

CURVE ESTIMATION
AND SIGNAL DISCRIMINATION
IN SPATIAL PROBLEMS

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Declaration

Unless otherwise specified in the text, this thesis describes my own work, carried out under the supervision of Professor Peter Hall, who is credited for the formulation of Theorems 2.1, 3.1, 4.1 and 4.2, as well as a major part of the theoretical arguments in Sections 2.4, 3.5 and 4.3. For those parts of the above proofs, and other remarks, which draw on the material in the Appendix, I estimate my contribution at no less than 75 per cent on average. I also acknowledge the contributions of Professors Don Poskitt (Monash University, Melbourne) and Brett Presnell (University of Florida), whose joint work with Peter Hall formed the basis for Subsection A.5.3. A suite of MATLAB[®] code authored by Brett Presnell enabled me to tackle the dataset of Chapter 6 much more quickly than would otherwise have been possible, and a number of discussions with the previously mentioned individuals influenced the exposition in that chapter. The dataset used there was made available to me by Dr Danny Gibbins and Professor Doug Gray (Cooperative Research Centre for Sensor Signal and Information Processing, Adelaide), who helped me a great deal to understand the nature of the data and its inherent problems during three visits and through other communication.

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Abstract

In many instances arising prominently, but not exclusively, in imaging problems, it is important to condense the salient information so as to obtain a low-dimensional approximant of the data. This thesis is concerned with two basic situations which call for such a dimension reduction. The first of these is the statistical recovery of smooth edges in regression and density surfaces. The edges are understood to be contiguous curves, although they are allowed to meander almost arbitrarily through the plane, and may even split at a finite number of points to yield an edge graph. A novel locally-parametric nonparametric method is proposed which enjoys the benefit of being relatively easy to implement via a ‘tracking’ approach. These topics are discussed in Chapters 2 and 3, with pertaining background material being given in the Appendix. In Chapter 4 we construct concomitant confidence bands for this estimator, which have asymptotically correct coverage probability. The construction can be likened to only a few existing approaches, and may thus be considered as our main contribution.

Chapter 5 discusses numerical issues pertaining to the edge and confidence band estimators of Chapters 2–4. Connections are drawn to popular topics which originated in the fields of computer vision and signal processing, and which surround edge detection. These connections are exploited so as to obtain greater robustness of the likelihood estimator, such as with the presence of sharp corners.

Chapter 6 addresses a dimension reduction problem for spatial data where the ultimate objective of the analysis is the discrimination of these data into one of a few pre-specified groups. In the dimension reduction step, an instrumental role is played by the recently developed methodology of functional data analysis. Relatively standard non-linear image processing techniques, as well as wavelet shrinkage, are used prior to this step. A case study for remotely-sensed navigation radar data exemplifies the methodology of Chapter 6.

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