OPERATIONAL ASPECTS OF DECISION FEEDBACK EQUALIZERS

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Declaration

The contents of this thesis are the result of original research and have not been submitted for a higher degree to any other university or institution.

Much of the work in this thesis has been published or has been submitted for publication as journal papers. These papers are:

- R.A. Kennedy, and B.D.O. Anderson, "Error Recovery of Decision Feedback Equalizers on Exponential Impulse Response Channels," *IEEE Trans. on Communications,* vol.COM-35, pp.846-848, August 1987.
- R.A. Kennedy, and B.D.O. Anderson, "Recovery Times of Decision Feedback Equalizers on Noiseless Channels," *IEEE Trans. on Communications,* vol.COM-35, pp.1012-1021, October 1987.
- R.A. Kennedy, B.D.O. Anderson, and R.R. Bitmead, "Tight Bounds on the Error Probabilities of Decision Feedback Equalizers ," *IEEE Trans. on Communications,* vol.COM-35, pp.1022-1029, October 1987.
- R.A. Kennedy, G. Pulford, B.D.O. Anderson, and R.R. Bitmead, "When has A Decision-Directed Equalizer Converged?," *IEEE Trans. on Communications,* (accepted for publication).
- R.A. Kennedy, B.D.O. Anderson, and R.R. Bitmead, "Channels Leading to Rapid DFE Error Recovery," *IEEE Trans. on Communications,* (accepted for publication).
- R.A. Kennedy, B.D.O. Anderson, and R.R. Bitmead, "Blind Adaptation of Decision Feedback Equalizers: Gross Convergence Properties," *International Journal of Adaptive Control and Signal Processing,* (submitted for publication).

Papers published in Conference Proceedings, but in some cases containing material overlapping with the above are:

- B.D.O. Anderson, R.A. Kennedy, and R.R. Bitmead, "Decision Feedback Equalizers: Concepts Towards Design Guidelines," *Proc. ISSPA87,* pp.1-7, Brisbane Australia, August 1987.
- R.A. Kennedy, B.D.O. Anderson, and R.R. Bitmead, "Stochastic Analysis of Non-Adaptive Decision Feedback Equalizers," *Proc. ISSPA87,* pp.842-847, Brisbane Australia, August 1987.

- R.A. Kennedy, B.D.O. Anderson, and R.R. Bitmead, "Channels Leading to Rapid DFE Error Recovery: Passivity Analysis," *Proc. 27th IEEE Conf. on Decision and Control,* Austin, Texas, December 1988 (to appear).
- R.A. Kennedy, B.D.O. Anderson, and R.R. Bitmead, "Stochastic Dynamics of Blind Decision Feedback Equalizer Adaptation," *Proc. ACASP'89,* Glasgow, Scotland, April 1989 (to appear).
- R.A. Kennedy, "Some Difficulties with Blind Regressor Based Channel Equalizers in Digital Communications," *Proc. 1989 Australian Symposium on Signal Processing and Applications,* Adelaide, Australia, April 1989 (to appear).

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Abstract

The central theme is the study of *error propagation* effects in decision feedback equalizers (DFEs). The thesis contains: a *stochastic analysis* of error propagation in a tuned DFE; an analysis of the effects of error propagation in a *blindly adapted* DFE; a *deterministic analysis* of error propagation through input-output stability ideas; and *testing procedures* for establishing correct tap convergence in blind adaptation. To a lesser extent, the decision directed equalizer (DDE) is also treated.

Characterizing error propagation using finite state Markov process (FSMP) techniques is first considered. We classify how the channel and DFE parameters affect the FSMP model and establish tight bounds on the error probability and mean error recovery time of a tuned DFE. These bounds are shown to be too conservative for practical use and highlight the need for imposing stronger hypotheses on the class of channels for which a DFE may be effectively used.

In *blind* DFE adaptation we show the effect of decision errors is to distort the adaptation relative to the use of a *training sequence.* The mean square error surface in a LMS type setting is shown to be a concatenation of quadratic functions exposing the possibility of false tap convergence to undesirable DFE parameter settings. Averaging analysis and simulation are used to verify this behaviour on some examples.

Error propagation in a tuned DFE is also examined in a *deterministic* setting. A finite error recovery time problem is set up as an input-output stability problem. Passivity theory is invoked to prove that a DFE can be effectively used on a channel satisfying a simple frequency domain condition. These results give performance bounds which relate well with practice.

Testing for false tap convergence in blind adaptation concludes our study. Simple statistic *output tests* are shown to be capable of discerning correct operation of a DDE. Similar tests are conjectured for the DFE, supported by proofs for the low dimensional cases.

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Glossary of Definitions

DFE	decision feedback equalizer
DDE	decision directed equalizer
FIR	finite impulse response
IIR	infinite impulse response
FSMP	finite state Markov process
$\mathbb{C}$	complex plane
$\mathbb{R}$	real numbers
$\mathbb{R}^{N\times P}$	real $N \times P$ matrices
$\mathbb{Z}_+$	natural numbers
$\mathbb{Z}$	integers
$\boldsymbol{A}'$	transpose of real matrix $\boldsymbol{A}$
q^{-1}	delay operator: $q^{-1}x_k \triangleq x_{k-1}$
$\widetilde{g}(z)$	Z-transform: If $g \triangleq \{g_0,\ g_1,\ g_2,\ \ldots\}$ then $\widetilde{g}(z) \triangleq \sum_{i=0}^{\infty} g_i z^{-i}$
$\operatorname{sgn}(\cdot)$	signum function: $\operatorname{sgn}(x) \triangleq 0$ for $x = 0$, $\operatorname{sgn}(x) \triangleq x/\lvert x\rvert$ for $x \neq 0$.
$\mathcal{M}$	M-ary alphabet: $\mathcal{M} \triangleq \{1-M,\ 3-M,\ \ldots,\ M-1\}$, $M \in \{2,\ 4,\ 6,\ \ldots\}$
$\mathcal{Q}_M(\cdot)$	M-ary quantizer: $\mathcal{Q}_M(x) \triangleq \sum_{k=1-M/2}^{M/2-1} \operatorname{sgn}(x+2k) \in \mathcal{M}$
$E\{\cdot\}$	expectation operator
$Pr(\cdot)$	probability function
$Re(\cdot)$	real part
$Im(\cdot)$	imaginary part
l_p	l_p-space: $x \triangleq \{x_0,\ x_1,\ x_2,\ \ldots\} \in l_p$ if $\sum_{i=0}^{\infty} \lvert x_i\rvert^p < \infty,\ \ p \geq 1$.
l_p^e	extended l_p-space: $x \in l_p^e$ if $\sum_{i=0}^{k} \lvert x_i\rvert^p < \infty,\ \forall k < \infty,\ \ p \geq 1$.
$\lVert \cdot \rVert$	l_2-norm: $\lVert x\rVert \triangleq \sqrt{\sum_{i=0}^{\infty} \lvert x_i\rvert^2}$
$\lVert \cdot \rVert_1$	l_1-norm: $\lVert x\rVert_1 \triangleq \sum_{i=0}^{\infty} \lvert x_i\rvert$
$\langle \cdot,\cdot \rangle$	real inner product: $\langle x,y\rangle \triangleq \sum_{i=0}^{\infty} x_i y_i$
$\langle \cdot,\cdot \rangle_T$	truncated real inner product: $\langle x,y\rangle_T \triangleq \sum_{i=0}^{T} x_i y_i$
$\circledast$	convolution: If $z = x \circledast y$ then $z_k \triangleq \sum_{i=0}^{k} x_{k-i} y_i$.
$O(\cdot)$	$f(x) = O\big(g(x)\big)$ as $x \to l$ if $\lim_{x\to l} \{f(x)/g(x)\} \to k$.
$o(\cdot)$	$f(x) = o\big(g(x)\big)$ as $x \to l$ if $\lim_{x\to l} \{f(x)/g(x)\} \to 0$.