

Advanced Interferometry for Gravitational Wave Detection

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To

Jan, Ann and Gran.

Declaration

This thesis is an account of research undertaken between March 1997 and September 2000 at *the Department of Physics, Faculty of Science, Australian National University, Canberra, Australia*.

Except where acknowledged in the customary manner, the material presented in this thesis is, to the best of my knowledge, original and has not been submitted in whole or part for a degree in any university.

Daniel Anthony Shaddock
November, 2000

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Abstract

In this thesis we investigate advanced techniques for the readout and control of various interferometers. In particular, we present experimental investigations of interferometer configurations and control techniques to be used in second generation interferometric gravitational wave detectors. We also present a new technique, tilt locking, for the readout and control of optical interferometers.

We report the first experimental demonstration of a Sagnac interferometer with resonant sideband extraction (RSE). We measure the frequency response to modulation of the length of the arms and demonstrate an increase in signal bandwidth of by a factor of 6.5 compared to the Sagnac with arm cavities only. We compare Sagnac interferometers based on optical cavities with cavity-based Michelson interferometers and find that the Sagnac configuration has little overall advantage in a cavity-based system.

A system for the control and signal extraction of a power recycled Michelson interferometer with RSE is presented. This control system employs a frontal modulation scheme requiring a phase modulated carrier field and a phase modulated subcarrier field. The system is capable of locking all 5 length degrees of freedom and allows the signal cavity to be detuned over the entire range of possibilities, in principle, whilst maintaining lock. We analytically investigate the modulation/demodulation techniques used to obtain these error signals, presenting an introductory explanation of single sideband modulation/demodulation and double demodulation.

This control system is implemented on a benchtop prototype interferometer. We discuss technical problems associated with production of the input beam modulation components and present several solutions. Operation of the interferometer is demonstrated for a wide range of detunings. The frequency response of the interferometer is measured for various detuned points and we observe good agreement with theoretical predictions. The ability of the control system to maintain lock as the interferometer is detuned is experimentally demonstrated.

Tilt locking, a new technique to obtain an error signal to lock a laser to an optical cavity, is presented. This technique produces an error signal by efficient measurement of the interference between the TEM_{00} and TEM_{10} modes. We perform experimental and theoretical comparisons with the widely used Pound-Drever-Hall (PDH) technique. We derive the quantum noise limit to the sensitivity of a measurement of the beam position, and using this result calculate the shot noise limited sensitivity of tilt locking. We show that tilt locking has a quantum efficiency of 80%, compared to 82% for the PDH technique. We present experimental demonstrations of tilt locking in several applications including frequency stabilisation, continuous-wave second harmonic generation, and injection locking of a Nd:YAG slab laser. In each of these cases, we demonstrate that the performance of tilt locking is not the limiting factor of the lock stability, and show that it achieves similar performance to the PDH based system.

Finally, we discuss how tilt locking can be effectively applied to two beam interferometers. We show experimentally how a two beam interferometer typically gives excellent isolation against errors arising from changes in the photodetector position, and exper-

imentally demonstrate the use of tilt locking as a signal readout system for a Sagnac interferometer.

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