. This is its sole justification. The project brings together two aspects of nuclear physics which have often been pursued as two different lines of thought: *nuclear structure* and *nuclear reactions*.

The experimental results demonstrate that nuclear reactions at Coulomb barrier energies sensitively depend on the internal structure of the participating nuclei. These reactions manifest an important special case of the generalized barrier problem, which attempts to describe the motion of a particle coupled to a many-particle environment over a potential barrier. The coupling can hinder or assist this motion depending on the properties of the environment. Nuclear reactions are unique in nature in enabling a comparison of the effects on the barrier problem of different environmental couplings within the same physical system. Thus their study benefits our general understanding of the barrier problem, which is of fundamental importance in nature.

The larger part of the experiments were carried out in collaboration with Dr. J.R. Leigh, Dr. D.J. Hinde, Dr. M. Dasgupta, Prof. J.O. Newton, Dr. C.R. Morton, Mr. J.C. Mein and Dr. R.C. Lemmon at the Nuclear Physics Department of the Research School of Physical Sciences and Engineering of the Australian National University in Canberra. The nuclear reactions $^{40}\text{Ca} + ^{90,96}\text{Zr}$ were studied in collaboration with Prof. A.M. Stefanini, Dr. L. Corradi, Dr. D. Ackermann, Dr. S. Beghini, Dr. J.H. He, Dr. G. Montagnoli, Dr. F. Scarlassara and Dr. G.F. Segato at the Legnaro Laboratories of the National Italian Nuclear Physics Institute in Padua, Italy. The experiments were set up using existing and modified equipment. The author contributed to the construction of the fission fragment spectrometer which was employed in the measurement of the fusion excitation function for $^{32}\text{S} + ^{208}\text{Pb}$. Some theoretical aspects of this thesis have been developed together with Dr. N. Rowley of the University of Manchester, England.

The work presented in this thesis has been or will be published in the following papers:

- *Probing Fusion Barrier Distributions with Quasi-Elastic Scattering*, H. Timmers, M. Dasgupta, D.J. Hinde, J.R. Leigh, R.C. Lemmon, J.C. Mein, C.R. Morton, J.O. Newton, N. Rowley, Nuclear Physics A **584** (1994) 190.

- *Nuclear Multi-Particle Systems in Changing Environments*,
H. Timmers, Australian & New Zealand Physicist **32**, No. 3 (1995) 39.
- *Barrier Distributions from Elastic Scattering*,
N. Rowley, H. Timmers, J.R. Leigh, M. Dasgupta, D.J. Hinde, J.C. Mein,
C.R. Morton, J.O. Newton, Physics Letters B **373** (1996) 23.
- *Strong Transfer Couplings in the Fusion of $^{40}\text{Ca} + ^{96}\text{Zr}$* ,
H. Timmers, L. Corradi, A.M. Stefanini, N. Rowley, D. Ackermann, S. Beghini,
J.H. He, G. Montagnoli, F. Scarlassara, G.F. Segato, to be submitted to
Physics Letters B.
- *A Case Study of Collectivity, Transfer and Fusion Enhancement*,
H. Timmers, L. Corradi, A.M. Stefanini, N. Rowley, D. Ackermann, S. Beghini,
J.H. He, G. Montagnoli, F. Scarlassara, G.F. Segato, to be submitted to
Nuclear Physics A.

The author has received the *Award for Excellence in Postgraduate Research 1994* by the New South Wales branch of the Australian Institute of Physics following a lecture about this project at the University of Sydney.

The results of this work have also been reported in seminars at
 the Congress of the Association of Asia Pacific Physical Societies, Brisbane,
 the Gesellschaft für Schwerionenforschung, Darmstadt,
 the Legnaro National Laboratories, Padua,
 the Nuclear Science Centre, New Delhi,
 the National Superconducting Cyclotron Laboratory, East Lansing,
 the Argonne National Laboratory, Chicago
 and the Nuclear Physics Laboratory of the University of Washington, Seattle.

The contents of this thesis are the original work of the author. No part of this thesis has been submitted for a degree to another university.

The thesis is structured as follows:

In Chapter 1, *Introduction*, the generalized barrier problem is presented. This is followed by a description of the fusion and scattering of the nuclear binary system at Coulomb barrier energies. At the end of the chapter the objectives of this work are stated. Chapter 2, *Experimental Methods*, illustrates the experimental techniques which have been applied. In Chapter 3, *Probing Barrier Distributions with Quasi-Elastic Scattering*, a new method to extract information about the distribution of potential barriers from quasi-elastic scattering is introduced and tested. An alternative approach based on elastic scattering is derived and discussed in Chapter 4, *Barrier Distributions from Elastic Scattering*. In Chapter 5, *Collectivity, Transfer and Fusion Enhancement*, the relative importance of collective excitations and of neutron transfer for fusion is investigated for the reactions $^{40}\text{Ca} + ^{90,96}\text{Zr}$. This investigation includes the search for signatures of neutron-flow. Chapter 6, *The Fusion of $^{32}\text{S} + ^{208}\text{Pb}$* , discusses the importance of double phonon excitations and one and two neutron transfer reactions for the fusion of this system. Finally, the results of this work are summarized in Chapter 7, *Summary and Conclusions*.

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List of Symbols

a_0	diffuseness of the real potential
a_{tr}	diffuseness of the transfer coupling strength
a_w	diffuseness of the imaginary potential
A_c	combined mass number of the system
A_p	atomic mass number of the projectile nucleus
A_t	atomic mass number of the target nucleus
α_i	generalized environmental coordinates
b_k	eigen-value of eigen-channel k
B_0	barrier height in the one-dimensional model
B_{Bass}	barrier height predicted by the global model of Bass
B_k	barrier height in eigen-channel k
$\beta_{ji}, \beta_\lambda$	deformation parameter
B	magnetic field strength
c	velocity of light
$d\sigma^{el}$	differential elastic scattering cross section
$d\sigma^{qel}$	differential quasi-elastic scattering cross section
$d\sigma^R$	differential Rutherford scattering cross section
$d\sigma^{trans}$	differential transfer cross section
$d\Omega$	differential solid angle
D_∞	asymptotic barrier
$D(E, B_k)$	distribution of potential barriers
$D^{el}(E)$	representation of $D(E, B_k)$ extracted from elastic scattering
$D^{fus}(E)$	representation of $D(E, B_k)$ extracted from fusion
$D^{qel}(E)$	representation of $D(E, B_k)$ extracted from quasi-elastic scattering
$D^{trans}(E)$	representation of $D(E, B_k)$ extracted from transfer
δ	relative experimental uncertainty

δ_{ji}	Kronecker symbol
ΔB	FWHM of a single peak in $D^{fus, qel, el}(E)$
ΔE	energy step length
e	electron charge
E	energy
E_{cm}	energy in the centre-of-mass system
E_{lab}	energy in the laboratory system
E_{res}	residual energy of a reaction product
ϵ_i	excitation energies of internal degrees of freedom
\mathcal{E}	electric field strength
$f_i(\tau)$	coupling form factor
$f_j(E, \theta)$	scattering amplitude in channel j
$f^{qel}(E, \theta)$	quasi-elastic scattering amplitude
$f^R(E, \theta)$	Rutherford scattering amplitude
F	Fano factor
F_{ji}, F_0	transfer coupling strength
$G^{el}(E, B_k)$	functional form of a single peak in $D^{el}(E)$
$G^{fus}(E, B_k)$	functional form of a single peak in $D^{fus}(E)$
$G^{qel}(E, B_k)$	functional form of a single peak in $D^{qel}(E)$
$G^{trans}(E, B_k)$	functional form of a single peak in $D^{trans}(E)$
$\gamma(\tau)$	friction form factor
$h_i(\alpha_i, \Pi_i)$	environmental Hamiltonians
\hbar	Planck's constant divided by 2π
$\hbar\omega_0$	barrier curvature for $\ell = 0$
H	Hamiltonian
i	labels environmental degrees of freedom; also $\sqrt{-1}$
I	current of incident nuclei
j	labels reaction channels
J	total angular momentum quantum number
k	labels eigen-channels; also wave number
ℓ	angular momentum quantum number
$\ell_{gr}\hbar$	grazing angular momentum
λ	reduced de Broglie wave length
λ_i	coupling constants

λ_{ji}	multipolarity of collective transitions
m	mass
m_N	nucleon mass
M_{ji}	element of coupling matrix
μ	reduced mass
n	Sommerfeld parameter
p	momentum; also pattern parameter
$\phi_j(r)$	radial wave functions in the reaction channels
Π_i	generalized environmental momenta
$\Psi(r, \alpha_i)$	total radial wave function
q	charge state
Q_j	Q -value in reaction channel j
r	inter-nuclear separation
r_0	radius parameter
R_0	barrier position in the one-dimensional model
R_C	Coulomb radius
R_n	radius of the nuclear potential
$\mathcal{R}(E)$	reflection coefficient
S_j^l	element of scattering matrix
σ^{fus}	fusion cross section
σ_j^{reac}	reaction cross section for channel j
σ^{tot}	total reaction cross section
t	time
T	temperature; also kinetic energy operator
$T(E)$	transmission coefficient
θ	scattering angle
θ_{cm}	scattering angle in the centre-of-mass system
θ_{lab}	scattering angle in the laboratory system
U	voltage
U_{jk}	unitary transformation matrix
$U(r)$	complex interaction potential
$v_i^{coup}(r, \alpha_i)$	coupling potentials
V_0	depth of the real potential
$V(r)$	interaction potential

$V_{cent}(r, \ell)$	centrifugal potential
$V_C(r)$	Coulomb potential
$V_n(r)$	nuclear potential
W_0	depth of the imaginary potential
W_k	weights of the eigen-barriers
$W(r)$	volume imaginary potential
x_e	effective fissility
$\chi_i(\alpha_i)$	internal wave functions
$Y_k(r)$	eigen-channel wave functions
Z_c	combined nuclear charge number of the system
Z_p	nuclear charge number of the projectile nucleus
Z_t	nuclear charge number of the target nucleus

Units are expressed following the recommendations from 1975 of The Royal Society.

