Long-Range Dependence of Markov Processes

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To Lolo and Lola

Declaration

Unless otherwise stated in the thesis, this study is my own work done under the supervision of Prof. Daryl J. Daley. The results 2.2.4, 2.2.6, 2.2.7 and 2.2.8 come from a manuscript written jointly with him, Lemmas 4.1.1 and 4.1.2 in their present form are due to him and so is finding the order of the funtion $\rho(t)$ in (3.14).

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Abstract

Long-range dependence in discrete and continuous time Markov chains over a countable state space is defined via embedded renewal processes brought about by visits to a fixed state. In the discrete time chain, solidarity properties are obtained and long-range dependence of functionals are examined. On the other hand, the study of LRD of continuous time chains is defined via the number of visits in a given time interval. Long-range dependence of Markov chains over a non-countable state space is also carried out through positive Harris chains. Embedded renewal processes in these chains exist via visits to sets of states called proper atoms.

Examples of these chains are presented, with particular attention given to long-range dependent Markov chains in single-server queues, namely, the waiting times of GI/G/1 queues and queue lengths at departure epochs in M/G/1 queues. The presence of long-range dependence in these processes is dependent on the moment index of the lifetime distribution of the service times. The Hurst indexes are obtained under certain conditions on the distribution function of the service times and the structure of the correlations. These processes of waiting times and queue sizes are also examined in a range of M/P/2 queues via simulation (here, P denotes a Pareto distribution).

Contents

Notation and Terminology

Mathematical objects and operations

$\mathbb{R}$	real numbers, 2
$\mathbb{Z}$	integers, 2
$\mathbb{N}$	natural numbers or non-negative integers, 2
H	Hurst index, 6
d	strength of long-memory in FARIMA processes, 8
$I_N(\cdot)$	periodogram of a dataset of length N, 9
$\overline{F}(x)$	tail of the distribution function $F(x)$, 10
κ	moment index, 10
$N(A)$	number of points in a Borel set A, 11
$N(0,t]$	number of renewals in $(0,t]$, 11
$\mathcal{X}$	state space, 12
$*$	convolution operator, 18
$\mathbf{1}_C(\cdot)$	indicator function of the set C, 21
$(E,\mathcal{E})$	measurable state space, 31
$\mathcal{E}_+$	set of non-negative measurable functions on $(E,\mathcal{E})$, 32
$\mathcal{M}_+$	set of measures on $(E,\mathcal{E})$, 34
$\mathcal{M}^+$	set of positive measures on $(E,\mathcal{E})$, 34

Discrete time Markov chain notation

p_{ij}^n	n-step transition probability, 19
T_{ij}	return time from i to j, 19
m_{ij}	$\mathbf{E}(T_{ij})$ = first moment of T_{ij}, 19
$m_{ij}^{(n)}$	nth moment of T_{ij}, 19
$\{\pi_i\}$	stationary distribution, 20
${}_H p_{ij}^n$	taboo probability, 20
f_{ij}^n	first passage probability = ${}_j p_{ij}^n$, 19
$N_i(0,n]$	number of visits to i in $(0,n]$, 20
I_{ni}	$\mathbf{1}_{\{i\}}(X_n)$, 21
Q_{ij}^n	$\sum_{r=1}^n (p_{ij}^r - \pi_j)$, 23
R_{ij}^n	$\sum_{r=1}^n Q_{ij}^r$, 23
I_{nA}	$\mathbf{1}_A(X_n)$, 26

Harris chain notation

$P^n(x,A)$	n-step transition probability, 31
$\mathbf{P}_x$	probability conditional on $\{X_0 = x\}$, 32
$\mathbf{E}_x$	expectation conditional on $\{X_0 = x\}$, 32
τ_A	return time to the set A, 32
φ	irreducibility measure, 32
ψ	maximal irreducibility measure, 32
$N(A)$	occupation time on A, 32
$\mathcal{E}_+$	set of non-negative measurable functions on $(E,\mathcal{E})$, 32
π	invariant measure, 33

α	proper atom, 34
$\{X_{mn}\}_{n\in\mathbb{N}}$	m-skeleton or m-step chain, 35
$\{(X_n, Y_n)\}$	split chain, 36
λ^*	split measure on $\mathcal{E}$, 36
$\widetilde{P}(x_i, \cdot)$	split transition probability, 36
$N_\alpha(0, n]$	number of visits to α in $(0, n]$, 38
$T_{\alpha\alpha}$	return time from α to α, 38
${}_AP^n(x, B)$	taboo probability, 39

Continuous time Markov chain notation

$\{\nu_i\}$	stationary distribution of the jump chain, 12
$M_{ij}(t)$	expected number of visits to j in $[0, t]$ starting at i, 12
λ_j	rate of visits to state j, 12
$M(t)$	$\sum_{i\in\mathcal{X}} \sum_{j\in\mathcal{X}} \nu_i M_{ij}(t)$, 12
λ	$\sum_{j\in\mathcal{X}} \lambda_j$, 12
$p_{ij}(t)$	transition probability, 46
q_i	rate at which the chain departs from i, 46
$q_{ij}, i \neq j$	rate at which the chain enters j from i, 46
$\tau_j(n)$	nth return time on j, 48
$\tau'_j(n)$	nth exit time from j, 48
$\mathbf{P}_i$	probability conditional on $\{X_0 = i\}$, 48
$\mathbf{E}_i$	expectation conditional on $\{X_0 = i\}$, 48
$N_j(t)$	number of visits to j in $(0, t]$, 48

Queueing notation

$\{T_n\}$	sequence of interarrival times, 12
$\{S_n\}$	sequence of service times, 12
$\{W_n\}$	sequence of waiting times, 12
$\{D_n\}$	sequence of interdeparture times, 12
ρ_n	correlation coefficient of $\{W_n\}$, 60
$\{Q_n\}$	sequence of queue sizes, 29
r_n	correlation coefficient of $\{Q_n\}$, 67
$T_i = T_{i0}$	first time the queue size hits 0 provided $Q_0 = i$, 69
$\boldsymbol{W}_n$	workload vector, 76

Acronyms and Terminology

LRD	long-range dependence, 1
LRD(ACF)	definition of LRD via the autocorrelation function, 2
LRD(SD)	definition of LRD via the spectral density function, 2
LRD(AV)	definition of LRD via the Allen variance, 2
SαS	Symmetric α-stable, 4
H-sssi	self-similar process with stationary increments and index H, 7
FBM	fractional Brownian motion, 7
FGN	fractional Gaussian noise, 6
FARIMA	fractional autoregressive integrated moving average, 6
i.i.d.	independent identically distributed, 8
LRiD	long-range interval dependence, 11

LRcD	long-range count dependence, 11
MSE	mean square error, 14
PDF	probability distribution function, 54
FIFO	first in first out, 57
BGT	Bingham, Goldie and Teugels, 61
DFA	detrended fluctuation analysis, 83
GPH	Geweke Porter-Hudak, 87