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Abstract

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Key Words: *the marginal social cost or public funds; the shadow value of government revenue; the marginal excess burden of taxation; tax inefficiency; cost-benefit analysis*

There is considerable interest in knowing what the value of the marginal social cost of public funds (MCF) is because it plays an important role in determining the size of government. Based on the conventional Harberger (1964) measure, which is one plus the marginal excess burden of taxation (MEB), there is a general expectation that the MCF will exceed unity whenever revenue is raised with distorting taxes.[1] And while this conjecture has considerable empirical support, there are generally large differences in estimates of the MCF, not just for the same taxes across countries, but the same taxes within countries. Evidence of this is provided in Table 1 which summarises estimates of the MCF for wage taxes in a selection of countries.[2]

Table 7.2. Estimates of the MCF for Wage Taxes

Country	Study	Estimate
United States	Ballard and Fullerton (1992)	1.047 - 1.315
	Fullerton (1991)	1 - 1.25
	Ballard, Shoven and Whalley (1985)	1.16 - 1.31
	Browning (1976,1987)	1.32 - 1.47
	Stuart (1984)	1.07 - 1.57
Canada	Dahlby (1994)	1.38
New Zealand	Diewert and Lawrence (1996)	1.18
Australia	Campbell and Bond (1997)	1.19 - 1.24
	Findlay and Jones (1982)	1.275 - 1.550

They range in value from 1 to 1.57, which means taxpayers lose, at most, $1.57 in private surplus on the last dollar of revenue the government collects. There are a number of reasons for the variations in these estimates, and while it is not the aim of this paper to explain them, it is worth noting why they occur. First, they use different structural parameters, including, different income tax schedules in each country, and different demand-supply elasticities on taxed activities. Fullerton examines the US estimates by removing differences in their modelling specifications to find that they also use different conceptual measures of the MCF. For example, Ballard, Shoven and Whalley compute the change in tax inefficiency by deducting the *actual* change in tax revenue from the cost to private surplus measured using the equivalent variation.[3] In contrast, Stuart computes it as the difference between ″compensating surplus″ and the change in actual (uncompensated) tax revenue, while Browning (1987) estimates the MEB as the compensated tax inefficiency per dollar change

[1] The MEB is computed as the tax inefficiency per dollar of revenue raised.

[2] All these studies measure the MCF for taxes on labour income in countries with progressive marginal tax rates. Most raise all marginal tax rates by the same proportion by increasing a weighted average marginal tax rate. Some studies obtain a much wider range of estimates than are reported in Table 1. Those reported here are for preferred parameter values, where such a preference is stated in the study. Browning (1987), for example, computes estimates that range from 1.10 to 4.30 but prefers the structural parameters for the estimates reported in Table 1.

[3] When computing the MEB we divide the change in tax inefficiency from marginally raising a tax by the change in tax revenue.

in *actual* tax revenue.[4] A number of papers examine these conceptual differences and argue in favour of their relative merits. In this paper we derive uncompensated and compensated measures of the MCF to show how they are defined by the equilibrium closure of the economy adopted in applied welfare analysis. Welfare measures in full equilibrium are (actual) dollar changes in utility when the government balances its budget, where the appropriate MCF is the uncompensated measure. In contrast, compensated welfare changes are changes in surplus revenue the government can collect at constant utility, where the appropriate MCF is the compensated measure. Clearly, the difference between them is due solely to income effects, which are irrelevant in single (aggregated) consumer economies. Having said that, however, the MCF must be consistent with the welfare measure adopted. In particular, the uncompensated MCF cannot be used in a compensated welfare analysis and vice versa.

Another reason for the variation in the estimates of the MCF in Table 1 is the inclusion of a spending effect identified by Diamond and Mirrlees (1971) and Stiglitz and Dasgupta (1971). When governments spend revenue to provide goods and services like defence, healthcare and education they raise the real income of consumers who change their demands for taxed goods. If tax activities expand there is a positive spending effect that makes government spending less costly to fund at the margin by partially undoing the excess burden of taxation. All the US estimates, except those made by Browning, include the spending effect in a modified measure of the MCF. But by doing so they make it project specific, where for each tax there is a separate modified MCF for every possible way the government can spend the revenue. Clearly, this is not the same measure of the MCF as the conventional Harberger measure which is independent of the way extra revenue is spent.[5] From a practical point of view it is important to know which measure of the MCF is being reported. Otherwise they may be used inappropriately in applied work. Both measures of the MCF are derived in this paper to show how they should be used in a cost-benefit analysis.

Still further differences arise when distributional effects are included in the MCF because they are based on subjectively chosen distributional weights. While they are not included in estimates reported in Table 1, they are important, particularly for setting progressive marginal tax rates on income. We do not include them in the following analysis, but examples of the MCF with distributional effects are provided in Batina (1990), Dahlby

[4] Hicks (1954) distinguishes between ″compensating surplus″ and ″compensating variation″. The former holds utility constant by transferring a numeraire good to consumers, while the latter holds utility constant with lump-sum transfers of revenue. None of these measures coincide with the compensated measures in Auerbach (1985) and Diamond and McFadden (1974) who use compensated (rather than actual) changes in tax revenue. Ultimately, the approach adopted will be dictated by the equilibrium concept employed in the analysis. In full equilibrium, which is also referred to as a balanced equilibrium because the government always balances its budget, the MEB is the actual change in tax inefficiency, measured as a dollar change in utility, divided by the actual (uncompensated) change in tax revenue. However, in a compensated equilibrium, which is used to isolate potential welfare changes, the changes in tax inefficiency and tax revenue are both measured as compensated welfare changes. In the final analysis, they must represent changes that can be supported as equilibrium outcomes, where the compensated equilibrium is what would prevail if actual compensation payments were made to consumers.

[5] The modified MCF is examined in Mayshaar (1990,1991) and Snow and Warren (1996).

(1998), Jones (2005) and Slemrod and Yitzhaki (2001).[6] Their analysis demonstrates the potentially large data requirements involved.

All these issues make it difficult to interpret and use empirical estimates of the MCF in applied work. Ideally they are used to determine the optimal level of government spending and to set Ramsey optimal taxes. Even when differences in demand-supply elasticities and other modelling specifications are taken into account we need to know whether the MCF is based on compensated or uncompensated welfare changes, whether it includes distributional effects, and whether it also includes the spending effect. Much of this information is not reported with empirical estimates of the MCF, in particular with respect to the inclusion of the spending effect. And without it they cannot be used properly in applied work to identify welfare improving policy changes.

Unfortunately, these problems are further compounded by the fact that the MCF is not the same as the shadow value of government revenue. Both play separate but related roles in policy evaluation, however, no clear distinction is made between them in the public finance literature. For example, Liu (2004) and Triest (1990) refer to the MCF as a shadow price, while Dahlby (1998), Sieper (1981) and Tsuneki (2002) obtain the shadow value of government revenue and refer to it as the MCF.[7] This paper will examine the separate roles of the MCF and the shadow value of government revenue in policy evaluation for a tax-distorted economy with constant producer prices and a single (aggregated) consumer.[8] In public finance applications the MCF is used to adjust revenue transfers made with distorting taxes to balance the government budget. For each project we compute its net impact on the government budget and then multiply it by the MCF to account for changes in tax inefficiency. Thus, the MCF is a scaling coefficient on revenue transfers made in full equilibrium, and there are no income effects from these transfers because they are offset by income effects from the government spending they fund inside projects.[9]

In contrast, the shadow value of government revenue is the change in welfare from endowing a dollar of revenue on the government when its budget is balanced by adjusting taxes. In policy evaluation it is used as a scaling coefficient to convert compensated welfare

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[6] To remove distributional effects we will assume all consumers are identical and then aggregate them. Another approach would allow heterogenous consumers but assign the same distributional weight to them. This is the "dollar-is-a-dollar" assumption employed in a conventional Harberger (1971) analysis. But distributional effects do matter in this setting whenever consumers have different marginal propensities to consume income. When the government balances its budget it can choose the share of revenue each consumer contributes through the taxes it changes. Different revenue shares will impact on the final equilibrium outcome.

[7] Dahlby finds the MCF for a lump-sum tax can differ from unity due to income effects from the revenue transfers that can impact on taxed activities. But these income effects are included in the shadow value of government revenue and not the MCF. Thus, when taxed activities expand the shadow value of government revenue exceeds unity. However, the conventional MCF is always unity because there is no MEB for a lump-sum tax.

[8] Producer prices are held constant by assuming the economy has a linear production possibility frontier. The main findings in this paper will also apply with variable producer prices, but the analysis becomes more complicated. These issues are examined in Jones.

[9] In a compensated equilibrium the government makes revenue transfers to hold consumer utilities constant. When distorting taxes are used to make these transfers they are multiplied by the compensated measure of the MCF to account for the compensated change in tax inefficiency.

changes into actual dollar changes in utility. Thus, it isolates income effects in project evaluation. For example, a project that generates an efficiency gain creates potential surplus revenue the government can collect at no cost to private utility. In other words, it is the foreign aid payment the project would fund without reducing private utility.[10] Once this surplus revenue is transferred to the domestic economy, each dollar will raise utility by the shadow value of government revenue. Auerbach, Anderson and Martin (2004), Diewert (1983), Dixit (1985), Fane (1991), Hatta (1977), Jones, Sieper and Tsuneki all isolate income effects from marginal policy changes using this coefficient. Anderson and Martin (2004) and Jones explicitly recognise it is not the MCF, while Sieper and Tsuneki refer to it as the MCF.[11]

It is not appropriate to use the shadow value of government revenue in the same way as the MCF. Both are derived in this paper to demonstrate the difference between them, where we show that when a unit of revenue (dR) is endowed on the government budget which is balanced using distorting tax d, the shadow value of government revenue, is:

$$\theta_d = mcf_d \left(1 + \frac{dT}{dR} \right),$$

where mcf_d is the conventional measure of the MCF for tax d, and the terms inside the brackets the net change in the government budget surplus which is larger than unity when tax revenue (T) rises endogenously due to income effects. If taxed activities are normal a unit of surplus revenue will increase private real income by more than unity, with $1 + dT/dR > 1$. In fact, the term inside the brackets is the shadow value of government revenue when the government budget is balanced using lump-sum transfers (with $\theta_L = 1 + dT/dR$). Each dollar of this surplus revenue must be scaled up by the MCF to account for the change in tax inefficiency when distorting tax d is used instead. Whenever the MCF exceeds unity, the final change in utility is larger due to the marginal excess burden of taxation (with $\theta_d > \theta_L$).

1. Welfare Comparisons using Partial Equilibrium Analysis

Before proceeding to a formal analysis we illustrate the MCF and the shadow value of government revenue in price-quantity diagrams using partial equilibrium analysis to provide intuition for the formal derivations in following sections. Also, each welfare change is defined, where for the MCF, we have:

Definition: *The conventional Harberger measure of the marginal social cost of public funds for any tax d is the direct cost to private surplus from the government raising a dollar of revenue to balance its budget.*

It is illustrated in Figure 1 for taxed good d which is produced at constant marginal cost. Using a conventional Harberger analysis we have $mcf_d = 1 + meb_d$, where $meb_d = b/(a - b)$ is the tax inefficiency (b) per dollar change in tax revenue ($a - b$). After substitution we have

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[10] In making this analogy we need to assume the utility of domestic consumers is unaffected by these foreign aid payments.

[11] Anderson and Martin refer to the shadow value of government revenue as the shadow exchange rate. They are identical when foreign exchange is chosen as the numeraire good. In general, however, the shadow value of government revenue is the shadow price of the numeraire good because it is the good the government holds as surplus revenue and uses to make compensating transfers. In other words, it is the real unit of account.

$mcf_d = a/(a - b)$, which measures the cost to private surplus (a) from transferring revenue ($a - b$) to balance the government budget. It is important to note that the change in tax inefficiency (b) is computed as the welfare change when tax revenue is returned to consumers as lump-sum transfers. In Harberger's terminology it is the welfare change for a "substitution effect only" problem in a balanced equilibrium. No resources are withdrawn from the economy by the revenue transfer to balance the government budget because it is used to fund government spending. Thus, it is one side of a resource reallocation within the economy.[12] Since there is no income effect from the revenue transfer the MCF for a lump-sum tax must be unity. And this applies even in a general equilibrium setting with variable producer prices.

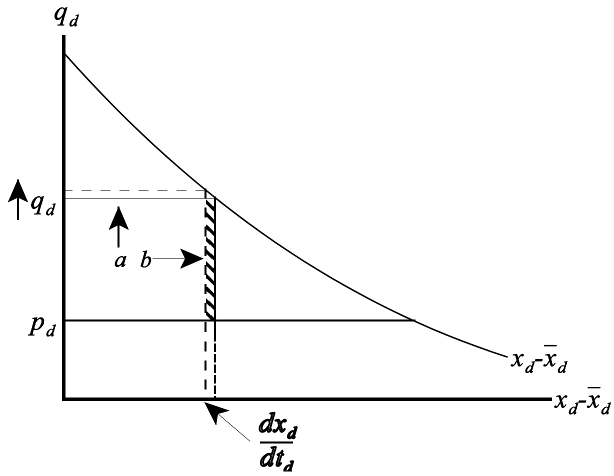

Figure 1: The Conventional MCF

But this is quite different to the derivation of the shadow value of government revenue, where:

Definition: *The shadow value of government revenue is the change in utility (measured in units of the numeraire good) from endowing a unit of surplus revenue on the government who balances its budget by lowering one or a combination of taxes.*

Now there are income effects from the initial revenue transfer because it represents an increase in resources available to the economy. The shadow value of government revenue is illustrated in Figure 2 where it is assumed the income effect falls solely on good d. After a

[12] This is the reason why lump-sum transfers are used in a conventional Harberger analysis to separate the welfare effects of each policy variable. For example, the welfare effects from a marginal increase in government spending are isolated by funding it with lump-sum transfers. When the revenue is raised with a distorting tax the change in tax inefficiency is isolated by marginally raising the tax and returning the revenue as a lump-sum transfer. Since the tax change must balance the government budget the lump-sum transfer of revenue from the tax change exactly offsets the lump-sum transfer of revenue to fund the government spending. In other words, the lump-sum transfers are neutralised inside the project when the government balances its budget. The main advantage of a conventional welfare analysis is that it allows separate agencies to compute the welfare effects of each component of a project. For example, treasury and finance departments can compute the MCF for each tax without knowing how the revenue will be spent. Equally, spending agencies can isolate the benefits from their spending programs without knowing how the revenue will be raised. Once the spending and tax changes are brought together inside a project the lump-sum transfers for each policy variable will offset each other when the government balances its budget.

unit of surplus revenue is transferred to the private economy as a lump-sum transfer the income effect moves the demand schedule for good d to the right. This raises utility (measured in units of the numeraire good) by $\theta_L = 1 + b'$. It is the unit increase in consumption expenditure (1), which falls entirely on good d by assumption, plus the extra tax revenue in b'. Since this budget surplus is transferred by lowering tax d (assuming this reduces tax revenue, with $a - b < 0$) then each unit must be scaled up by the MCF to account for the reduction in tax inefficiency in b, where the shadow value of government revenue becomes:

$$\theta_d = mcf_d \, \theta_L = \frac{b}{a-b}(1 + b').$$

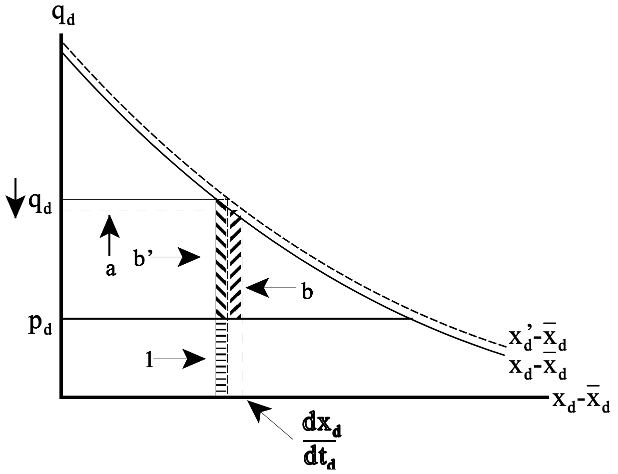

Figure 2: The Shadow Value of Government Revenue

In project evaluation the shadow value of government revenue converts efficiency gains into dollar changes in utility. Indeed, the efficiency gain for a marginal policy change (G) is the surplus revenue the government can raise after making compensating transfers to hold utility constant ($\tilde{\pi}_G$). When it is transferred to domestic consumers by lowering any distorting tax d, the shadow value of government revenue will convert the surplus revenue into utility, where the actual welfare change (π_G) can be decomposed as $\pi_G = \theta_d \tilde{\pi}_G$. All the income effects are isolated in θ_d, which is independent of the policy change.[13]

This welfare decomposition is particularly useful because it makes income effects irrelevant in applied welfare analysis in single (aggregated) consumer economies. Moreover, if the shadow value of government revenue is positive there must be efficiency gains whenever policy changes raise utility. And it seems reasonable to expect $\theta_d > 0$.[14] The same decomposition also applies in heterogeneous consumer economies, where the shadow value of government becomes the distributional-weighted sum of the personal shadow value of government revenue for consumers. This provides a convenient way of isolating

[13] For (large) discrete policy changes, however, the shadow value of government revenue will not in general be independent of the policy choices made.

[14] Hatta (1977) and Foster and Sonnenschein (1970) show how $\theta_d < 0$ can occur in a tax-distorted economy due to the possibility of multiple equilibrium outcomes. They appeal to stable equilibrium adjustment mechanisms to rule this out. While it is theoretically possible for extra real income to make the consumer worse off, it seems reasonable to expect that this case is unlikely in practice.

distributional effects for marginal policy changes.[15]

2. A Conventional Welfare Equation with Distorting Taxation

The results in the previous section will now be formalised in a model of the economy that is common to the public finance literature.[16] It has a single price-taking consumer who chooses a vector of (N) private consumption goods x and a pure public good G to maximise the utility function $u(x,G)$.[17] Private goods trade at a vector of consumer prices q, where the value of consumption is constrained by a vector of endowments $\bar{x}$ plus a lump-sum revenue transfer from the government budget (L), with $qx = q\bar{x} + L$.[18] We choose good 1 as numeraire and normalise its price at one dollar. A vector of specific consumption taxes t can drive wedges between the consumer and producer prices of all goods except good 1, where $q_i = p_i + t_i \;\forall_{i\in N-1}$, with $t_i > 0$ for net demands ($x_i - \bar{x}_i > 0$) and $t_i < 0$ for net supplies ($x_i - \bar{x}_i < 0$).[19] These goods are produced by private firms with constant marginal costs, where the vector of net outputs y has elements $y_i > 0$ for outputs and $y_i < 0$ for inputs. Since firms operate in competitive markets they will earn no profits, where $py = 0$. Thus, with unchanged production costs all price changes must result from changes in specific taxes, with $dq_i = dt_i \;\forall_{i\in N-1}$. In equilibrium there is market clearing, with $x_i - \bar{x}_i = y_i \;\forall_{i\in N}$. To make the analysis less complicated we assume the public good G is produced solely by the government at constant marginal cost to revenue of MRT.[20] In these circumstances, the government budget constraint is $L = T + R - MRT{\cdot}G$, where $T = t(x - \bar{x})$ is tax revenue and R an exogenous gift of revenue from outside the economy.

In a competitive equilibrium the demands for private goods can be solved as functions of the vectors of exogenous policy variables t, G and R, the vector of endowments $\bar{x}$, preferences and production technologies. Since the only comparative statics considered in following sections are for changes in the policy variables we can write the consumer's indirect utility

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[15] The shadow value of government revenue and the MCF are derived with distributional effects in Jones. If governments distribute surplus revenue by choosing a combination of tax changes to make the personal shadow value of government revenue positive for every consumer, there are strict Pareto improvements whenever policy changes have efficiency gains. This extends the welfare test in Bruce and Harris (1982) which relies on lump-sum transfers instead of distorting taxes.

[16] See for examples, Atkinson and Stern (1974), Ballard and Fullerton, Mayshaar (1991), Snow and Warren, Tsuneki and Wildasin (1984).

[17] A pure public good is perfectly non-rivalrous and non-excludable. Also, we assume the utility function provides ordinal rankings over consumption bundles based on preference orderings that are complete, reflexive, transitive, continuous and monotone.

[18] In the following analysis the government uses distorting taxes to balance its budget. We use the lump-sum transfers in a conventional manner to separate the welfare effects of each individual component of project. These transfers are purely notional because they are offset by changes in the distorting taxes to balance the government budget. Indeed, this is the how the shadow prices for individual goods are computed.

[19] We reverse the sign of t_i for specific subsidies.

[20] Liu and Jones consider circumstances where public goods are produced using private inputs. It makes the welfare changes slightly more complicated without affecting the separate roles of the MCF and the shadow value of government revenue. Once we allow private provision of the public good there can be strategic interactions that result in private provision being completely crowded out by public production. This is examined in Bergstrom, Blume and Varian (1986).

function as $v(t,G,R)$. Then by totally differentiating this function we obtain a dollar measure of the change in private surplus, of:

$$\text{(1)} \qquad \frac{dv}{\lambda} = MRS_G dG - (x - \bar{x})dt + dL,$$

where λ is the marginal utility of income, and $MRS_G = v_G / \lambda$ the marginal consumption benefits from the public good.[21] It is comprised, respectively, of consumption benefits endowed on the consumer from increases in the public good, reductions in consumer surplus from higher taxes, plus revenue transfers from the government budget. But this is not the final welfare change because the transfer of private surplus from tax changes are offset when the government balances its budget, where in full equilibrium we have:

$$\text{(2)} \qquad dL = (x - \bar{x})dt + tdx + dR - MRT \cdot dG.$$

A conventional welfare equation is obtained by substituting (2) into (1), as:

$$\text{(3)} \qquad \frac{dv}{\lambda} = MRS_G dG + tdx + dR - MRTdG,$$

where the dollar changes in utility are determined by changes in activity. Once the public sector budget constraint binds it undoes the transfers of private surplus between the private and public sectors of the economy where the final welfare changes are determined by changes in consumption expenditure. Now we are in a position to consider the public sector project that produces one extra unit of the public good when tax d is used to balance the government budget. The dollar change in utility is obtained from (3), with $dR = dt_{-d} = 0$, when the change in tax d is solved using (2) with $dL = 0$, as:

$$\text{(4)} \qquad \pi_G = \frac{dv}{dG}\frac{1}{\lambda} = MRS_G - mcf_d\left(MRT - t\frac{dx}{dG}\right),^{22}$$

where $mcf_d = \dfrac{x_d - \bar{x}_d}{(x_d - \bar{x}_d) + t\dfrac{dx}{dt_d}}$ is the conventional measure of the MCF for tax d.

A revised Samuelson (1954) condition for the optimal supply of the public good is obtained be setting $\pi_G = 0$, where:

$$\text{(5)} \qquad MRS_G = mcf_d\left(MRT - \frac{dT}{dG}\right).$$

At a social optimum the marginal consumption benefit from extra output of the public good

[21] This change in private surplus is obtained by totally differentiating the Lagrangean function:

$$\mathcal{L} = u(x,G) + \lambda[q(x - \bar{x}) - L],$$

and using the envelope theorem.

[22] The tax change to balance the government budget is solved using (2) with $dL = 0$, as:

$$\frac{dt_d}{dG} = \frac{MRT \cdot dG - t\dfrac{dx}{dG}}{(x_d - \bar{x}_d) + t\dfrac{dx}{dt_d}}.$$

The notation dt_{-d} is used to denote the vector of tax changes excluding the tax on good d.

is equated to the social cost of producing it.[23] The terms inside the brackets measure the impact the project has on the government budget deficit; it is the production cost (MRT) less the spending effect (dT/dG) identified by Diamond and Mirrlees, and Stiglitz and Dasgupta. When the government funds this deficit using tax d each dollar reduces private surplus by mcf_d, where, consistent with intuition provided by Pigou (1947), the public good is more costly to supply at the margin whenever $mcf_d > 1$. If extra output of the good expands taxed activities, a positive spending effect ($dT/dG > 0$) will reduce the size of the budget deficit and make the project less costly to fund. Ballard and Fullerton consider a special case where the spending effect exactly offsets the marginal excess burden of taxation so that (5) collapses to $MRS_G = MRT$. It leads them to conclude that the MCF must be unity in these circumstances. The basis for their claim is examined in section 4.2 where a modified measure of the MCF is derived. Before doing so, however, we isolate the role of the shadow value of government revenue in policy evaluation.

3. Using the Shadow Value of Government Revenue to Isolate Efficiency Effects

It is clear from the welfare analysis in the previous section that the MCF is used as a scaling coefficient to adjust revenue transfers made to balance the government budget for changes in tax inefficiency. Since it applies to transfers in a balanced equilibrium the MCF is not a shadow price. Thus, it cannot be used to convert efficiency gains into utility; that role is performed by the shadow value of government. We confirm this by deriving the change in foreign aid payments that would offset the change in utility from the public project evaluated in the previous section, where:

(6) $$\frac{dv}{dG} = v_G + v_R \frac{d\hat{R}}{dG} = 0.$$

The symbol ^ is used to indicate a compensated welfare change with utility held constant at its initial level (u_0). After dividing (6) by the marginal utility of income we obtain the welfare decomposition:

(7) $$\pi_G = \theta_d\, \hat{\pi}_G,$$

where $\pi_G = v_G/\lambda$ is the dollar change in utility computed in (4), $\theta_d = v_R/\lambda$ the shadow value of government revenue, and $\hat{\pi}_G = -d\hat{R}/dG$ the compensated welfare change for the project. If there is an efficiency gain from the project (with $\hat{\pi}_G > 0$) it will generate surplus revenue the government could pay as foreign aid at no cost to domestic utility. When this surplus is instead returned to the domestic economy by lowering tax d, each dollar will raise utility by the shadow value of government revenue (θ_d). Since θ_d converts the efficiency gain into utility (π_G) it must isolate all the income effects from the project, and as an independent scaling coefficient it makes them irrelevant in policy evaluation. Thus, we can identify the optimal level of government spending using actual or compensated welfare changes. Whenever good G is optimally supplied there is no marginal efficiency gain ($\hat{\pi}_G = 0$) to raise utility (so that $\pi_G = 0$), irrespective of the sign of the shadow value of government revenue. Furthermore, if $\theta_d > 0$, there must be efficiency gains ($\hat{\pi}_G > 0$) whenever the project does raise utility ($\pi_G > 0$).

As noted earlier in the introduction the welfare decomposition in (7) is used elsewhere in the literature. While no other studies refer to the coefficient that isolates income effects as the

[23] In economies with a large number of identical consumers MRS_G is the sum of their marginal consumption benefits.

shadow value of government revenue, most do not distinguish it from the MCF.[24] To see that the two welfare measures are different we will use the welfare equation (3) to compute the change in utility from endowing a unit of revenue on the government when tax d is used to balance the government budget, as:

$$\text{(8)} \qquad \theta_d = \frac{dv}{dR}\frac{1}{\lambda} = mcf_d\left(1 + t\frac{dx}{dR}\right).^{25}$$

The term inside the brackets is the shadow value of government revenue when the government budget is balanced using lump-sum transfers ($\theta_L = 1 + dT/dR$); it is the budget surplus generated by the initial unit of revenue endowed on the government. If taxed activities are normal this surplus revenue will exceed unity. When the government budget is balanced by lowering tax d, the budget surplus must be scaled up by the MCF to account for the reduction in tax inefficiency.

4. Which Measure of the MCF to Use in Policy Evaluation

We are now in a position to determine how the MCF should be measured. As noted earlier in the introduction it is measured in many different ways in the public finance literature. For example, some studies compute an uncompensated measure, while others compute a compensated measure. In some cases the marginal excess burden of taxation is computed as a combination of uncompensated and compensated welfare changes. The welfare analysis in the previous section will allow us to choose the appropriate measure to use. It will also allow us to compute the modified measure of the MCF, which has become so popular in recent applied work, and to compare it to the conventional measure of the MCF. By doing so it is possible to reconcile the two cost-benefit approaches that use them.

4.1 Uncompensated vs. Compensated Measures of the MCF

A perplexing issue that analysts confront when choosing how to measure the MCF results from a conflict between their desire to report actual welfare changes, on the one hand, and to accurately measure them, on the other. It is widely accepted that compensated welfare changes are reliable measures of aggregate changes in real income. By holding utility constant they avoid distributional issues which arise in economies with heterogeneous consumers. Also, for large (discrete) policy changes they are path independent. The problem, however, is they are based on hypothetical equilibrium outcomes which require compensating transfers that are rarely made. Consequently, they are based on changes in activity that are unlikely to eventuate, and this makes them difficult to explain to policy makers. Actual welfare changes are preferred in so far as they are based on actual changes in activity, but they include subjectively determined distributional effects and are in general path dependent for large policy changes.

[24] Anderson and Martin examine the separate roles of the coefficient θ_d and the MCF. However, their decomposition is slightly different because they compute θ_d by endowing a unit of surplus revenue directly on the private sector of the economy as a lump-sum transfer. This is because they isolate compensated welfare changes for projects by using lump-sum transfers to hold consumer utilities constant. In this paper lump-sum transfers are ruled out, which is why the compensating transfers are also made with distorting taxes. It is based on the view that the compensated equilibrium should be computed as an actual equilibrium outcome. In other words, it should be the realised outcome when compensating transfers are made with distorting rather than lump-sum taxes.

[25] The change in tax d is solved using (2) with $dL = 0$.

In single (aggregated) consumer economies where distributional effects play no role in marginal welfare analysis, the choice between the uncompensated and compensated welfare measures of the MCF is irrelevant. This is confirmed by the decomposition in (7) which makes income effects irrelevant in project evaluation. However, the MCF must be consistent with the equilibrium concept employed. In full equilibrium the uncompensated MCF is used to scale revenue transfers made to balance the government budget, while in the compensated equilibrium the compensated MCF is used to discount revenue transfers made to hold utility constant.

The role of the uncompensated MCF was demonstrated by the derivation of the actual welfare change from the project evaluated in equation (4). To determine the role of the compensated MCF we compute the welfare change for the project as the change in foreign aid revenue in equation (2) when the change in tax d is solved to hold utility constant using (1), with $dv/\lambda = 0$ and $dL = 0$, as:

$$\hat{\pi}_G = -\frac{d\hat{R}}{dG} = \frac{MRS_G}{m\hat{c}f_d} - \left(MRT - \frac{d\hat{T}}{dG}\right), \tag{9}$$ [26]

where $m\hat{c}f_d = \dfrac{x_d - \bar{x}_d}{(x_d - \bar{x}_d) + t\dfrac{d\hat{x}}{dt_d}}$ is the conventional measure of the compensated MCF for tax d.

Notice that this measure of the MCF, which is based solely on substitution effects, discounts the marginal consumption benefit from the public good. When another unit of the non-excludable public good is endowed on the economy it confers consumption benefit MRS_G on the consumer. It is the amount of revenue that must be transferred away to hold utility constant. Since all other welfare changes from the project impact directly on the government budget they do not require compensating transfers. By using tax d to make the compensating transfer the government collects revenue of $MRS_G / m\hat{c}f_d$, which is less than MRS_G due to the increase in tax inefficiency.

At a social optimum, with $\hat{\pi}_G = 0$, the welfare change in (9) provides a compensated version of the revised Samuelson condition, of:

$$\frac{MRS_G}{m\hat{c}f_d} = MRT - \frac{d\hat{T}}{dG}. \tag{10}$$

It can be rearranged in the same way as the uncompensated condition obtained previously in (5), but that would conceal the role of the MCF in the compensated equilibrium. Comparing the two revised conditions in (5) and (10) makes it clear what role the MCF plays, and what measure of the MCF to use, in each equilibrium closure of the economy.

Some studies combine uncompensated and compensated welfare changes in their measure of the MCF. For example, empirical estimates in Table 1 for the US by Browning and Stuart

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[26] The surplus revenue from the project is computed using (2), as:

$$\hat{\pi}_G = -\frac{d\hat{R}}{dG} = \left(x_d - \bar{x}_d + t\frac{d\hat{x}}{dt_d}\right)\frac{d\hat{t}}{dG} - MRT + t\frac{d\hat{x}}{dG}.$$

Since the tax change must hold utility constant it is solved using (1), with $dv/\lambda = 0$ and $dL = 0$, as:

$$\frac{d\hat{t}_d}{dG} = \frac{MRS_G}{x_d - \bar{x}_d}.$$

After substitution we obtain the welfare change in (9).

compute the MEB by normalising the compensated change in tax inefficiency over the actual change in tax revenue.[27] But this is a hybrid of the two measures of the MCF used in (4) and (9), and it will not arise naturally in either of the uncompensated or compensated equilibrium outcomes. As noted previously, this practice appears to result from the conflict between wanting to report actual welfare changes, which are observable, and the desire to compute hypothetical welfare changes, which are reliable. Changes in tax inefficiency are measured in compensated terms to determine the fall in real income, while changes in tax revenue are frequently measured in actual terms because they will be observed (in the absence of any outside compensation). While Ballard (1988) and Fullerton (1991) find these differences may not impact significantly on empirical estimates of the MCF, it is nevertheless important to have a sound conceptual understanding of the different welfare measures to clarify how estimates of the two measures of the MCF should be used in applied welfare analysis.

Either of the two measures of the MCF in (4) and (9) can be used to find the optimal level of government spending in single consumer economies because income effects play no role in applied welfare analysis. Once the equilibrium closure of the economy is chosen it determines how the welfare changes are measured, as either actual or compensated, and this in turn determines which measure of the MCF to use. Based on the welfare decomposition in (7), both approaches will lead to the same optimal level of government spending.

4.2 A Modified Measure of the MCF

In recent times a number of studies have used a modified measure of the MCF in project evaluation. It differs from the conventional measure used in (4) by including the spending effect (dT/dG) identified by Diamond and Mirrlees, and Stiglitz and Dasgupta. Snow and Warren demonstrate how it is obtained by explicitly writing the welfare change in equation (5), as:

$$MRS_G = mcf_d^* \cdot MRT, \tag{11}$$

where $mcf_d^* = mcf_d \left(1 - \frac{dT}{dG} \frac{1}{MRT} \right)$ is the modified MCF.

In most empirical estimates of this measure of the MCF the spending effect is positive, which makes it less than the conventional measure (mcf_d). As noted earlier, all the estimates of the MCF for the US in Table 1, with the exception of those made by Browning, are for the modified MCF. While it is not incorrect to rearrange the welfare changes in this way, it is important that anyone using these estimates realises they are for the modified measure of the MCF. They can only be used for projects with the same type of government spending, and in the manner illustrated by the welfare analysis in equation (11). Errors are likely to occur when they are used inadvertently as the conventional measure of the MCF. In particular, they will understate the true social cost of funding a budget deficit when the spending effect is positive.

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[27] In this paper the compensated welfare changes are derived at the initial level of utility where the actual and compensated changes in tax revenue differ when income effects impact on taxed activities. Some studies, for example, Ballard, Shoven and Whalley, hold utility constant at its new level where the actual and compensated changes in tax revenue are equal in economies with constant producer prices. However, when producer prices change endogenously with the compensating transfers there are in general be differences between these two measures of the change in tax revenue.

We can see from the welfare change in (4) that the spending effect is not ignored in a conventional analysis. It separates the welfare effects of each component of the project using notional (hypothetical) lump-sum transfers where the spending effect is included with the welfare effects from extra output of the public good, and the welfare effects from using the distorting tax in the MCF.[28] Since the lump-sum transfers are non-distorting they do not impact on the final welfare change for the project because they are offset when the government balances its budget. Clearly, the modified MCF cannot be used in the same way as the conventional MCF.

Before proceeding it is worth considering the special case examined by Ballard and Fullerton where the modified MCF is unity for a single distorting tax on labour income. By making utility additive in the public good the combined effects of a project that marginally raises the public good and the wage tax has no net impact on employment. This makes the modified MCF unity because the spending effect exactly offsets the change in tax inefficiency, and results in the traditional first best Samuelson condition of $MRS_G = MRT$ at a social optimum.[29]

5. Isolating Income Effects in the (uncompensated) MCF

In a widely cited paper Atkinson and Stern (1974) isolate a revenue effect in the conventional measure of the MCF that can offset the distortionary effect of taxation; it lowers the MCF and makes government spending less costly to fund at the margin. Dahlby (1998) argues that the revenue effect is the reason why, in the presence of distorting taxes, the uncompensated measure of the MCF for a lump-sum tax can be less than unity. They argue it is the endogenous change in tax revenue when income effects from the revenue transfers impact on taxed activities. The intuition runs as follows - when the government transfers a unit of revenue from the private sector with a lump-sum tax it reduces the real income of consumers. If taxed activities are inferior this income effect will increase the amount of tax revenue raised, thereby making the MCF less than a unity.

This is somewhat puzzling to anyone who is familiar with the conventional Harberger measure of the MCF because it should always be unity for a non-distorting tax. And the reason for this intuition is based on the way the change in tax inefficiency is computed in a conventional analysis; it is the welfare change from marginally raising a tax when the revenue is returned to taxpayers as lump-sum transfers. In fact, these transfers undo the income effects from the dollar of tax revenue initially raised, where the MCF for any non-distorting tax should be unity. Atkinson and Stern, and Dahlby obtain a revenue effect because they are actually computing the shadow value of government revenue, which is the welfare change from endowing revenue on the economy. Thus, it contains income effects that can make it different from unity for a lump-sum tax.

The best way to confirm this conjecture is to isolate any income effects in the conventional measure of the MCF and the shadow value of government revenue. And we do so by first

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[28] This separation allows us to compute shadow prices for the individual inputs and outputs to the project. Jones and Tsuneki obtain the revised Samuelson condition using shadow prices, and they extend the analysis by allowing variable producer prices and including distributional effects.

[29] In these circumstances: $t(dx/dt_d) + t(dx/dG) = 0$ and $x_d - \bar{x}_d = MRT$. After substituting these relationships into (5) we have $MRS_G = MRT$. Atkinson and Stern (1974) caution that this optimality condition is unlikely to result in the same level of government spending as the level in the first-best economy with no tax distortions.

noting that the consumer demands for each good i are functions of the vector of consumer prices and money income, with $x_i = x_i(q, I)\ \forall_i$. In full equilibrium these prices and money income are ultimately functions of the exogenous variables, where aggregate income for the economy is obtained by combining the private and public sector budget constraints, as $I = q\bar{x} + T + R - MRT \cdot G$.[30] Using this information, we can decompose the income and substitution effects in the conventional MCF in (4), as:

(12) $$mcf_d = \frac{1}{1 + \theta_L \dfrac{t}{x_d - \bar{x}_d} \dfrac{\partial \hat{x}}{\partial q_d}},^{31}$$

with $\theta_L = \dfrac{1}{1 - t\dfrac{\partial x}{\partial I}}$.

Notice how the income effect is due solely to the change in tax inefficiency; it is the scaling coefficient (θ_L) on the efficiency loss in the denominator of (12). Thus, with no income effect from the revenue transfer, we must have $mcf_d = 1$ for a non-distorting tax.

By following the same approach we can isolate the income effects in the shadow value of government revenue in equation (8), where:

(13) $$\theta_d = \frac{1}{1 - t\dfrac{\partial x}{\partial I} + \dfrac{t}{x_d - \bar{x}_d} \dfrac{\partial \hat{x}}{\partial q_d}}.^{32}$$

This is the decomposition of the change in tax inefficiency obtained by Atkinson and Stern, where they refer to $t(\partial x/\partial I)$ as the revenue effect. If taxed activities are normal the revenue effect raises the shadow value of government revenue by expanding taxed activities, where for a non-distorting tax, we must have $\theta_d > 1$. This is consistent with the finding in Dahlby.

There are two points to make about the role of the revenue effect. First, it is not present in the MCF, as is confirmed by the decomposition in (12), and second, it will play no role in determining the optimal level of government spending by the welfare decomposition in (7).[33]

6. Setting Ramsey Optimal Taxes

The welfare analysis used in previous sections provides a straightforward way of deriving the well know proposition that in single consumer economies Ramsey optimal taxes will minimise the excess burden of taxation by equating their compensated marginal excess burdens. This important result was formalised in Diamond and Mirrlees and it can be obtained by marginally raising any tax j when some other tax d is used to balance the government budget, where from equation in (3), we have:

[30] This is sometimes referred to as virtual income.

[31] Workings are provided in the Appendix.

[32] Sieper extends this decomposition by allowing producer prices to change endogenously, while Tsuneki also includes distributional effects. Both refer to it as the MCF but they use it to convert compensated welfare changes into actual welfare changes in the manner described in (7).

[33] At a social optimum (with $\hat{\pi}_G = 0$) the income effects from the combined changes in the public good and tax d must offset each other in the revised Samuelson condition in (5).

(14) $$\frac{dv}{dt_j}\frac{1}{\lambda} = t\frac{dx}{dt_j} + t\frac{dx}{dt_d}\frac{dt_d}{dt_j}.$$

The change in tax d is solved using (2), with $dL = 0$, as:

(15) $$\frac{dt_d}{dt_j} = -\frac{(x_j - \bar{x}_j) + t\dfrac{dx}{dt_j}}{(x_d - \bar{x}_d) + t\dfrac{dx}{dt_d}}.$$

We obtain the rule for setting Ramsey optimal taxes by substituting (15) into (14), where:

(16) $$\frac{dv}{dt_j}\frac{1}{\lambda} = (meb_j - meb_d)\frac{dT}{dt_j} = 0,$$

with $meb_i = \dfrac{-t\dfrac{dx}{dt_i}}{(x_i - \bar{x}_i) + t\dfrac{dx}{dt_i}} \quad \forall_{i \in N-1}$ being the marginal excess burden of taxation.

To obtain the result in Diamond and Mirrlees we isolate the income effects in the marginal excess burden, with $meb_i = \theta_i\, m\hat{e}b_i\ \forall_{i \in N-1}$.[34] When taxes are Ramsey optimal the shadow value of government revenue is independent of the tax used to balance the government budget, so that $\theta_i = \theta\ \forall_{i \in N-1}$, where this allows us to rewrite (16), as:

(17) $$\frac{dv}{dt_j}\frac{1}{\lambda} = \theta(m\hat{e}b_j - m\hat{e}b_d)\frac{dT}{dt_j} = 0,$$

with $m\hat{e}b_i = -\dfrac{t}{(x_i - \bar{x}_i)}\dfrac{d\hat{x}}{dt_i} \quad \forall_{i \in N-1}$ being the compensated marginal excess burden of taxation.

At a social optimum there is no welfare loss from marginally raising one tax (j) and at the same time lowering any other tax (d) to hold utility constant. And without an efficiency loss from these tax changes to affect real income there cannot be any change in utility.[35]

7. Conclusion

There are a variety of factors that can impact on the empirical estimates of the MCF. They include the demand-supply elasticities of tax activities and other modelling specifications, whether welfare changes are uncompensated or compensated, the inclusion of distributional effects, and the inclusion of the spending effect. Unless users have this information they cannot use and interpret the empirical estimates correctly in applied work.

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[34] This is obtained by separating the income and substitution effects for the uncompensated MEB in (16), and using the welfare decomposition in (13), where:

$$meb_i = \frac{m\hat{e}b_i}{1 - t\dfrac{\partial x}{\partial I} - m\hat{e}b_i} = \theta_i\, m\hat{e}b_i \quad \forall_{i \in N-1}.$$

[35] Jones extends this result in Diamond and Mirrlees by allowing producer prices to change endogenously.

Further problems arise when the shadow value of government revenue is used in place of the MCF. This paper shows that they are distinctly different welfare measures with different roles to play in applied welfare analysis. The MCF is used as a scaling coefficient to account for changes in tax inefficiency on revenue transfers made to balance the government budget, while the shadow value of government revenue is used as a scaling coefficient to convert efficiency effects into actual changes in utility; it is the welfare gain from endowing a unit of surplus revenue on the government who transfers it to the private economy by adjusting taxes. We isolated a revenue effect in the shadow value of government revenue that is not present in the MCF. This allowed us to reconcile the finding in Atkinson and Stern, and Dahlby that the MCF for a lump-sum tax can differ from unity with the conventional understanding that it should always be unity. They compute the shadow value of government revenue and refer to it as the MCF. In this paper we show why it is important to recognise the difference between these two welfare measures. Failure to do so can lead to the MCF being incorrectly used in policy evaluation.

Appendix

To isolate the income effects in the conventional measure of the MCF in (4) we use the fact that uncompensated demands for consumption goods are functions of the consumer prices and money income, with $x_i = x_i(q, I)\ \forall_i$, and compensated demands are functions of the consumer prices and initial utility (u_0), with $\hat{x}_i = x_i(q, u_0)\ \forall_i$. By using these functional relationships we can decompose the change in tax inefficiency in (4), as:

$$t\frac{dx}{dt_d} = t\frac{\partial x}{\partial q}\frac{dq}{dt_d} + t\frac{\partial x}{\partial I}\frac{dI}{dt_d}.$$

Since producer prices are fixed we have $dq_i/dt_d = 0\ \forall_{i \neq d}$ and $dq_d/dt_d = 1$, while the change in income is solved using aggregate income $\theta_d > 1$, with:

$$\frac{dI}{dt_d} = x_d - \bar{x}_d + t\frac{dx}{dt_d}.$$

After substitution we can write the change in tax inefficiency as:

$$t\frac{dx}{dt_d} = t\frac{\partial x}{\partial q_d} + t\frac{\partial x}{\partial I}\left(x_d - \bar{x}_d + t\frac{dx}{dt_d}\right).$$

By using a Slutsky decomposition, with:

$$\frac{\partial \hat{x}_i}{\partial q_d} = \frac{\partial x_i}{\partial q_d} + (x_d - \bar{x}_d)\frac{\partial x_i}{\partial I}\quad \forall_i,$$

we can write the change in tax inefficiency, as:

$$t\frac{dx}{dt_d} = t\frac{\partial \hat{x}}{\partial q_d}\bigg/\left(1 - t\frac{\partial x}{\partial I}\right).$$

Once this is substituted into the MCF in equation (4) we obtain the decomposition in (12).

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