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THE HEALTH EXPECTANCIES OF OLDER AUSTRALIANS:

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SUMMARY

Based on Australian Bureau of Statistics surveys of 1981, 1988, 1993 and 1998 the paper presents estimates of health expectancies of the states Disability-free and Disabled for females and males aged 60 and over by cohort from 1980 and current for the survey years. Recently developed logistic regression techniques are used instead of the standard methods due to Sullivan, and a major aim of this report is to present these techniques in a readily useable form. An informal presentation of the approach is given in Section 2, with a more rigorous methodological exposition in Section 4. Section 3 contains graphical and numerical results. Amongst our findings is that the results of the three later surveys are broadly similar and differ in important respects from those of the 1981 survey. Based on the last three surveys our estimates support the view that, depending on age, roughly two thirds or more of the increase in life expectancy over the decade 1988-1998 is taken in a state of disability. Also, our findings do not support rectangularisation of the survival or disability-free survival curve.

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CONTENTS

1 INTRODUCTION

1.1 Background

Australian health expectancies in 1981, 1988 and 1993 were discussed by Mathers (1996) following an earlier treatment using data for the first two of the three survey years (Mathers, 1991). His calculations were based on population surveys by the Australian Bureau of Statistics (ABS). The ABS carried out a fourth survey in 1998 and a consolidated account of the four surveys is given in ABS (2001). The ABS report contains minor revisions and changes for the sake of compatibility and gives health expectancies by five year age groups for the two states Disability-free and Disabled. Importantly, the publication makes possible the calculation of estimates, comparisons and projections using data that is consistent for 1981, 1988, 1993 and 1998.

The ABS provided us with estimates of population prevalences of the two states Disability-free and Disabled for the years of the last three surveys by sex and age in single year and 5 year age groups from 60 to 99. The ABS also provided us with the mid-year current estimated resident population for these and neighbouring years. Prevalences for 1981 were obtained from Mathers (1991). Together with current ABS life tables we use this data to model and estimate cohort health expectancies for persons who in 1980 were aged 60 to 77 and current health expectancies for the years of the surveys for ages 60 to 99. Implicit here is our working definition of ‘older’ in the title.

The methods used are modifications and developments of those in Davis et al. (2001), based on regression techniques, and are different from the commonly used Sullivan method, which is employed, for example, in the cited work by Mathers. A mathematical presentation of the method is given in Section 4. A point that should be made at the outset is that, with a few exceptions, actuaries and demographers have tended to neglect regression methodology, and one aim of the paper is to remedy the situation.

Health expectancies are the future durations that an individual expects to spend in different states of health before the inevitable outcome of death and as such are components of life expectancies. Of present interest are the two mutually exclusive and exhaustive ‘alive’ states Disability-free, labelled 1, and Disabled, which is labelled 2. The only other possibility, Death, is referred to as state 3, and its complement, Alive, will sometimes be referred to as state 0 (zero). The precise nature of what is meant by Disability need not concern us since the methods used are not specific to health statistics and the practical parts of the paper are based totally on ABS definitions and data; see in particular the table on page 4 of ABS (2001) and the summary at the end of that publication. Suffice it to say that by Disability is meant a functional impairment that persists for at least six months. More detailed discussions can be found in Mathers (1991), Mathers et al. (1994) and other publications of REVES (French initials for the International Network on Health Expectancy), and World Health Organisation (1980).

A distinction should be drawn between *marginal* and *conditional* health expectancies. The latter depend on the initial state, that is, whether a person is disabled or not at the age for which the expectancy is calculated; longitudinal data is required in this case. On the other hand, marginal expectancies are those lacking such information, and in this paper these are the quantities with which we shall be concerned. This is because the ABS surveys are cross-sectional without a longitudinal component. Under a Markov condition the basic results necessary for estimating conditional health expectancies and constructing a multistate life table are given in Davis et al. (2002).

It is convenient to phrase the discussion in terms of a cohort of like individuals who, as they age, can move between states 1 and 2 (Disability-free and Disabled) before being absorbed in state 3. Then one can think of members of the cohort generating replicates of a stochastic process taking values 1, 2 or 3 as each unit of time passes, that is, as age increases. Our approach is to estimate the probabilities associated with this process by logistic regression adapted to aggregate data as made available by official agencies such as the ABS. A qualification is that our preference is for data that has not been graduated. What smoothing we do is inherent in fitting a regression to data as it arises and this should be contrasted with the standard actuarial practice of smoothing before constructing demographic measures which are, in the present case, functions of the marginal totals of a multistate life table. As it happens we do not have access to the original data and hence are obliged to use published figures and life tables. Consequently it should be borne in mind that the standard errors presented are too small.

Regression methods have previously been used in the study of ordinary life tables by Heligman and Pollard (1980), Kostaki (1992), Heathcote and McDermid (1994) and by Heathcote and Higgins (2001). The Heligman-Pollard approach is to fit a regression by ordinary least squares to the logistic transform of the age-specific probability of death. In some ways the Heathcote-Higgins method is more general since it models the probability of death specific to both age and year and develops a large sample weighted least squares technique for fitting what are called mortality surfaces. These are parameterised measures of mortality indexed by age and year whose random element is generated by stochastic processes defined on the diagonals of the Lexis plane.

Generally we consider only persons aged 60 and over. A major reason for restricting discussion to this part of the population is that we wish to treat health expectancies for cohorts. The youngest considered is the 1920 birth cohort whose surviving lives were aged 60 in 1980. There seemed to be substantial difficulties in extending consideration to younger cohorts, given the data to hand. A consequence is that discussion of current expectancies is also restricted to ages 60 and over. Younger ages will be dealt with elsewhere.

Some notation and definitions follow in Section 1.2, and in Section 2 there is an informal exposition of the methodolgy. Section 3 presents results for females and males aged 60 and over using the ABS data. Presentation of the statistical argument is deferred until Section 4. Forecasting is not attempted. In the terminology of Heathcote and Higgins (2001) attention is confined to descriptive, not predictive, models. However, we do note that the results of the 1981 survey are not consistent with those for later years. Consequently it would be prudent to base forecasts only on the 1988, 1993 and 1998 surveys.

It is well known that life expectancy has been increasing, and a proposition that has attracted some support is that the extra years are ones of ill health; see page 321 of Bebbington (1988) for a succinct summary of alternative views, and also Crimmins et al. (1994) and Verbrugge (1984) for a wider discussion. Our findings are that the situation is complex but, depending on age, about two thirds or slightly more of the increased years are spent in a disabled state.

An important aim of this report is to present the large sample multiple logistic regression technique developed in Davis et al. (2001). The method is designed for the modelling and estimation of time series aggregate cross-sectional data of health, employment and like sequences produced by official agencies such as ABS. The Markov property is not required for the estimation of the expectancies considered here. For expository purposes, and at the risk of some repitition, the introduction to the approach in Section 2 is presented with minimum formalism, with a more rigorous discussion in Section 4.

1.2 Statement of the problem

For the four states Alive (0), Disability-free (1), Disabled (2) and Dead (3) defined above, let $p_j(x,y)$ denote the conditional probability that an individual known to be alive at age x is in state j at the subsequent age y (j = 0,1,2,3). Then

$$p_0(x,y) = p_1(x,y) + p_2(x,y) = 1 - p_3(x,y). \tag{1.1}$$

Next let $l_j(x)$ denote the number of lives at exact age x in state j. Then, as in Chapter 1 of Bejamin and Pollard (1993),

$$p_j(x,y) = l_j(y)/l_0(x). \tag{1.2}$$

Generally we shall be considering a cohort of lives at the same age x. It will be assumed that the number $l_0(x)$ alive at the initial age x is known, but that counts $l_j(y)$ at later ages are unknown and must be estimated from the data to hand. Note also that $l_3(y)$ denotes the cumulative number of deaths to age y in the cohort, and that for all $y > x$,

$$l_1(y) + l_2(y) + l_3(y) = l_0(x). \tag{1.3}$$

We are therefore ignoring the effects of migration.

As a function of y the conditional probability $p_0(x,y)$ defines the survival curve for those lives known to have been alive at the fixed age x. Put another way, with $\mu(v)$ denoting the force of mortality at age v, the survival curve giving the probability of surviving to age y conditional on being alive at prior age x is

$$p_0(x,y) = \exp\left\{-\int_x^y \mu(v)dv\right\}, \quad x < y < \infty.$$

Life expectancy $e_0(x)$ at age x is the area under this curve, namely

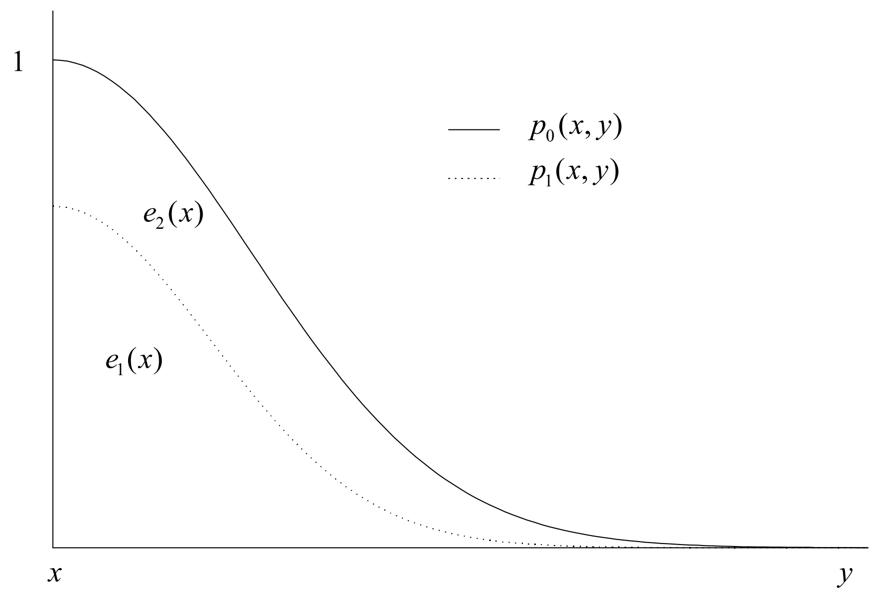
$$e_0(x) = \int_x^{\infty} p_0(x,y)dy\,. \qquad (1.4)$$

Similarly, the health expectancies of the alive states (j = 1,2) at age x are the areas

$$e_j(x) = \int_x^{\infty} p_j(x,y)dy\,, \qquad (1.5)$$

with $e_0(x) = e_1(x) + e_2(x)$, by (1.1). These relationships are illustrated in Figure 1.1. We will sometimes refer to $p_1(x,y)$, $x < y$, as the Disability-free survival curve.

Figure 1.1 Life and health expectancies as areas



Discrete approximations are customarily used to find the said areas. Thus, with $l_0(y)$ denoting the number of survivors at age y, a formula for life expectancy at age x is obtained from (1.25) on page 27 of Benjamin and Pollard (1993), or by rewriting (6) on page 42 of Pollard et al. (1990), as follows:

$$e_0(x) = \frac{1}{l_0(x)} \sum_{y=x}^{\infty} \frac{1}{2}\{l_0(y) + l_0(y+1)\}$$

$$= \frac{1}{l_0(x)}\left\{\frac{1}{2}l_0(x) + \sum_{y=x+1}^{\infty} l_0(y)\right\}$$

$$= \frac{1}{2} + \sum_{y=x+1}^{\infty} p_0(x,y), \qquad (1.6)$$

where, as in (1.2), $p_0(x,y) = l_0(y)/l_0(x)$. The corresponding health expectancies are given by

$$e_j(x) = \frac{1}{l_0(x)}\sum_{y=x}^{\infty}\frac{1}{2}\left\{l_j(y) + l_j(y+1)\right\}$$

$$= \frac{1}{2}\pi_j(x) + \sum_{y=x+1}^{\infty} p_j(x,y), \quad j = 1,2, \qquad (1.7)$$

where $\pi_j(x) = p_j(x,x) = l_j(x)/l_0(x)$ is the prevalence of the alive state *j* at age *x*.

Note that (1.7) differs slightly from the health expectancies estimated in Davis et al. (2001) in which the factor 1/2 is missing. This, together with the use of revised data, means that the results below are not strictly comparable with those of the earlier work. Furthermore, we now directly evaluate estimates of the integrals defining the expectancies using the S-PLUS function `integrate` (see Section 4.5) and obtain standard errors by Monte Carlo methods. As it transpires the numerical differences are not large.

When it is necessary to distinguish marginal from conditional probabilities we use the notation

$$p_j(y) = l_j(y)/l_0(0), \quad j = 1,2,3,$$

to define the marginal probability of an individual being in state *j* at age *y*. It is conditional only on knowledge of the size $l_0(0)$ of the cohort at age 0. It follows that

$$p_0(y) = p_1(y) + p_2(y) = l_0(y)/l_0(0)$$

is the marginal probability of being alive at age *x* + *y*. For life expectancy $e_0(0)$ at birth the term 1/2 in (1.6) may be replaced by a different constant (see, for example, Chapter 9 of Chiang, 1968).

It is the estimation of the health expectancies (1.5) and (1.7) that is our major concern. Recall that they are marginal expectancies because the initial health state is not known. The only data to hand are the marginal totals of a multistate life table which for $y = x, x+1, ...$ can be represented by the sequence of tables one of which is shown in Table 1.1. With $l_{ij}(y, y+1)$ denoting the number of the cohort in state *i* at age *y* and in *j* at *y* + 1, observation of their values as shown in the body of the tables permits the estimation of the expectancies conditional on the initial health state. The health expectancy of state *j* at age *x*, conditional on being in alive state *i* at *x*, is then

$$e_{ij}(x) = \int_x^\infty p_{ij}(x, y)dy \, ,$$

with $p_{ij}(x, y)$ the conditional probability of being in *j* at *y*, given having been in *i* at *x*. A multiple logistic method for modelling the $p_{ij}(x, y)$ is given in Davis et al. (2002). However, $l_{ij}(y, y+1)$ is not available from the cross-sectional ABS surveys and we are constrained to marginal totals.

Table 1.1 Transition frequencies and marginal totals for ages *y* > *x*

	1	2	3	
1	$l_{11}(y, y+1)$	$l_{12}(y, y+1)$	$l_{13}(y, y+1)$	$l_1(y)$
2	$l_{21}(y, y+1)$	$l_{22}(y, y+1)$	$l_{23}(y, y+1)$	$l_2(y)$
3	0	0	$l_{33}(y, y+1)$	$l_3(y)$
	$l_1(y+1)$	$l_2(y+1)$	$l_3(y+1)$	$l_0(x)$

The sequence of probabilities $p_j(x, y)$, $y = x+1, x+2, ...,$ $j = 1, 2, 3$, form what is sometimes called a *compositional time series* (Aitchison, 1986, Brunsden and Smith, 1998) since their sum is constrained, in this case via $\sum_{j=1}^{3} p_j(x, y) = 1$. However, a theory beyond that of the multiple logistic transform and central limit theorem is unnecessary for our purposes and we refer to these special time series as of only passing interest. We also note that our approach is different to that usually adopted in the study of multistate life tables, as for example by Gill (1992), Hoem and Jensen (1982) and Land et al. (1994). First, we base discussion on stochastic processes in discrete rather than continuous time and argue that this makes the modelling of probabilities much easier. For example, the parameterisation, and hence age dependence, is determined by an exploratory analysis of the log odds and not determined beforehand. Secondly, for marginal expectancies a Markov condition is not required. Thirdly, our estimates are determined by standard regression procedures which are straightforward and widely understood.

The estimation of the probabilities $p_j(x,y)$, and hence the health expectancies $e_j(x)$, proceeds by first choosing any convenient state, coincidentally with Brunsden and Smith (1998), called the *reference state*, and forming the relative log odds as follows. If state 3 is chosen as the reference state, define

$$\xi_j(x,y) = \log\left\{\frac{p_j(x,y)}{p_3(x,y)}\right\} = \log\left\{\frac{l_j(y)}{l_3(y)}\right\}, \quad j = 1,2. \tag{1.8}$$

Given observations on the frequencies $l_j(y)$ for a sequence of ages $y = x+1, x+2, \ldots$, these log odds will be modelled and estimated by regression methods. This yields estimates of the probabilities since they satisfy the following logistic form:

$$\begin{aligned} p_3(x,y) &= \left\{1 + \sum_{j=1}^{2} e^{\xi_j(x,y)}\right\}^{-1} \\ p_j(x,y) &= p_3(x,y) e^{\xi_j(x,y)}, \quad j = 1,2. \end{aligned} \tag{1.9}$$

There are other multiple logistic transforms that could have been used (see, for example, Glonek and McCullagh, 1995) but we found the versions (1.8) and (1.9) convenient. An informal description of the argument is given in Section 2, with a more detailed treatment in Section 4.

1.3 The data

As mentioned earlier, the empirical part of this study is based on ABS data. This data includes life tables from 1978 to 2000, the Current Estimated Resident Population (CERP) for ages 60 to 99 from 1976 to 2000, and prevalences of Disability for each of the last three survey years, as follows:

1988: ages 60 to 79 by single years and 80 to 99 by 5 year age groups (i.e., 80-84, 85-89, etc.)
1993: ages 60 to 84 by single years and 85 to 99 by 5 year age groups
1998: ages 60 to 84 by single years and 85 to 99 by 5 year age groups.

Spline interpolation was used to obtain prevalences by single years for the older ages. Current ABS records for the 1981 survey are apparently incomplete and the prevalences given in Mathers (1991) were used for that year.

Table 1.2 shows the numerical values used for the prevalence of Disability. Prevalences for the other alive state, Disability-free, are obtained by subtracting these numbers from one. Tables 1.3-1.6 (one table for each survey year) exhibit the following frequencies by sex for ages $y = 61, 62, ..., 99$:

$l_1(y)$ = number in state 1 (Disability-free)
$l_2(y)$ = number in state 2 (Disabled)
$l_3(y)$ = number in state 3 (Dead).

The numbers alive at age 60 (obtained from the CERP) are also shown. Use of life table age-specific probabilities of survival for one year gave the hypothetical numbers of survivors at subsequent ages, and the age-specific numbers in states 1 and 2 were then found using the prevalences. Note that each row sum equals this quantity, $l_0(60) = \sum_{j=1}^{3} l_j(y)$. For each of the survey years, we will interpret the listed frequencies $l_j(y)$ as realisations of a sequence of hypothetical multinomial distributions with parameters $l_0(60)$, $p_j(60, y)$, j = 1,2,3, where $p_j(60, y)$ is the conditional probability that an individual alive at age 60 is in state *j* at age *y*.

Tables 1.3-1.6 show the frequencies used as data from which to estimate expectancies current for the survey years 1981, 1988, 1993 and 1998. The corresponding frequencies used to estimate cohort health expectancies for selected ages in 1980 are given in Tables 1.7-1.10. Since prevalences for a given birth cohort are observed on only four occasions some data augmentation procedure is required in order to use regression techniques. We follow the device introduced on page 1102 of Davis et al. (2001) of assuming that a given cohort and the four older ones have the same prevalences. This is a computational device, not dissimilar in principle to the use of moving averages, and prevalences from five neighbouring cohorts are used to estimate expectancies for only one cohort. The advantage is that there are now twenty, rather than four, data vectors on which to base the estimation. For example, for a cohort of known size aged x in 1980 we can now calculate frequencies of the three states Disability-free, Disabled and Dead for ages $x + 1$, $x + 2$, $x + 3$, $x + 4$, $x + 5$ from the 1981 survey, for ages $x + 8$, $x + 9$, $x + 10$, $x + 11$, $x + 12$ from the 1988 survey, for ages $x + 13$, ..., $x + 17$ from the 1993 survey, and for ages $x + 18$, ..., $x + 22$ from the fourth survey in 1998. Only a selection of these data vectors are tabulated here: ages 60, 65, 70 and 75 in 1980, as shown in Tables 1.7-1.10.

Table 1.2 Prevalences of Disability (state 2) by sex, age and year of survey

	Females				Males			
Age	1981	1988	1993	1998	1981	1988	1993	1998
60	0.22150	0.24010	0.25422	0.31896	0.33930	0.37786	0.43381	0.45497
61	0.23180	0.27160	0.29201	0.27809	0.35000	0.39450	0.38562	0.47829
62	0.23730	0.28130	0.23747	0.23418	0.36070	0.42285	0.39471	0.38555
63	0.25330	0.31199	0.25585	0.24911	0.36920	0.50317	0.44075	0.35976
64	0.27790	0.29165	0.28143	0.36554	0.36750	0.49580	0.42458	0.48390
65	0.28430	0.30131	0.33503	0.34684	0.36240	0.46159	0.45925	0.45668
66	0.29410	0.32061	0.32867	0.33406	0.36420	0.42785	0.42491	0.42133
67	0.31600	0.39977	0.36141	0.31017	0.34790	0.44746	0.40020	0.51757
68	0.32050	0.35163	0.37538	0.45787	0.33920	0.47028	0.49872	0.41600
69	0.31090	0.44413	0.35909	0.34669	0.34980	0.57617	0.42413	0.44397
70	0.33470	0.38441	0.39615	0.43506	0.35740	0.49731	0.52753	0.53866
71	0.35690	0.43505	0.45726	0.42973	0.36490	0.46513	0.57564	0.49042
72	0.36780	0.50119	0.49191	0.42173	0.40870	0.54019	0.59510	0.52246
73	0.37920	0.46839	0.57838	0.51126	0.42600	0.52440	0.62060	0.62986
74	0.42210	0.52061	0.54804	0.47423	0.44410	0.50588	0.61311	0.58511
75	0.43740	0.45644	0.48606	0.47170	0.45010	0.46109	0.54641	0.64163
76	0.45650	0.53031	0.48830	0.59585	0.46960	0.56042	0.59523	0.61111
77	0.47930	0.57390	0.57630	0.50779	0.46050	0.54470	0.63777	0.67032
78	0.50660	0.58994	0.52024	0.62169	0.47650	0.54734	0.51345	0.65725
79	0.51380	0.57442	0.67811	0.65658	0.49580	0.56669	0.73257	0.68097
80	0.52080	0.62707	0.47628	0.57640	0.51100	0.60199	0.71266	0.64610
81	0.53420	0.65331	0.58836	0.59311	0.51060	0.62223	0.65918	0.72179
82	0.56200	0.68071	0.62200	0.70278	0.52510	0.64358	0.84042	0.80872
83	0.57390	0.70901	0.67862	0.73970	0.53630	0.66578	0.87159	0.66780
84	0.58920	0.73757	0.74394	0.76358	0.54020	0.68887	0.63189	0.53344
85	0.61430	0.76570	0.70591	0.75865	0.55240	0.71288	0.79006	0.79061
86	0.59590	0.79274	0.72610	0.78021	0.57670	0.73787	0.80858	0.81566
87	0.59570	0.81799	0.74574	0.80042	0.56880	0.76388	0.82720	0.84249
88	0.61370	0.84107	0.76481	0.81919	0.61770	0.79079	0.84590	0.87032
89	0.63280	0.86212	0.78341	0.83660	0.69700	0.81828	0.86468	0.89775
90	0.65720	0.88131	0.80162	0.85271	0.73530	0.84603	0.88353	0.92335
91	0.69150	0.89883	0.81957	0.86758	0.79340	0.87371	0.90245	0.94569
92	0.71420	0.91486	0.83734	0.88129	0.80460	0.90102	0.92142	0.96342
93	0.76210	0.92950	0.85504	0.89392	0.84290	0.92786	0.94044	0.97651
94	0.82490	0.94279	0.87274	0.90556	0.85800	0.95435	0.95950	0.98600
95	0.89110	0.95475	0.89055	0.91632	0.93420	0.98060	0.97859	0.99298
96	0.95460	0.96542	0.90855	0.92629	0.96570	1.00000	0.99769	0.99851
97	0.99010	0.97484	0.92681	0.93558	1.00000	1.00000	1.00000	1.00000
98	1.00000	0.98324	0.94530	0.94428	1.00000	1.00000	1.00000	1.00000
99	1.00000	0.99091	0.96394	0.95253	1.00000	1.00000	1.00000	1.00000

Table 1.3 Observed current frequencies by sex and age in 1981

Age	Females $l_0(60) = 75412$			Males $l_0(60) = 69162$		
y	$l_1(y)$	$l_2(y)$	$l_3(y)$	$l_1(y)$	$l_2(y)$	$l_3(y)$
61	57469.2	17341.0	601.8	44212.2	23806.6	1143.2
62	56556.8	17596.6	1258.6	42698.6	24091.0	2372.3
63	54835.5	18601.6	1974.9	41297.6	24171.0	3693.4
64	52465.2	20191.2	2755.6	40510.7	23537.8	5113.5
65	51392.3	20414.7	3604.9	39864.2	22658.1	6639.7
66	50037.3	20847.1	4527.7	38709.4	22173.6	8279.1
67	47800.8	22083.4	5527.8	38554.8	20569.2	10038.0
68	46750.7	22050.9	6610.3	37824.4	19415.9	11921.7
69	46603.8	21026.2	7782.0	35910.2	19319.3	13932.5
70	44150.6	22211.3	9050.1	34116.6	18974.9	16070.5
71	41793.1	23193.8	10425.1	32283.7	18548.8	18329.5
72	40141.8	23353.6	11916.6	28654.8	19805.9	20701.3
73	38413.6	23464.0	13534.4	26396.1	19590.1	23175.7
74	34746.0	25378.6	15287.4	24138.6	19283.9	25739.5
75	32760.6	25470.1	17181.3	22428.4	18357.9	28375.7
76	30540.4	25651.7	19220.0	20205.3	17889.1	31067.6
77	28120.4	25884.6	21407.0	19078.8	16285.0	33798.2
78	25493.1	26175.1	23743.8	17073.2	15540.4	36548.4
79	23911.0	25268.4	26232.6	15057.6	14806.7	39297.7
80	22300.3	24236.2	28875.5	13269.7	13866.7	42025.5
81	20373.9	23365.7	31672.4	11965.9	12484.3	44711.8
82	17869.0	22927.8	34615.2	10366.7	11462.5	47332.8
83	16074.4	21650.0	37687.6	8949.7	10351.0	49861.3
84	14194.2	20358.4	40859.5	7766.0	9124.0	52272.0
85	12080.7	19240.8	44090.5	6542.6	8074.4	54545.0
86	11346.9	16732.5	47332.6	5290.1	7207.2	56664.6
87	10039.5	14792.3	50580.2	4535.3	5982.5	58644.2
88	8352.9	13269.9	53789.2	3325.2	5372.8	60464.0
89	6793.4	11707.1	56911.6	2137.4	4916.8	62107.8
90	5318.7	10196.8	59896.4	1481.8	4116.2	63564.0
91	3923.7	8794.9	62693.4	895.8	3440.0	64826.2
92	2927.9	7316.8	65167.3	645.7	2658.9	65857.4
93	1928.2	6177.0	67306.7	389.7	2091.1	66681.2
94	1102.6	5194.5	69114.9	260.8	1576.0	67325.1
95	523.2	4280.9	70608.0	88.4	1255.4	67818.2
96	163.4	3436.2	71812.4	33.4	939.8	68188.8
97	26.2	2623.7	72762.1	0.0	698.8	68463.2
98	0.0	1917.8	73494.2	0.0	498.1	68663.9
99	0.0	1365.6	74046.4	0.0	352.6	68809.4

Table 1.4 Observed current frequencies by sex and age in 1988

Age	Females $l_0(60) = 74790$			Males $l_0(60) = 75590$		
y	$l_1(y)$	$l_2(y)$	$l_3(y)$	$l_1(y)$	$l_2(y)$	$l_3(y)$
61	54086.9	20167.6	535.5	45133.3	29405.2	1051.5
62	52947.5	20723.4	1119.1	42355.2	31031.7	2203.1
63	50249.2	22786.6	1754.2	35839.1	36295.8	3455.1
64	51246.7	21099.7	2443.6	35688.4	35093.3	4808.3
65	50024.6	21573.0	3192.4	37326.0	31999.7	6264.3
66	48091.3	22694.4	4004.3	38771.6	28993.6	7824.8
67	41958.3	27945.4	4886.3	36519.8	29574.3	9495.9
68	44702.2	24243.1	5844.7	34064.6	30241.7	11283.7
69	37744.4	30157.1	6888.5	26443.8	35949.3	13196.9
70	41099.5	25665.3	8025.2	30336.4	30012.2	15241.5
71	37018.4	28507.3	9264.4	31112.9	27055.8	17421.3
72	32011.4	32163.8	10614.8	25681.6	30170.9	19737.5
73	33332.7	29369.0	12088.3	25397.5	28003.0	22189.5
74	29289.4	31807.8	13692.8	25110.7	25707.9	24771.4
75	32261.3	27090.3	15438.4	25930.6	22186.5	27472.9
76	26987.1	30470.6	17332.3	19916.6	25391.9	30281.5
77	23608.5	31797.9	19383.5	19308.3	23100.0	33181.7
78	21811.1	31379.1	21599.8	17851.0	21584.9	36154.1
79	21620.5	29182.0	23987.5	15778.2	20635.3	39176.5
80	17991.4	30251.6	26546.9	13281.1	20087.9	42221.0
81	15780.8	29738.0	29271.2	11459.5	18875.3	45255.2
82	13617.1	29030.9	32142.1	9746.0	17597.8	48246.2
83	11540.2	28117.7	35132.1	8164.4	16263.7	51161.9
84	9601.5	26984.9	38203.6	6725.7	14891.0	53973.3
85	7841.9	25628.1	41320.1	5435.0	13494.5	56660.5
86	6288.8	24053.7	44447.5	4294.3	12088.4	59207.3
87	4941.2	22206.3	47642.6	3297.7	10668.9	61623.4
88	3802.5	20123.6	50863.9	2449.5	9259.1	63881.4
89	2857.8	17868.9	54063.3	1750.6	7883.0	65956.4
90	2089.3	15514.1	57186.6	1195.1	6567.0	67827.9
91	1478.4	13135.6	60176.0	771.5	5337.6	69480.9
92	1014.9	10905.2	62869.9	467.1	4252.2	70870.7
93	672.8	8870.2	65247.0	258.0	3318.1	72013.9
94	428.6	7062.7	67298.7	121.3	2535.6	72933.1
95	260.7	5500.0	69029.3	37.5	1897.2	73655.3
96	149.9	4185.1	70455.0	0.0	1380.6	74209.4
97	80.2	3108.9	71600.8	0.0	965.6	74624.4
98	38.4	2253.1	72498.5	0.0	661.9	74928.1
99	14.6	1591.9	73183.5	0.0	444.9	75145.1

Table 1.5 Observed current frequencies by sex and age in 1993

Age	Females $l_0(60) = 69825$			Males $l_0(60) = 70366$		
y	$l_1(y)$	$l_2(y)$	$l_3(y)$	$l_1(y)$	$l_2(y)$	$l_3(y)$
61	49128.7	20262.7	433.6	42737.1	26824.7	804.3
62	52552.9	16365.9	906.2	41570.8	27108.2	1687.0
63	50902.9	17501.1	1421.0	37868.0	29844.6	2653.3
64	48749.8	19093.3	1981.9	38356.5	28301.2	3708.3
65	44706.8	22525.0	2593.2	35423.9	30084.7	4857.5
66	44687.1	21877.8	3260.1	36954.8	27304.5	6106.7
67	42043.1	23794.3	3987.7	37730.7	25174.7	7460.7
68	40626.3	24415.8	4783.0	30799.1	30642.4	8924.5
69	41128.3	23043.5	5653.2	34474.5	25390.5	10501.1
70	38174.9	25044.0	6606.2	27485.3	30687.9	12192.8
71	33744.9	28430.2	7649.9	23919.4	32446.4	14000.3
72	31009.4	30022.3	8793.3	22043.4	32398.0	15924.6
73	25205.4	34576.9	10042.7	19880.6	32519.8	17965.6
74	26402.9	32015.8	11406.3	19438.5	30804.0	20123.5
75	29259.9	27673.2	12891.9	21757.0	26209.5	22399.5
76	28305.2	27011.0	14508.8	18446.1	27125.5	24794.4
77	22693.3	30866.1	16265.6	15598.2	27463.3	27304.5
78	24781.8	26872.5	18170.7	19677.3	20764.8	29923.9
79	15964.3	33630.5	20230.2	10089.0	27637.0	32640.0
80	24813.0	22565.0	22447.1	10037.0	24893.1	35435.9
81	18525.7	26478.6	24820.7	10933.2	21145.9	38286.9
82	16056.3	26421.0	27347.7	4659.6	24539.8	41166.6
83	12793.3	27013.9	30017.8	3380.6	22946.1	44039.3
84	9477.0	27533.7	32814.3	8649.5	14847.6	46868.9
85	10031.5	24079.0	35714.4	4355.9	16391.9	49618.3
86	8527.1	22605.6	38692.3	3466.9	14644.7	52254.5
87	7136.4	20931.3	41757.3	2692.8	12890.4	54782.8
88	5868.3	19083.3	44873.4	2033.0	11160.0	57173.0
89	4727.8	17099.9	47997.3	1484.4	9485.3	59396.3
90	3718.7	15027.1	51079.2	1041.1	7897.6	61427.3
91	2843.8	12917.3	54063.9	694.6	6425.7	63245.7
92	2118.8	10907.0	56799.2	439.6	5155.3	64771.1
93	1533.0	9042.0	59250.0	258.6	4082.8	66024.6
94	1072.8	7357.2	61395.1	134.9	3196.3	67034.8
95	722.0	5874.9	63228.1	54.1	2472.5	67839.4
96	463.5	4604.4	64757.2	4.4	1888.8	68472.8
97	279.8	3542.6	66002.6	0.0	1400.7	68965.3
98	154.9	2676.9	66993.2	0.0	1022.8	69343.2
99	74.3	1987.5	67763.2	0.0	736.6	69629.4

Table 1.6 Observed current frequencies by sex and age in 1998

Age	Females $l_0(60) = 79167$			Males $l_0(60) = 79808$		
y	$l_1(y)$	$l_2(y)$	$l_3(y)$	$l_1(y)$	$l_2(y)$	$l_3(y)$
61	56824.6	21889.6	452.8	41218.4	37788.3	801.3
62	59903.2	18318.3	945.6	48003.2	30121.0	1683.8
63	58333.6	19352.0	1481.4	49396.9	27756.2	2654.9
64	48918.5	28184.5	2064.0	39269.0	36818.7	3720.3
65	49946.3	26522.1	2698.6	40705.1	34214.6	4888.3
66	50462.5	25313.8	3390.6	42615.4	31027.7	6164.9
67	51751.0	23269.1	4146.9	34855.8	37394.7	7557.5
68	40222.4	33971.0	4973.6	41309.4	29426.0	9072.6
69	47879.7	25408.6	5878.8	38417.1	30675.1	10715.8
70	40843.7	31453.7	6869.6	31055.4	36260.5	12492.2
71	40610.5	30601.7	7954.8	33327.7	32074.4	14405.9
72	40492.9	29531.5	9142.6	30252.0	33097.8	16458.3
73	33588.5	35135.5	10443.0	22637.5	38521.0	18649.5
74	35383.4	31915.3	11868.3	24407.7	34421.9	20978.4
75	34728.4	31008.2	13430.3	20199.4	36164.6	23444.0
76	25875.4	38148.9	15142.8	20907.6	32854.6	26045.7
77	30590.5	31558.5	17018.0	16822.1	34203.6	28782.2
78	22736.4	37362.9	19067.7	16506.4	31652.2	31649.4
79	19872.2	37993.8	21301.0	14409.6	30757.9	34640.5
80	23485.9	31957.2	23723.8	14886.7	27177.7	37743.5
81	21495.7	31333.9	26337.4	10813.3	28054.7	40940.0
82	14869.4	35158.6	29139.0	6810.2	28793.3	44204.5
83	12245.7	34798.7	32122.7	10730.9	21572.1	47505.0
84	10376.3	33512.8	35277.9	13532.5	15472.1	50803.4
85	9793.8	30784.7	38588.5	5391.6	20357.4	54058.9
86	8162.2	28974.5	42030.3	4162.3	18417.6	57228.1
87	6705.4	26891.4	45570.2	3077.7	16462.2	60268.0
88	5424.8	24578.5	49163.7	2161.8	14508.3	63137.9
89	4316.2	22099.0	52751.8	1432.1	12574.1	65801.8
90	3373.2	19528.5	56265.3	887.5	10691.6	68228.9
91	2586.7	16947.6	59632.8	511.3	8904.4	70392.2
92	1944.4	14435.2	62787.3	275.6	7258.5	72273.9
93	1431.5	12062.4	65673.1	139.5	5799.4	73869.1
94	1031.0	9885.7	68250.3	64.7	4554.1	75189.2
95	725.8	7947.8	70493.4	25.0	3528.5	76254.5
96	499.2	6273.0	72394.9	4.0	2704.8	77099.2
97	334.9	4863.2	73968.9	0.0	2044.0	77764.0
98	218.5	3702.6	75245.9	0.0	1525.2	78282.8
99	137.8	2765.8	76263.4	0.0	1126.1	78681.9

Table 1.7 Observed cohort frequencies by sex aged 60 in 1980

Age y	Year	Females $l_1(y)$	Females $l_2(y)$	Females $l_3(y)$	Males $l_1(y)$	Males $l_2(y)$	Males $l_3(y)$
61	1981	50433.3	15217.9	546.8	40140.1	21613.9	1068.0
62	1982	45516.7	14161.7	1057.6	34671.7	19562.1	2012.2
63	1983	44420.8	15068.7	1719.5	33974.6	19885.0	3260.4
64	1984	42720.9	16441.1	2433.0	34349.8	19958.2	4737.9
65	1985	43402.4	17240.9	3289.7	33332.3	18945.4	6154.3
68	1988	39310.6	21319.1	5568.3	27846.2	24721.1	10254.7
69	1989	30381.2	24274.1	6080.7	19260.0	26183.3	10802.7
70	1990	33226.2	20748.7	7234.1	22297.2	22058.9	12763.9
71	1991	30012.0	23111.7	8471.3	23474.5	20413.4	15158.1
72	1992	26852.4	26980.3	10100.3	19024.7	22350.3	17057.0
73	1993	23321.5	31992.6	10883.9	16760.7	27416.5	18644.8
74	1994	22304.2	27045.8	11386.1	14511.8	22996.7	18737.5
75	1995	24757.0	23414.4	13037.6	16273.4	19603.8	21242.8
76	1996	23944.3	22849.5	14801.1	14048.5	20658.7	24338.8
77	1997	19795.4	26924.5	17213.1	11559.9	20353.2	26518.8
78	1998	18158.6	29840.3	18199.1	11736.8	22506.3	28578.9
79	1999	14454.3	27635.2	18646.4	9025.7	19265.7	27954.7
80	2000	17064.0	23218.9	20926.2	9287.0	16954.6	30878.4
81	2001	15567.7	22692.8	23334.4	6824.3	17705.3	34516.4
82	2002	11065.6	26164.6	26702.8	4153.1	17559.5	36719.4

Table 1.8 Observed cohort frequencies by sex aged 65 in 1980

Age y	Year	Females $l_1(y)$	Females $l_2(y)$	Females $l_3(y)$	Males $l_1(y)$	Males $l_2(y)$	Males $l_3(y)$
66	1981	41762.4	17399.5	778.0	33386.4	19124.4	1503.2
67	1982	40013.0	18485.5	1653.5	33353.4	17794.3	3124.4
68	1983	37517.8	17696.0	2524.2	30970.0	15897.4	4664.5
69	1984	36474.6	16456.2	3470.2	29001.8	15602.6	6418.6
70	1985	34552.9	17382.9	4599.2	26712.1	14856.7	8163.3
73	1988	27569.8	24291.4	8078.8	19313.5	21294.9	13405.7
74	1989	24246.5	26331.3	9574.2	19049.4	19502.5	15720.2
75	1990	25557.1	21460.6	10720.3	18503.7	15831.9	17196.5
76	1991	20815.9	23502.9	12082.2	13921.0	17748.0	19354.0
77	1992	18178.5	24484.3	13872.1	12984.3	15534.1	21213.6
78	1993	21236.5	23028.1	15675.4	14829.5	15649.1	23535.4
79	1994	13637.1	28728.0	17786.9	7512.2	20578.3	26181.5
80	1995	20193.5	18364.0	19180.5	6961.8	17266.4	27303.8
81	1996	14606.5	20877.0	20917.5	7354.9	14225.0	29443.1
82	1997	12567.6	20680.3	23287.1	2986.4	15727.9	31017.7
83	1998	8916.4	25337.8	25685.8	6605.9	13279.6	34128.5
84	1999	7496.8	24212.9	28442.2	8167.4	9338.1	36766.5
85	2000	6708.0	21085.3	29944.7	3007.6	11355.9	37168.5
86	2001	5388.0	19126.6	31886.4	2233.9	9884.6	38904.5
87	2002	4371.7	17532.3	34631.0	1561.7	8353.1	39817.2

Table 1.9 Observed cohort frequencies by sex aged 70 in 1980

Age y	Year	Females $l_1(y)$	Females $l_2(y)$	Females $l_3(y)$	Males $l_1(y)$	Males $l_2(y)$	Males $l_3(y)$
71	1981	30162.9	16739.4	1022.7	23954.0	13762.9	1711.2
72	1982	27965.8	16269.9	2061.3	20663.7	14282.5	3422.9
73	1983	26362.7	16103.0	3227.4	18625.5	13823.1	5162.3
74	1984	22235.5	16240.9	4248.6	16473.5	13160.4	6958.1
75	1985	21714.0	16881.8	5740.2	14991.2	12270.5	8707.2
78	1988	15457.2	22237.9	10229.9	11282.4	13642.3	14503.3
79	1989	14754.5	19914.8	11627.7	9608.2	12565.9	16194.9
80	1990	12058.0	20274.9	13360.1	7842.4	11861.8	17906.8
81	1991	9824.2	18513.2	14387.6	6481.7	10676.3	19434.0
82	1992	8739.3	18631.8	16964.9	5341.9	9645.6	20981.5
83	1993	9158.2	19338.2	19428.5	2016.5	13686.6	23724.9
84	1994	6488.1	18850.0	20958.9	4923.2	8451.1	24994.7
85	1995	6693.7	16067.2	22932.1	2381.8	8963.2	26266.0
86	1996	5242.0	13896.7	23586.3	1799.6	7601.8	27190.7
87	1997	4485.4	13155.7	26694.9	1341.1	6420.1	28207.8
88	1998	3219.3	14585.9	30119.8	1013.7	6802.9	31611.4
89	1999	2439.8	12491.7	31365.6	637.0	5593.4	32138.6
90	2000	1854.9	10738.5	33099.6	377.3	4545.8	32687.9
91	2001	1306.8	8562.0	32856.2	205.1	3571.0	32816.0
92	2002	999.5	7420.4	35916.1	105.1	2768.6	33095.2

Table 1.10 Observed cohort frequencies by sex aged 75 in 1980

Age y	Year	Females $l_1(y)$	Females $l_2(y)$	Females $l_3(y)$	Males $l_1(y)$	Males $l_2(y)$	Males $l_3(y)$
76	1981	18479.5	15521.5	1250.0	12688.0	11233.6	1734.3
77	1982	16055.1	14778.6	2463.4	11380.7	9714.2	3335.0
78	1983	13874.5	14245.7	3740.7	9734.6	8860.7	4893.7
79	1984	12740.6	13463.9	5110.5	8182.8	8046.5	6295.7
80	1985	13029.4	14160.5	7620.2	7319.4	7648.7	8506.0
83	1988	6724.0	16383.0	12144.0	4153.9	8274.8	13227.3
84	1989	5244.3	14738.9	13313.8	3205.8	7097.9	14126.3
85	1990	4051.7	13241.3	14568.1	2438.2	6053.8	14997.0
86	1991	3152.7	12058.4	16103.9	1806.5	5085.2	15633.3
87	1992	2689.6	12087.3	20033.2	1386.5	4485.6	17601.9
88	1993	3319.3	10794.0	21137.7	929.0	5099.7	19627.3
89	1994	2482.8	8979.9	21834.3	627.9	4012.5	19789.6
90	1995	1828.9	7390.5	22641.5	409.8	3109.0	19970.2
91	1996	1341.6	6093.7	23879.7	253.3	2343.0	19928.7
92	1997	1068.6	5501.0	28240.4	158.0	1852.4	21463.7
93	1998	656.6	5533.0	29061.4	48.2	2003.1	23604.7
94	1999	432.8	4150.0	28714.2	20.3	1427.4	22982.4
95	2000	280.2	3068.0	28512.8	7.1	1007.7	22474.2
96	2001	180.9	2273.0	28861.2	1.0	699.8	21824.1
97	2002	126.4	1836.2	32847.4	0.5	508.8	22964.7

2 ESTIMATION

2.1 Preliminaries

Health expectancies are usually estimated by a method due to Sullivan (1971). Bebbington (1988) and Mathers et al. (1994), amongst others, discuss the approach, which can be summarised as follows. From a life table obtain $L(y)$, the total number of years lived in $(y, y+1)$. Combine this with an estimate of the prevalence $\pi_j(y)$ of state j at age y to obtain what is called the Sullivan estimate of the health expectancy of state j at age x:

$$e_j(x)^{(S)} = \sum_{y=x}^{\infty} \frac{\pi_j(y)L(y)}{l_0(x)}.$$

The essential difference in our approach is that the summands $\pi_j(y)L(y)/l_0(x)$ are replaced by parameterised probabilities which are then estimated by regression techniques.

The methods used in this paper are a development of those in Davis et al. (2001). In particular the formulae now used to calculate estimates of cohort health expectancies are, we believe, more accurate than the earlier ones. Also, as mentioned, use of revised figures and the fourth ABS survey mean that estimates of cohort health expectancies obtained here and in the earlier paper are therefore not strictly comparable. We consider current (period) expectancies using a variation of the methodology.

Given a cohort of a known number $l_0(x)$ of lives at age x, at a future age y they will be distributed over the states 1 (Disability-free), 2 (Disabled) and 3 (Dead). For j = 1,2,3 introduce the sequence of random variables

$$\tilde{l}_j(y) = \text{number of the } l_0(x) \text{ in state } j \text{ at age } y,\ \ y = x+1, x+2, \ldots$$

For each y it is then true that

$$\tilde{l}_1(y) + \tilde{l}_2(y) + \tilde{l}_3(y) = l_0(x),$$

with $\tilde{l}_3(y)$ nondecreasing since Death is an absorbing state. Movement in either direction between states 1 and 2 is possible before the inevitable passage into state 3. Migration is not considered. However, it is worth commenting that, at the cost of some complication, if suitable data were available then migration could be built into the model.

The simplest, and in many ways the most natural, assumption is that for fixed y the three random variables $\tilde{l}_1(y)$, $\tilde{l}_2(y)$ and $\tilde{l}_3(y)$ satisfy a multinomial distribution. This holds if all $l_0(x)$ individuals in the cohort are independent and identically distributed as far as the states Disability-free, Disabled and Dead are concerned and is a plausible approximation in many applications. It is an immediate generalisation of the assumption of a binomial distribution for the number of survivors in a cohort life table. The general theory presented in Section 4 does not require this condition.

Under the multinomial distribution of frequencies the expectation and variance of $\tilde{l}_j(y)$ are, respectively:

$$E\tilde{l}_j(y) = l_0(x)p_j(x,y) = l_j(y)$$

$$Var\tilde{l}_j(y) = l_0(x)p_j(x,y)\{1-p_j(x,y)\}, \qquad (2.1)$$

with covariances

$$Cov\{\tilde{l}_i(y),\tilde{l}_j(y)\} = -l_0(x)p_i(x,y)p_j(x,y).$$

The maximum likelihood estimator (MLE) of $p_j(x,y)$ is

$$\tilde{p}_j(x,y) = \tilde{l}_j(y)/l_0(x),$$

and we use vector logistic regression to model and estimate a parameterised $p_j(x,y)$ based on realised values of the random variables $\tilde{l}_j(y)$.

It is important to distinguish between random variables and unknown but fixed quantities. A superscript tilde will always signify a random variable, as in $\tilde{p}_j(x,y)$. The random variable's expectation, or more generally what it is estimating, will be denoted by the same symbol without the superscript tilde, as in $p_j(x,y)$.

We found it convenient to take state 3 (Death) as reference, although the reasons were not compelling. In that case,

$$\tilde{\xi}_j(x,y) = \log\left\{\frac{\tilde{p}_j(x,y)}{\tilde{p}_3(x,y)}\right\} = \log\left\{\frac{\tilde{l}_j(y)}{\tilde{l}_3(y)}\right\}, \quad j = 1,2, \tag{2.2}$$

estimates the log odds

$$\xi_j(x,y) = \log\left\{\frac{p_j(x,y)}{p_3(x,y)}\right\}$$

of (1.8). It is shown in Davis (2001) and in Section 4 that for a large cohort (roughly greater than a few hundred in number) these estimates are approximately jointly normally distributed with means $\xi_j(x,y)$, $j = 1,2$, and covariance matrix

$$U(x,y) = \begin{pmatrix} \dfrac{1}{l_1(y)} + \dfrac{1}{l_3(y)} & \dfrac{1}{l_3(y)} \\ \dfrac{1}{l_3(y)} & \dfrac{1}{l_2(y)} + \dfrac{1}{l_3(y)} \end{pmatrix}. \tag{2.3}$$

Suppose that the log odds are parameterised, in particular as polynomials in y, and perhaps also x, together with possibly other predictors describing socioeconomic variables. To be specific, let

$$\xi_j(x,y) = \xi_j(x,y;\beta^{(j)}) = z^{(j)}(x,y)'\beta^{(j)} = \sum_{r=1}^{k(j)} z_r^{(j)}(x,y)\beta_r^{(j)}, \tag{2.4}$$

where the form of the right hand side is determined by exploratory analysis and the $\beta^{(j)} = (\beta_1^{(j)},...,\beta_{k(j)}^{(j)})'$ are parameter vectors to be estimated in relation to the chosen covariate vectors $z^{(j)}(x,y) = (z_1^{(j)}(x,y),...,z_{k(j)}^{(j)}(x,y))'$. It is convenient to rewrite (2.4) in vector notation as

$$\xi(x,y) = \xi(x,y;\beta) = \begin{pmatrix} \xi_1(x,y) \\ \xi_2(x,y) \end{pmatrix} = Z'\beta,$$

where:

$$Z = Z(x,y) = \begin{pmatrix} z^{(1)} & 0 \\ 0 & z^{(2)} \end{pmatrix}, \qquad z^{(j)} = z^{(j)}(x,y)$$

$$\beta = \begin{pmatrix} \beta^{(1)} \\ \beta^{(2)} \end{pmatrix} = (\beta_1^{(1)},...,\beta_{k(1)}^{(1)},\beta_1^{(2)},...,\beta_{k(2)}^{(2)})' \equiv (\beta_1,...,\beta_{k(1)+k(2)})' .$$

For a large population we have for $y = x+1, x+2,...$ the vector linear model

$$\xi(x,y) = \begin{pmatrix} \xi_1(x,y) \\ \xi_2(x,y) \end{pmatrix} = Z'\beta + \varepsilon(x,y) , \qquad (2.5)$$

with the error vectors $\varepsilon(x,y) = (\varepsilon_1(x,y), \varepsilon_2(x,y))'$ being approximately normally distributed with zero means and covariance matrices $U(x,y)$ of (2.3). Ignoring possible autocorrelations for the moment the parameter vector β can now be estimated by minimising the weighted least squares loss function

$$L(\beta) = \sum_{y=x+1}^{w} \{\tilde{\xi}(x,y) - Z'\beta\}' U(x,y)^{-1} \{\tilde{\xi}(x,y) - Z'\beta\} . \qquad (2.6)$$

These arguments give estimates of $\beta^{(1)}$ and $\beta^{(2)}$, which in turn yield estimates of the log odds and thence estimates of the $p_j(x,y)$ and the health expectancies. If $\hat{\beta}^{(1)}$ and $\hat{\beta}^{(2)}$ are the least squares estimates of $\beta^{(1)}$ and $\beta^{(2)}$, the other estimates are:

$$\hat{\xi}_j(x,y) = z^{(j)}(x,y)'\hat{\beta}^{(j)} , \quad j = 1,2$$

$$\hat{p}_3(x,y) = \left\{ 1 + \sum_{j=1}^{2} e^{\hat{\xi}_j(x,y)} \right\}^{-1} \qquad (2.7)$$

$$\hat{p}_j(x,y) = \hat{p}_3(x,y) e^{\hat{\xi}_j(x,y)} , \quad j = 1,2.$$

These formulae are appropriate when state 3 is the reference state, and different but corresponding expressions would hold if either states 1 or 2 had been selected as reference. What favours state 3 is that with this choice all the ratios $l_j(y)/l_3(y)$ are generally bounded. However, the ratios approach zero as y increases, with consequent instability at very old ages; but this is a problem that seems unavoidable.

The estimators given by (2.7) are consistent for the corresponding population quantities. However, the calculation of standard errors depends on whether one is dealing with current (period) data or birth cohort data. In the latter case standard errors cannot be obtained by the usual arguments because the sequence $\tilde{\xi}_j(x,y)$, $y = x+1, x+2, \ldots$, turns out to be autocorrelated with a correlation structure that cannot be estimated from cross-sectional data. Special methods must be used to calculate the standard errors for cohorts, and these are described in Davis et al. (2001) and in Section 4. On the other hand, standard errors can be calculated by well known methods in the case of period data.

Finally, the health expectancies $e_1(x)$ and $e_2(x)$ at age *x* are estimated from (2.7) by

$$\hat{e}_j(x) = \int_x^w \hat{p}_j(x,y)dy . \tag{2.8}$$

Given a formula for $\hat{p}_j(x,y)$, and numbers for *x* and the maximum age *w*, the integral is evaluated using the `integrate` function in S-PLUS (see Section 4.5). In the numerical results that follow we take *w* = 110 for current expectancies and *w* = 95 for the cohort case. Thus only partial expectancies are given for the latter, the reason being that it proved difficult to satisfactorily model $p_j(x,y)$ at high ages of a birth cohort, a point to which we return later.

Observe that having used (2.8) to estimate the expectancies at age *x*, it is possible to estimate $e_j(u)$ for an older age *u* > *x*, as follows. For the 'alive' states *j* = 1,2, and assuming *y* > *u*,

$p_j(x,y)$ = Pr(Individual is alive at *u* and in *j* at *y* | Alive at *x*)

= Pr(Alive at *u* | Alive at *x*) Pr(In *j* at *y* | Alive at *u*)

$$= \left\{ \sum_{i=1}^{2} p_i(x,u) \right\} p_j(u,y) .$$

It follows for each *u* between *x* and *w* that

$$\hat{e}_j(u) = \int_u^w \hat{p}_j(u,y)dy = \left\{ \sum_{i=1}^{2} \hat{p}_i(x,u) \right\}^{-1} \int_u^w \hat{p}_j(x,y)dy . \tag{2.9}$$

2.2 Current health expectancies

As is standard practice, the procedure followed in discussing health expectancies current at a given year is to estimate them for a synthetic cohort that has the same prevalences and mortality as the population existing in the year in question. The earlier theory therefore applies with an important simplification. This is that provided cohorts are stochastically independent the error vectors $\varepsilon(x,y)$ in the regression model (2.5) can also be taken as independent, in which case standard errors of estimates are given by well known expressions (see Section 4).

To illustrate the method consider females aged 60 in 1981. Numbers of lives were calculated using age-specific probabilities of death obtained from the 1981 ABS life table, with initial numbers at age 60 given by the CERP. Prevalences from Mathers (1991) were then used to obtain the observed frequencies of Table 1.3, as discussed in Section 1.3.

After some experimentation it was again found convenient to take state 3 (Death) as reference. A plot of the observed log odds $\xi_1(60,y)$ and $\xi_2(60,y)$ suggested fitting regressions that were quadratic for ages near 60 and in the nineties, and the models estimated were of the form:

$$\xi_1(60,y) = \beta_1 + \beta_2 y + \beta_3 g_3(y) + \beta_4 g_4(y)$$

$$\xi_2(60,y) = \beta_5 + \beta_6 y + \beta_7 g_7(y) + \beta_8 g_8(y),$$

where:

$$g_3(y) = g_7(y) = (y-66)^2 I(y<66)$$

$$g_4(y) = g_8(y) = (y-90)^2 I(y>90).$$

Here, $I(.)$ is the standard indicator function, so that, for example, $g_3(y)$ is the function which equals $(y-66)^2$ if $y < 66$ and 0 otherwise. For the case of females aged 60 in 1981 it was deemed appropriate to use the same quadratic function for both log odds. This was not so for all the regressions performed. The almost linear decline in the log odds for ages roughly 66 to 89 should be compared with the accelerated slopes of the end quadratics, indicating a different regime in the early sixties and after ninety years of age.

Weighted least squares estimates were found to be:

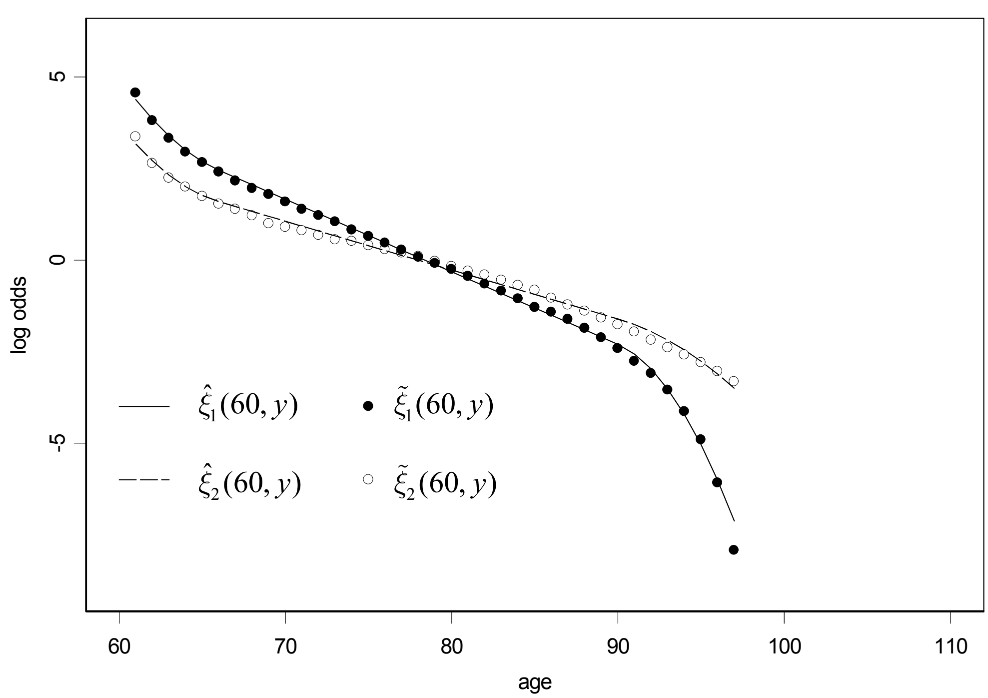
$$\hat{\xi}_1(60,y) = \underset{(0.0056)}{3.6409} - \underset{(0.0003)}{0.1981}y + \underset{(0.0011)}{0.0377}g_3(y) - \underset{(0.0010)}{0.0702}g_4(y)$$

$$\hat{\xi}_2(60,y) = \underset{(0.0057)}{2.3883} - \underset{(0.0003)}{0.1333}y + \underset{(0.0011)}{0.0367}g_7(y) - \underset{(0.0003)}{0.0196}g_8(y).$$

Here, the values in brackets under the eight parameter estimates are the associated standard errors. They are very small because, as already explained, only smoothed data was available for the analysis, and the closeness of the fit is illustrated in Figure 2.1.

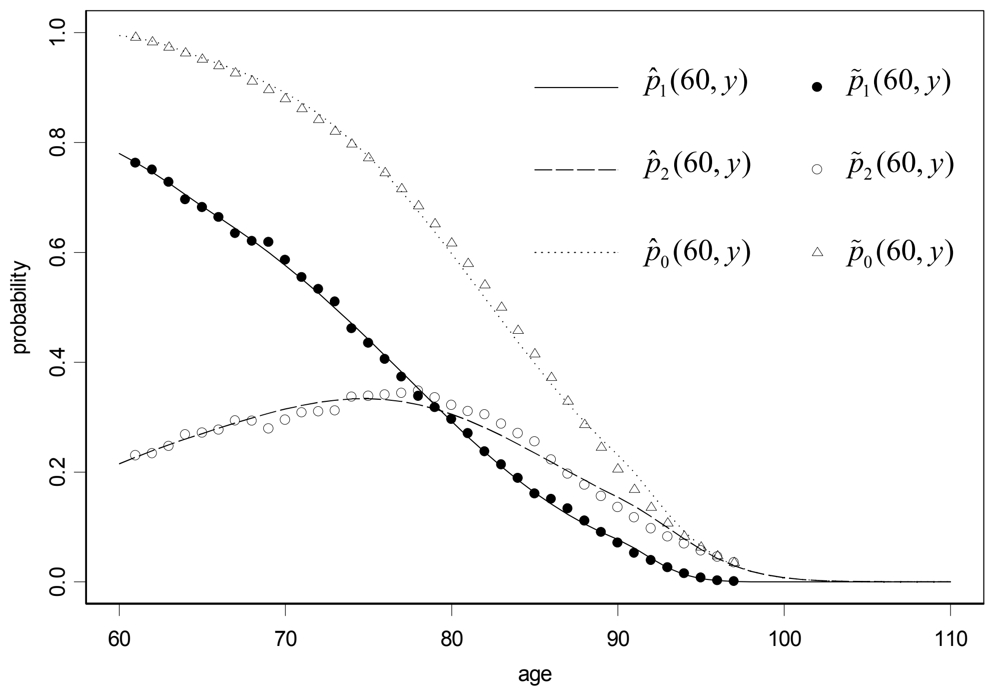
Figure 2.1 exhibits observed and estimated log odds. Note that a slightly different $g_3(y)$ is suggested by the observed $\xi_2(60,y)$, i.e. $\tilde{\xi}_2(60,y)$, but the improvement in fit is minimal and some computational convenience is gained by having the same tail quadratic for both log odds. Also, recall that we were obliged to use the already graduated prevalences of Mathers (1991). This means that if the original unsmoothed data had been used, the fits currently shown in Figure 2.1 would appear better, because the observed log odds would be more scattered.

Figure 2.1 Current log odds for females aged 60 in 1981



Estimates of the probabilities follow immediately from (2.7). Plots of observed and fitted $p_j(60, y)$, j = 1,2, together with their sum $p_0(60, y)$, are shown below.

Figure 2.2 Current probabilities for females aged 60 in 1981



Comparisons between the probabilities of the states Disability-free and Disabled can be made by examining their ratio. From the definition of log odds we find that

$$\log\left\{\frac{p_1(x, y)}{p_2(x, y)}\right\} = \xi_1(x, y) - \xi_2(x, y),$$

and hence

$$\frac{\hat{p}_1(x, y)}{\hat{p}_2(x, y)} = \exp\{\hat{\xi}_1(x, y) - \hat{\xi}_2(x, y)\}.$$

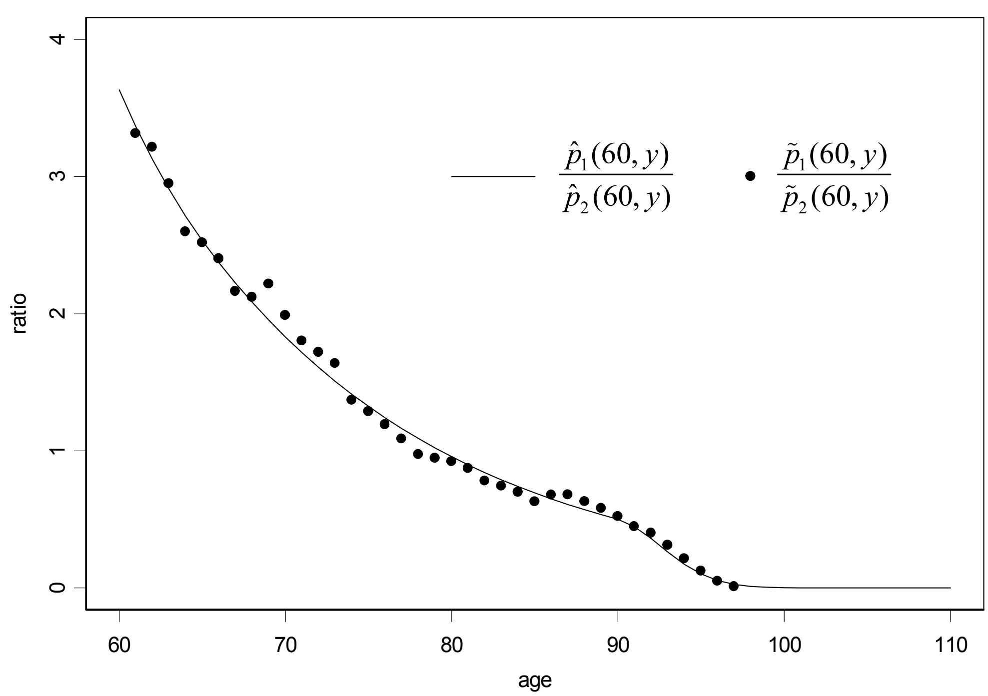
Thus the asymptotic normality of $\hat{\xi}_1(x,y)-\hat{\xi}_2(x,y)$ implies a log-normal distribution for the ratio of probability estimators. In particular, if a 95% confidence interval for $\xi_1(x,y)-\xi_2(x,y)$ is

$$(\hat{\xi}_1(x,y)-\hat{\xi}_2(x,y)\pm 1.96s)\equiv(a,b),$$

then a 95% confidence interval for $p_1(x,y)/p_2(x,y)$ is (e^a,e^b).

Figure 2.3 shows the ratio for the case x = 60 of current interest. Due to the use of smoothed prevalences, the 95% confidence band was very narrow and is not shown. Observe that the two probabilities are estimated to be equal at age about 79.5, some two and one half years less than the life expectancy of 82 for a 60 year old female in 1981. Observe also the change in slope at about age 92 due to an increased probability of disability.

Figure 2.3 Current ratio of probabilities for females aged 60 in 1981



Finally for 60 year old females in 1981, current health expectancies are estimated by

$$\hat{e}_j(60) = \int_{60}^{110} \hat{p}_j(60, y)dy .$$

Here the integral defining expectancies is to age 110 and yields

$$\hat{e}_1(60) = \underset{(0.01)}{13.18}, \qquad \hat{e}_2(60) = \underset{(0.01)}{9.00}, \qquad \hat{e}_0(60) = \underset{(0.01)}{22.18}.$$

It is interesting to truncate the summation to the life expectancy of 82 to obtain

$$\hat{e}_1(60,82) = 11.81, \quad \hat{e}_2(60,82) = 6.56, \quad \hat{e}_0(60,82) = 18.36.$$

Observe that

$$\frac{\hat{e}_1(60,82)}{\hat{e}_1(60)} = 0.90, \qquad \frac{\hat{e}_2(60,82)}{\hat{e}_2(60)} = 0.73,$$

indicating that the bulk of Disability-free life expectancy occurs before the life expectancy. Further results are presented in Section 3.

2.3 Cohort health expectancies

A common difficulty when estimating measures on a birth cohort is the sparseness of data. For regression methods to be useful one requires more than the four data points per birth cohort (the term will always be used in this sense) available from the surveys of 1981, 1988, 1993 and 1998. As noted in Section 1.3 we use the working device of Davis et al. (2001) and assume that, for a given age in 1980, the prevalences for the older four cohorts are applicable to the four subsequent ages of the cohort of interest. We therefore have a total of twenty points, rather than four, for each cohort and this is sufficient for the purposes of regression analysis. Inaccuracies are inevitably introduced, but there seems no alternative to augmenting the data in some way when dealing with cohort quantities, and the real differences in prevalences of health states between neighbouring cohorts are arguably small. This is a problem that does not arise when estimating current health expectancies.

Thus for a known number of females aged 60 in 1980, calculate the realised values $l_1(y)$, $l_2(y)$, $l_3(y)$ for ages y = 61,62,63,64,65 from the 1981 survey, for y = 68,...,72 from the 1988 survey, for y = 73,...,77 from the 1993 survey, and for y = 78,...,82 from the fourth survey in 1998. We assume that these data, shown in Table 1.7, provide a satisfactory surrogate for a sample of frequencies at the twenty specified ages of the cohort of females aged 60 in 1980. Next, use these counts to plot the observed log odds as a function of age as shown in the following figures. This suggests the model to be fitted. In the particular cases under immediate consideration we find a linear or quadratic fit to be appropriate, as shown.

A difficulty that arises when estimating cohort expectancies is that the stochastic process generating the multinomial frequencies is autocorrelated in an awkward way. More specifically, the log odds at two different ages are asymptotically correlated in a way that depends on transition probabilities that cannot be estimated from cross-sectional data. As in Davis et al. (2001) we use the method of Liang and Zeger (1986) to calculate standard errors and this is described in Section 4. This means that we estimate the probabilities and expectancies by the weighted least squares method of Section 2.2 but calculate the standard errors differently.

Partly for illustrative purposes, we now take state 1 (Disability-free) as reference and define the log odds as

$$\eta_j(x,y) = \log\left\{\frac{p_j(x,y)}{p_1(x,y)}\right\}, \quad j = 2,3. \tag{2.10}$$

These are estimated by

$$\tilde{\eta}_j(x,y) = \log\left\{\frac{\tilde{l}_j(y)}{\tilde{l}_1(y)}\right\}, \quad j = 2,3.$$

The earlier argument holds with (because of the changed reference state) the covariance matrix $U(x,y)$ of (2.3) replaced by

$$U_1(x,y) = \begin{pmatrix} \frac{1}{l_2(y)} + \frac{1}{l_1(y)} & \frac{1}{l_1(y)} \\ \frac{1}{l_1(y)} & \frac{1}{l_3(y)} + \frac{1}{l_1(y)} \end{pmatrix}. \tag{2.11}$$

With $U_1(x,y)$ instead of $U(x,y)$ and obvious notational changes, minimising the appropriate loss function (2.6) leads to estimates of the health expectancies.

As an illustration, consider the cohort of females aged 60 in 1980. Observed and fitted $\eta_2(60,y)$ and $\eta_3(60,y)$ are shown in Figure 2.4. The maximum age for which data is available from this cohort is 82, meaning that there is insufficient information to model log odds for high ages. This is apparent from the plot. A linear fit was used in the case of $\eta_2(60,y)$ and a lower tail quadratic superimposed on a straight line was used to model $\eta_3(60,y)$, giving the estimates:

$$\begin{aligned} \hat{\eta}_2(60,y) &= \underset{(0.0250)}{-1.2832} + \underset{(0.0057)}{0.0930}y \\ \hat{\eta}_3(60,y) &= \underset{(0.0486)}{-3.4008} + \underset{(0.0055)}{0.1892}y - \underset{(0.0025)}{0.0487}g_7(y) \end{aligned} \tag{2.12}$$

where $g_7(y) = (y-66)^2 I(y<66)$.

Estimates of the probabilities are:

$$\hat{p}_1(60,y) = \left\{1 + \sum_{j=2}^{3} e^{\hat{\eta}_j(60,y)}\right\}^{-1}$$

$$\hat{p}_j(60,y) = \hat{p}_1(60,y)e^{\hat{\eta}_j(60,y)}, \quad j = 2,3.$$

Figure 2.5 exhibits $\hat{p}_1(60,y)$, $\hat{p}_2(60,y)$ and their sum, $\hat{p}_0(60,y)$. Again it is clear that extrapolation as shown by the dotted line is inappropriate and in particular leads to an excessively heightened survival curve. Figure 2.6 shows the ratio $\hat{p}_1(60,y)/\hat{p}_2(60,y)$.

As mentioned earlier, lack of data at high ages means that health expectancies to age 95 are being estimated for cohorts, according to

$$\hat{e}_j(x) = \int_x^{95} \hat{p}_j(x,y)dy. \tag{2.13}$$

For the female cohort under consideration, $x = 60$ and

$$\hat{e}_1(60) = \underset{(0.31)}{11.96}, \qquad \hat{e}_2(60) = \underset{(0.38)}{11.99}, \qquad \hat{e}_0(60) = \underset{(0.13)}{23.95}.$$

Standard errors were calculated by the Liang-Zeger method described in Section 4.4.

Figure 2.4 Cohort log odds for females aged 60 in 1980

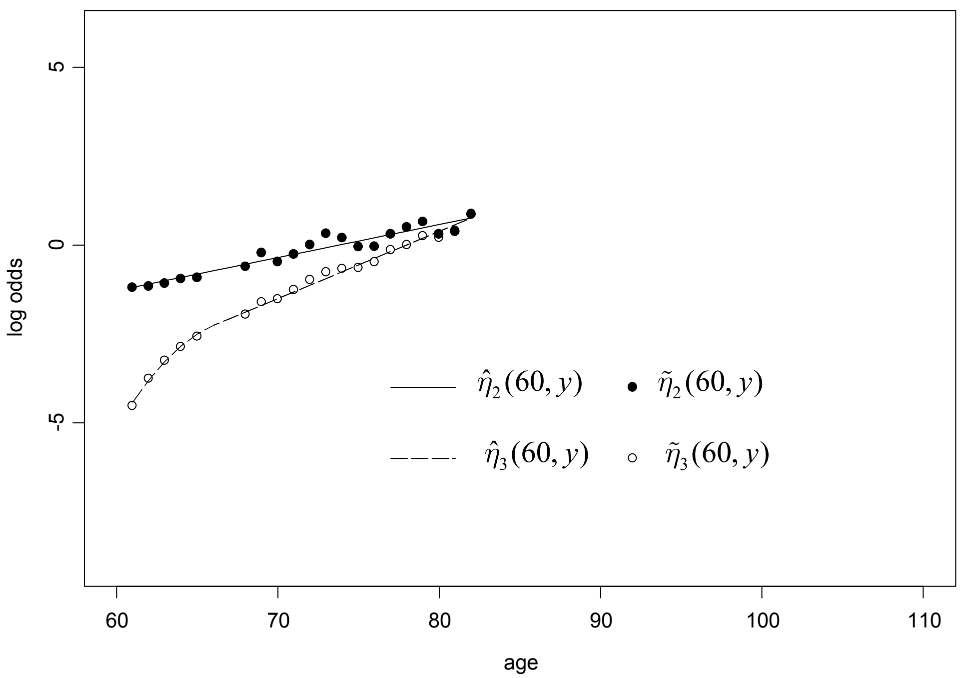


Figure 2.5 Cohort probabilities for females aged 60 in 1980

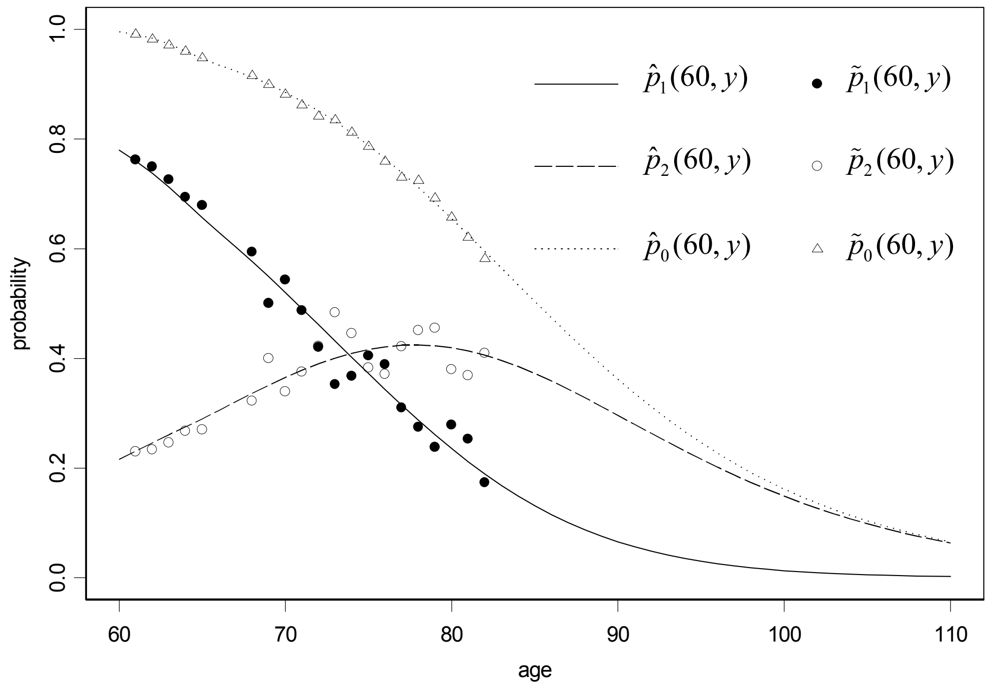
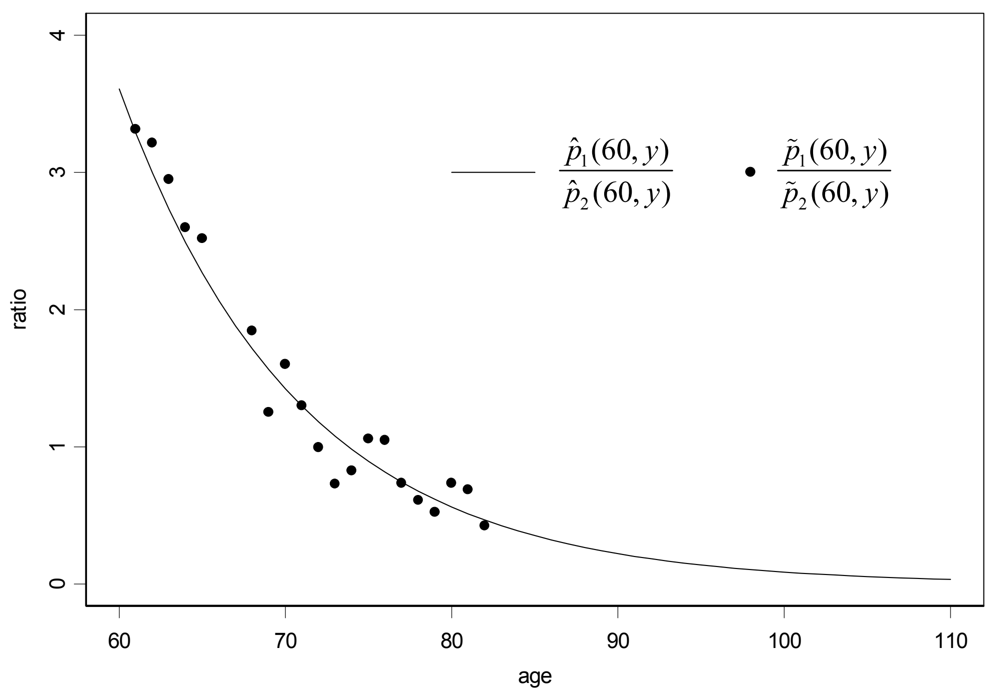


Figure 2.6 Cohort ratio of probabilities for females aged 60 in 1980



3 RESULTS

This section presents numerical results for Australian females and males aged 60 and over as derived from the data given in Section 1.3. It is the frequencies $l_1(y)$, $l_2(y)$ and $l_3(y)$ of Table 1.2-1.10 that provide the numbers on which all inferences are based.

3.1 Current health expectancies

The methods applied in Subsection 2.2 to obtain probabilities and expectancies for females aged 60 in 1981 are now used to estimate corresponding quantities for females and males aged 60 and over for the other survey years. Figures 3.1-3.3 plot, respectively, log odds, probabilities and ratios of probabilities for 60 year olds in the survey years, and correspond to Figures 2.1-2.3. Details of the regressions are shown in Table 3.1. The fitted log odds are of the form:

$$\begin{aligned}\hat{\xi}_1(60,y) &= \hat{\beta}_1 + \hat{\beta}_2 y + \hat{\beta}_3 g_3(y) + \hat{\beta}_4 g_4(y) \\ \hat{\xi}_2(60,y) &= \hat{\beta}_5 + \hat{\beta}_6 y + \hat{\beta}_7 g_7(y) + \hat{\beta}_8 g_8(y),\end{aligned} \tag{3.1}$$

with the $g_k(y)$ being quadratics similar to those in Section 2.2.

The quadratic parts of the log odds exhibit a decline much sharper than the steady decrease of the central linear component (see Figure 3.1 and Table 3.1), particularly in the case of $\hat{\xi}_1(60,y)$, the log odds of being in the Disability-free state relative to death. For females the reason seems to be that while disability-free numbers decline in a more or less linear manner until the late eighties, the rate of increase in the numbers of dead increases fairly slowly at first and accelerates through the eighties before flattening at very high ages. This is illustrated in Figure 3.2. For males under seventy the picture is complicated by the 'retirement effect'. Further investigation is called for into the decline of the log odds in the sixties and nineties. Note also that generally the estimated Disability-free survival curves $\hat{p}_1(60,y)$ have sharper and more uniform rates of decrease than the survival curves $\hat{p}_0(60,y)$, except for ages above the late eighties when the probability of death accelerates.

Estimates $\hat{p}_j(60,y)$, $j = 0,1,2$, which are plotted in Figure 3.2, are tabulated in Tables 3.2-3.5, and the health expectancies $\hat{e}_j(x)$ are presented in Tables 3.6-3.9 for ages $x = 60$ and above. Comparisons are made in Section 3.2.

Observe that Tables 3.2-3.5 have first row $\hat{p}_j(60,60)$, which is an estimate of the probability of being in state *j* at 60 conditional on being alive at 60. That is, $\hat{p}_j(60,60) = \hat{\pi}_j(60)$ is an estimate of the prevalence of state *j* at the initial age 60. It is given by

$$\hat{p}_j(60,60) = \frac{e^{\hat{\xi}_j(60,60)}}{e^{\hat{\xi}_1(60,60)} + e^{\hat{\xi}_2(60,60)}}.$$

The exact value of $p_0(60,60)$ is unity. Thus our estimates slightly undershoot the initial probability of being alive.

Coefficients $\hat{\beta}_2$ and $\hat{\beta}_6$ of the linear term provide useful information about log odds ratios. For example, between the two ages at which the quadratics commence,

$$\hat{\beta}_2 = \log\left(\frac{\hat{p}_1(60,y)}{\hat{p}_3(60,y)}\right) - \log\left(\frac{\hat{p}_1(60,y-1)}{\hat{p}_3(60,y-1)}\right),$$

so that

$$\frac{\hat{p}_1(60,y)}{\hat{p}_3(60,y)} = \frac{\hat{p}_1(60,y-1)}{\hat{p}_3(60,y-1)} e^{\hat{\beta}_2}.$$

For both sexes $\hat{\beta}_2$ is close to –0.2. Thus $e^{-0.2}$ = 0.82 gives a summary measure of how the odds of Disability-free relative to Death are changing from year to year. The corresponding number for Disability is $e^{\hat{\beta}_6} \approx e^{-0.13}$ = 0.88.

Figure 3.1 Current log odds for all survey years

(The legend and axes are the same as in Figure 2.1:
solid line = $\hat{\xi}_1(60, y)$, dashed line = $\hat{\xi}_2(60, y)$.)

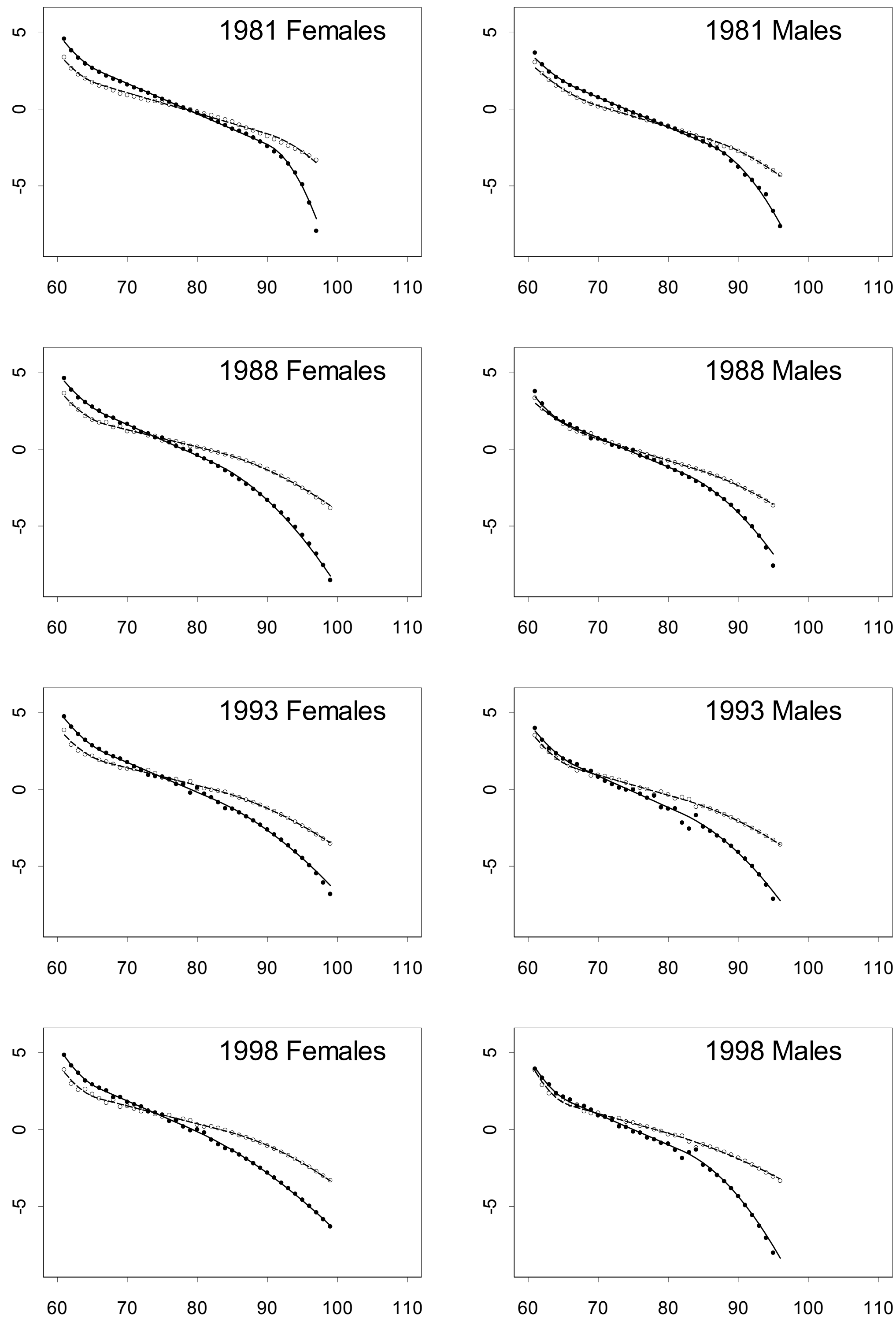


Figure 3.2 Current probabilities for all survey years

(The legend and axes are the same as in Figure 2.2:
solid line = $\hat{p}_1(60, y)$, dashed line = $\hat{p}_2(60, y)$, dotted line = $\hat{p}_0(60, y)$.)

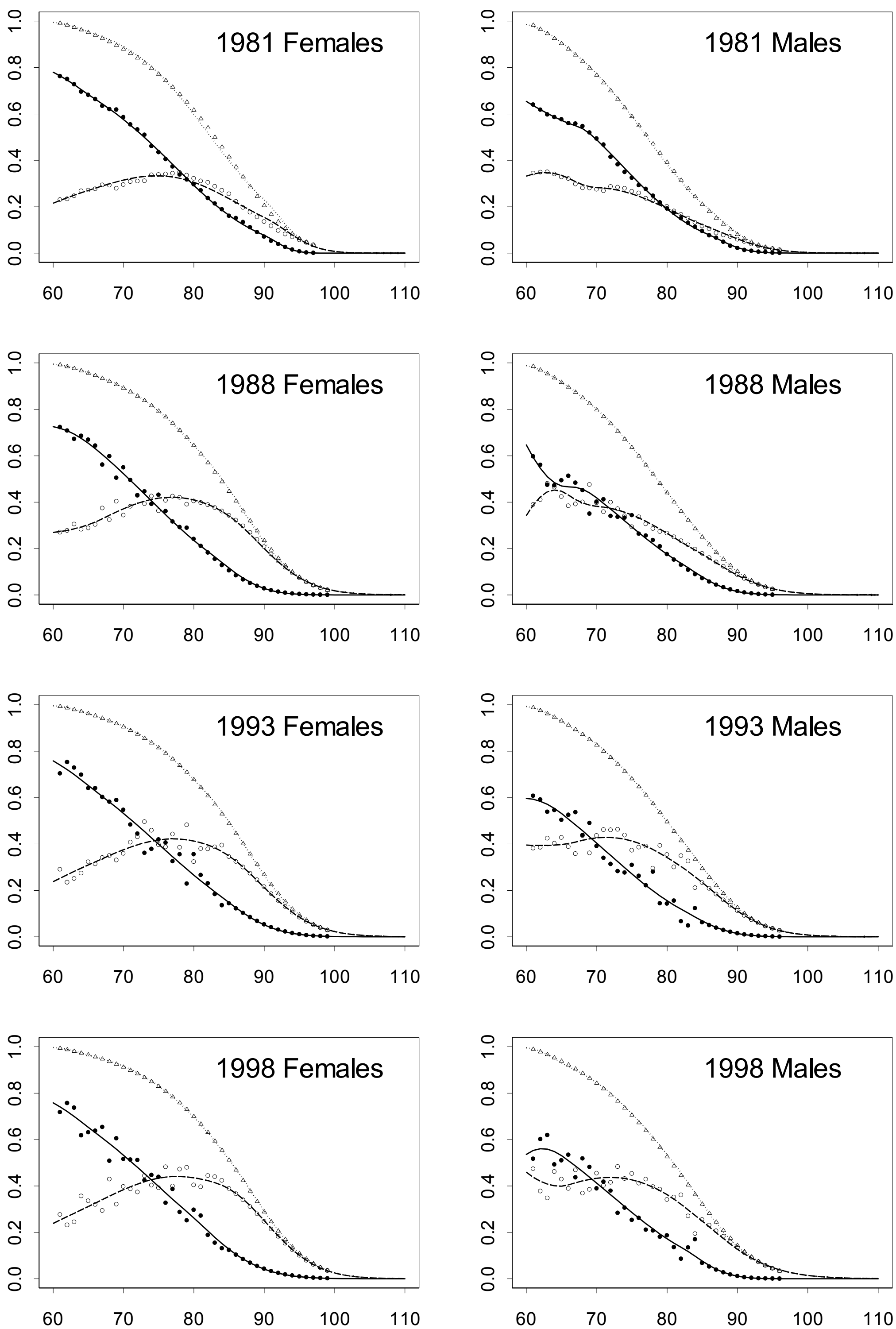


Figure 3.3 Current ratios of probabilities for all survey years

(The legend and axes are the same as in Figure 2.3:
solid line = $\hat{p}_1(60, y)/\hat{p}_2(60, y)$.)

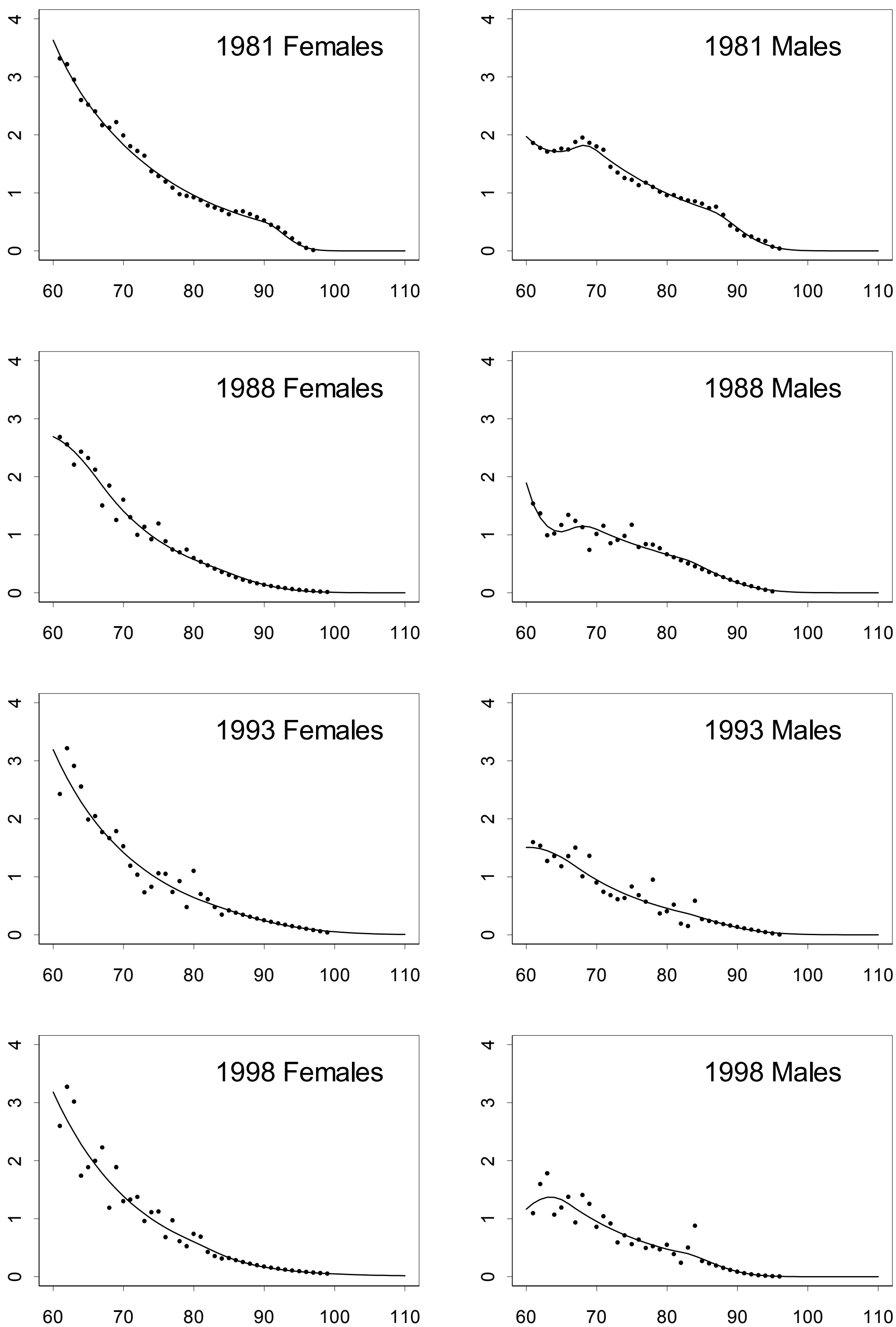


Table 3.1 Fitted log odds as given by (3.1) for all survey years

(Standard errors are given in parentheses below each parameter estimate.)

$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	$\hat{\beta}_5$	$\hat{\beta}_6$	$\hat{\beta}_7$	$\hat{\beta}_8$
Females 1981							
3.6409	–0.1981	0.0377	–0.0702	2.3883	–0.1333	0.0367	–0.0196
(0.0056)	(0.0003)	(0.0011)	(0.0010)	(0.0057)	(0.0003)	(0.0011)	(0.0003)
$g_3(y)=g_7(y)=(y-66)^2 I(y<66)$, $g_4(y)=g_8(y)=(y-90)^2 I(y>90)$							
Females 1988							
3.6086	–0.2012	0.0281	–0.0138	2.3687	–0.1114	0.0332	–0.0059
(0.0071)	(0.0004)	(0.0008)	(0.0001)	(0.0067)	(0.0003)	(0.0008)	(0.0000)
$g_3(y)=g_7(y)=(y-67)^2 I(y<67)$, $g_4(y)=g_8(y)=(y-82)^2 I(y>82)$							
Females 1993							
3.6812	–0.1937	0.0314	–0.0082	2.5424	–0.1147	0.0309	–0.0054
(0.0074)	(0.0004)	(0.0008)	(0.0001)	(0.0072)	(0.0004)	(0.0008)	(0.0000)
$g_3(y)=g_7(y)=(y-67)^2 I(y<67)$, $g_4(y)=g_8(y)=(y-82)^2 I(y>82)$							
Females 1998							
3.8420	–0.1991	0.0455	–0.0058	2.6864	–0.1162	0.0454	–0.0067
(0.0071)	(0.0004)	(0.0013)	(0.0001)	(0.0065)	(0.0003)	(0.0013)	(0.0001)
$g_3(y)=g_7(y)=(y-66)^2 I(y<66)$, $g_4(y)=g_8(y)=(y-79)^2 I(y>79)$							
Males 1981							
2.7017	–0.1941	0.0222	–0.0320	1.5954	–0.1382	0.0152	–0.0098
(0.0055)	(0.0003)	(0.0004)	(0.0005)	(0.0068)	(0.0004)	(0.0002)	(0.0002)
$g_3(y)=(y-67)^2 I(y<67)$, $g_7(y)=(y-70)^2 I(y<70)$, $g_4(y)=g_8(y)=(y-86)^2 I(y>86)$							
Males 1988							
2.6412	–0.1901	0.0380	–0.0166	2.0463	–0.1396	0.0132	–0.0062
(0.0057)	(0.0004)	(0.0006)	(0.0002)	(0.0063)	(0.0003)	(0.0002)	(0.0001)
$g_3(y)=(y-66)^2 I(y<66)$, $g_7(y)=(y-70)^2 I(y<70)$, $g_4(y)=g_8(y)=(y-82)^2 I(y>82)$							
Males 1993							
2.9096	–0.2042	0.0301	–0.0143	2.2321	–0.1310	0.0356	–0.0057
(0.0068)	(0.0004)	(0.0006)	(0.0002)	(0.0062)	(0.0003)	(0.0006)	(0.0001)
$g_3(y)=g_7(y)=(y-67)^2 I(y<67)$, $g_4(y)=g_8(y)=(y-82)^2 I(y>82)$							
Males 1998							
2.9592	–0.1977	0.0503	–0.0215	2.3109	–0.1279	0.0641	–0.0032
(0.0064)	(0.0004)	(0.0010)	(0.0002)	(0.0063)	(0.0004)	(0.0010)	(0.0000)
$g_3(y)=g_7(y)=(y-66)^2 I(y<66)$, $g_4(y)=(y-82)^2 I(y>82)$, $g_8(y)=(y-79)^2 I(y>79)$							

Table 3.2 Estimated probabilities for 1981 survey

Age y	Females $\hat{p}_1(60,y)$	Females $\hat{p}_2(60,y)$	Females $\hat{p}_0(60,y)$	Males $\hat{p}_1(60,y)$	Males $\hat{p}_2(60,y)$	Males $\hat{p}_0(60,y)$
60	0.78001	0.21472	0.99473	0.65369	0.33155	0.98524
61	0.76362	0.22686	0.99048	0.63558	0.34115	0.97673
62	0.74556	0.23854	0.98410	0.61849	0.34640	0.96489
63	0.72584	0.24959	0.97543	0.60227	0.34703	0.94930
64	0.70491	0.25997	0.96488	0.58688	0.34304	0.92992
65	0.68365	0.26986	0.95351	0.57253	0.33471	0.90724
66	0.66327	0.27963	0.94290	0.55965	0.32265	0.88230
67	0.64318	0.28932	0.93250	0.54895	0.30773	0.85668
68	0.62194	0.29849	0.92043	0.53575	0.29440	0.83015
69	0.59951	0.30699	0.90650	0.51541	0.28618	0.80159
70	0.57588	0.31463	0.89051	0.48882	0.28270	0.77152
71	0.55105	0.32123	0.87228	0.45891	0.28064	0.73955
72	0.52507	0.32657	0.85164	0.42812	0.27686	0.70498
73	0.49799	0.33047	0.82846	0.39673	0.27130	0.66803
74	0.46994	0.33273	0.80267	0.36508	0.26400	0.62908
75	0.44104	0.33318	0.77422	0.33351	0.25503	0.58854
76	0.41151	0.33169	0.74320	0.30243	0.24455	0.54698
77	0.38157	0.32815	0.70972	0.27219	0.23274	0.50493
78	0.35150	0.32252	0.67402	0.24315	0.21985	0.46300
79	0.32159	0.31483	0.63642	0.21562	0.20616	0.42178
80	0.29216	0.30517	0.59733	0.18985	0.19196	0.38181
81	0.26353	0.29370	0.55723	0.16603	0.17752	0.34355
82	0.23600	0.28062	0.51662	0.14427	0.16311	0.30738
83	0.20983	0.26622	0.47605	0.12461	0.14898	0.27359
84	0.18527	0.25079	0.43606	0.10704	0.13533	0.24237
85	0.16247	0.23465	0.39712	0.09148	0.12230	0.21378
86	0.14155	0.21813	0.35968	0.07782	0.11002	0.18784
87	0.12257	0.20152	0.32409	0.06406	0.09791	0.16197
88	0.10552	0.18511	0.29063	0.04947	0.08546	0.13493
89	0.09036	0.16912	0.25948	0.03577	0.07301	0.10878
90	0.07699	0.15376	0.23075	0.02418	0.06094	0.08512
91	0.06133	0.13745	0.19878	0.01526	0.04966	0.06492
92	0.04266	0.11871	0.16137	0.00900	0.03951	0.04851
93	0.02573	0.09837	0.12410	0.00495	0.03070	0.03565
94	0.01340	0.07786	0.09126	0.00255	0.02331	0.02586
95	0.00602	0.05877	0.06479	0.00123	0.01732	0.01855
96	0.00233	0.04234	0.04467	0.00055	0.01259	0.01314
97	0.00078	0.02916	0.02994	0.00023	0.00896	0.00919
98	0.00023	0.01922	0.01945	0.00009	0.00624	0.00633
99	0.00006	0.01214	0.01220	0.00003	0.00426	0.00429
100	0.00001	0.00735	0.00736	0.00001	0.00285	0.00286
101	0.00000	0.00427	0.00427	0.00000	0.00187	0.00187
102	0.00000	0.00239	0.00239	0.00000	0.00120	0.00120
103	0.00000	0.00128	0.00128	0.00000	0.00076	0.00076
104	0.00000	0.00066	0.00066	0.00000	0.00047	0.00047
105	0.00000	0.00033	0.00033	0.00000	0.00028	0.00028
106	0.00000	0.00016	0.00016	0.00000	0.00017	0.00017
107	0.00000	0.00007	0.00007	0.00000	0.00010	0.00010
108	0.00000	0.00003	0.00003	0.00000	0.00006	0.00006
109	0.00000	0.00001	0.00001	0.00000	0.00003	0.00003
110	0.00000	0.00001	0.00001	0.00000	0.00002	0.00002

Table 3.3 Estimated probabilities for 1988 survey

Age y	Females $\hat{p}_1(60,y)$	Females $\hat{p}_2(60,y)$	Females $\hat{p}_0(60,y)$	Males $\hat{p}_1(60,y)$	Males $\hat{p}_2(60,y)$	Males $\hat{p}_0(60,y)$
60	0.72556	0.26949	0.99505	0.64678	0.34149	0.98827
61	0.71821	0.27314	0.99135	0.59199	0.38828	0.98027
62	0.70749	0.27832	0.98581	0.54581	0.42322	0.96903
63	0.69315	0.28495	0.97810	0.50971	0.44466	0.95437
64	0.67521	0.29303	0.96824	0.48426	0.45233	0.93659
65	0.65400	0.30269	0.95669	0.46965	0.44700	0.91665
66	0.63022	0.31426	0.94448	0.46601	0.43010	0.89611
67	0.60475	0.32823	0.93298	0.46418	0.41068	0.87486
68	0.57828	0.34335	0.92163	0.45518	0.39641	0.85159
69	0.55088	0.35782	0.90870	0.43964	0.38701	0.82665
70	0.52267	0.37140	0.89407	0.41832	0.38221	0.80053
71	0.49377	0.38384	0.87761	0.39407	0.37869	0.77276
72	0.46435	0.39489	0.85924	0.36927	0.37322	0.74249
73	0.43459	0.40431	0.83890	0.34408	0.36575	0.70983
74	0.40468	0.41186	0.81654	0.31869	0.35630	0.67499
75	0.37484	0.41735	0.79219	0.29332	0.34491	0.63823
76	0.34530	0.42059	0.76589	0.26822	0.33172	0.59994
77	0.31629	0.42146	0.73775	0.24364	0.31691	0.56055
78	0.28804	0.41989	0.70793	0.21981	0.30072	0.52053
79	0.26078	0.41586	0.67664	0.19698	0.28342	0.48040
80	0.23469	0.40943	0.64412	0.17533	0.26534	0.44067
81	0.20995	0.40070	0.61065	0.15505	0.24679	0.40184
82	0.18671	0.38984	0.57655	0.13625	0.22809	0.36434
83	0.16355	0.37654	0.54009	0.11728	0.20995	0.32723
84	0.13958	0.36006	0.49964	0.09737	0.19268	0.29005
85	0.11581	0.34011	0.45592	0.07795	0.17519	0.25314
86	0.09322	0.31669	0.40991	0.06017	0.15679	0.21696
87	0.07267	0.29016	0.36283	0.04473	0.13796	0.18269
88	0.05479	0.26121	0.31600	0.03200	0.11927	0.15127
89	0.03990	0.23083	0.27073	0.02203	0.10130	0.12333
90	0.02806	0.20012	0.22818	0.01459	0.08453	0.09912
91	0.01905	0.17019	0.18924	0.00931	0.06933	0.07864
92	0.01249	0.14201	0.15450	0.00572	0.05592	0.06164
93	0.00791	0.11632	0.12423	0.00339	0.04439	0.04778
94	0.00484	0.09359	0.09843	0.00193	0.03469	0.03662
95	0.00287	0.07402	0.07689	0.00107	0.02671	0.02778
96	0.00165	0.05759	0.05924	0.00057	0.02027	0.02084
97	0.00091	0.04411	0.04502	0.00029	0.01517	0.01546
98	0.00049	0.03328	0.03377	0.00014	0.01119	0.01133
99	0.00026	0.02475	0.02501	0.00007	0.00815	0.00822
100	0.00013	0.01815	0.01828	0.00003	0.00585	0.00588
101	0.00006	0.01314	0.01320	0.00001	0.00415	0.00416
102	0.00003	0.00938	0.00941	0.00001	0.00291	0.00292
103	0.00001	0.00662	0.00663	0.00000	0.00201	0.00201
104	0.00001	0.00461	0.00462	0.00000	0.00137	0.00137
105	0.00000	0.00317	0.00317	0.00000	0.00092	0.00092
106	0.00000	0.00215	0.00215	0.00000	0.00061	0.00061
107	0.00000	0.00145	0.00145	0.00000	0.00040	0.00040
108	0.00000	0.00096	0.00096	0.00000	0.00026	0.00026
109	0.00000	0.00063	0.00063	0.00000	0.00017	0.00017
110	0.00000	0.00041	0.00041	0.00000	0.00011	0.00011

Table 3.4 Estimated probabilities for 1993 survey

Age y	Females $\hat{p}_1(60,y)$	Females $\hat{p}_2(60,y)$	Females $\hat{p}_0(60,y)$	Males $\hat{p}_1(60,y)$	Males $\hat{p}_2(60,y)$	Males $\hat{p}_0(60,y)$
60	0.75833	0.23757	0.99590	0.59693	0.39564	0.99257
61	0.74025	0.25243	0.99268	0.59273	0.39388	0.98661
62	0.72057	0.26723	0.98780	0.58407	0.39338	0.97745
63	0.69919	0.28175	0.98094	0.57071	0.39385	0.96456
64	0.67628	0.29585	0.97213	0.55283	0.39518	0.94801
65	0.65231	0.30951	0.96182	0.53117	0.39759	0.92876
66	0.62805	0.32293	0.95098	0.50699	0.40172	0.90871
67	0.60443	0.33649	0.94092	0.48180	0.40853	0.89033
68	0.58102	0.35005	0.93107	0.45630	0.41630	0.87260
69	0.55679	0.36303	0.91982	0.43028	0.42238	0.85266
70	0.53180	0.37524	0.90704	0.40384	0.42654	0.83038
71	0.50613	0.38649	0.89262	0.37713	0.42858	0.80571
72	0.47987	0.39657	0.87644	0.35031	0.42834	0.77865
73	0.45313	0.40525	0.85838	0.32355	0.42568	0.74923
74	0.42604	0.41235	0.83839	0.29707	0.42052	0.71759
75	0.39875	0.41767	0.81642	0.27107	0.41286	0.68393
76	0.37142	0.42103	0.79245	0.24576	0.40275	0.64851
77	0.34423	0.42229	0.76652	0.22136	0.39032	0.61168
78	0.31738	0.42135	0.73873	0.19806	0.37577	0.57383
79	0.29104	0.41816	0.70920	0.17604	0.35935	0.53539
80	0.26542	0.41270	0.67812	0.15544	0.34140	0.49684
81	0.24069	0.40502	0.64571	0.13636	0.32224	0.45860
82	0.21703	0.39523	0.61226	0.11887	0.30224	0.42111
83	0.19369	0.38283	0.57652	0.10185	0.28104	0.38289
84	0.17014	0.36708	0.53722	0.08472	0.25810	0.34282
85	0.14687	0.34788	0.49475	0.06830	0.23373	0.30203
86	0.12440	0.32537	0.44977	0.05330	0.20841	0.26171
87	0.10324	0.29988	0.40312	0.04022	0.18281	0.22303
88	0.08384	0.27203	0.35587	0.02933	0.15764	0.18697
89	0.06656	0.24262	0.30918	0.02066	0.13360	0.15426
90	0.05162	0.21261	0.26423	0.01406	0.11130	0.12536
91	0.03909	0.18298	0.22207	0.00925	0.09117	0.10042
92	0.02891	0.15465	0.18356	0.00588	0.07347	0.07935
93	0.02087	0.12838	0.14925	0.00362	0.05829	0.06191
94	0.01473	0.10474	0.11947	0.00216	0.04554	0.04770
95	0.01016	0.08402	0.09418	0.00125	0.03508	0.03633
96	0.00686	0.06632	0.07318	0.00070	0.02664	0.02734
97	0.00453	0.05156	0.05609	0.00038	0.01996	0.02034
98	0.00293	0.03950	0.04243	0.00020	0.01476	0.01496
99	0.00186	0.02984	0.03170	0.00010	0.01078	0.01088
100	0.00116	0.02224	0.02340	0.00005	0.00777	0.00782
101	0.00071	0.01637	0.01708	0.00002	0.00554	0.00556
102	0.00043	0.01190	0.01233	0.00001	0.00390	0.00391
103	0.00025	0.00855	0.00880	0.00001	0.00271	0.00272
104	0.00015	0.00607	0.00622	0.00000	0.00187	0.00187
105	0.00008	0.00426	0.00434	0.00000	0.00127	0.00127
106	0.00005	0.00296	0.00301	0.00000	0.00085	0.00085
107	0.00003	0.00203	0.00206	0.00000	0.00057	0.00057
108	0.00001	0.00138	0.00139	0.00000	0.00037	0.00037
109	0.00001	0.00092	0.00093	0.00000	0.00024	0.00024
110	0.00000	0.00061	0.00061	0.00000	0.00015	0.00015

Table 3.5 Estimated probabilities for 1998 survey

Age y	Females $\hat{p}_1(60,y)$	Females $\hat{p}_2(60,y)$	Females $\hat{p}_0(60,y)$	Males $\hat{p}_1(60,y)$	Males $\hat{p}_2(60,y)$	Males $\hat{p}_0(60,y)$
60	0.75858	0.23826	0.99684	0.53594	0.45951	0.99545
61	0.74081	0.25297	0.99378	0.55296	0.43711	0.99007
62	0.72117	0.26771	0.98888	0.56068	0.42002	0.98070
63	0.69962	0.28228	0.98190	0.55886	0.40779	0.96665
64	0.67653	0.29665	0.97318	0.54823	0.40050	0.94873
65	0.65277	0.31103	0.96380	0.53070	0.39897	0.92967
66	0.62952	0.32589	0.95541	0.50890	0.40466	0.91356
67	0.60649	0.34110	0.94759	0.48551	0.41400	0.89951
68	0.58259	0.35597	0.93856	0.46155	0.42203	0.88358
69	0.55788	0.37032	0.92820	0.43708	0.42857	0.86565
70	0.53243	0.38396	0.91639	0.41219	0.43341	0.84560
71	0.50631	0.39667	0.90298	0.38700	0.43636	0.82336
72	0.47962	0.40823	0.88785	0.36162	0.43724	0.79886
73	0.45248	0.41840	0.87088	0.33620	0.43591	0.77211
74	0.42503	0.42697	0.85200	0.31090	0.43228	0.74318
75	0.39741	0.43372	0.83113	0.28591	0.42628	0.71219
76	0.36980	0.43846	0.80826	0.26140	0.41794	0.67934
77	0.34237	0.44101	0.78338	0.23756	0.40730	0.64486
78	0.31531	0.44124	0.75655	0.21458	0.39452	0.60910
79	0.28881	0.43908	0.72789	0.19262	0.37976	0.57238
80	0.26194	0.43516	0.69710	0.17203	0.36255	0.53458
81	0.23404	0.42986	0.66390	0.15302	0.34250	0.49552
82	0.20582	0.42284	0.62866	0.13551	0.32006	0.45557
83	0.17799	0.41381	0.59180	0.11720	0.29654	0.41374
84	0.15126	0.40263	0.55389	0.09696	0.27261	0.36957
85	0.12658	0.38770	0.51428	0.07655	0.24807	0.32462
86	0.10449	0.36767	0.47216	0.05756	0.22302	0.28058
87	0.08501	0.34306	0.42807	0.04117	0.19783	0.23900
88	0.06810	0.31463	0.38273	0.02800	0.17305	0.20105
89	0.05365	0.28336	0.33701	0.01810	0.14930	0.16740
90	0.04154	0.25039	0.29193	0.01114	0.12711	0.13825
91	0.03160	0.21696	0.24856	0.00652	0.10688	0.11340
92	0.02359	0.18426	0.20785	0.00364	0.08884	0.09248
93	0.01729	0.15337	0.17066	0.00194	0.07306	0.07500
94	0.01244	0.12512	0.13756	0.00099	0.05949	0.06048
95	0.00880	0.10008	0.10888	0.00048	0.04800	0.04848
96	0.00611	0.07852	0.08463	0.00022	0.03838	0.03860
97	0.00417	0.06048	0.06465	0.00010	0.03044	0.03054
98	0.00280	0.04576	0.04856	0.00004	0.02394	0.02398
99	0.00185	0.03403	0.03588	0.00002	0.01869	0.01871
100	0.00121	0.02490	0.02611	0.00001	0.01448	0.01449
101	0.00078	0.01793	0.01871	0.00000	0.01113	0.01113
102	0.00049	0.01271	0.01320	0.00000	0.00850	0.00850
103	0.00031	0.00888	0.00919	0.00000	0.00645	0.00645
104	0.00019	0.00612	0.00631	0.00000	0.00485	0.00485
105	0.00012	0.00415	0.00427	0.00000	0.00363	0.00363
106	0.00007	0.00278	0.00285	0.00000	0.00270	0.00270
107	0.00004	0.00184	0.00188	0.00000	0.00199	0.00199
108	0.00002	0.00120	0.00122	0.00000	0.00146	0.00146
109	0.00001	0.00077	0.00078	0.00000	0.00106	0.00106
110	0.00001	0.00049	0.00050	0.00000	0.00077	0.00077

Table 3.6 Health expectancies for 1981 survey

(Standard errors are given in parentheses below each estimate.)

	Females			Males		
Age x	$\hat{e}_1(x)$	$\hat{e}_2(x)$	$\hat{e}_0(x)$	$\hat{e}_1(x)$	$\hat{e}_2(x)$	$\hat{e}_0(x)$
60	13.175	9.000	22.176	10.265	7.298	17.564
	(0.010)	(0.010)	(0.009)	(0.009)	(0.009)	(0.009)
61	12.453	8.816	21.269	9.695	7.017	16.712
	(0.009)	(0.010)	(0.010)	(0.009)	(0.009)	(0.010)
62	11.766	8.637	20.403	9.164	6.747	15.911
	(0.009)	(0.010)	(0.009)	(0.009)	(0.008)	(0.010)
63	11.117	8.463	19.580	8.672	6.492	15.164
	(0.009)	(0.010)	(0.009)	(0.009)	(0.009)	(0.010)
64	10.497	8.292	18.788	8.213	6.256	14.469
	(0.009)	(0.009)	(0.008)	(0.009)	(0.008)	(0.009)
65	9.894	8.113	18.007	7.780	6.038	13.818
	(0.008)	(0.009)	(0.008)	(0.008)	(0.008)	(0.009)
66	9.291	7.913	17.204	7.358	5.836	13.194
	(0.008)	(0.009)	(0.008)	(0.008)	(0.008)	(0.008)
67	8.694	7.696	16.390	6.931	5.643	12.574
	(0.008)	(0.009)	(0.008)	(0.008)	(0.008)	(0.009)
68	8.121	7.477	15.598	6.499	5.461	11.959
	(0.008)	(0.008)	(0.008)	(0.008)	(0.008)	(0.009)
69	7.572	7.258	14.830	6.074	5.294	11.368
	(0.008)	(0.008)	(0.008)	(0.008)	(0.008)	(0.009)
70	7.048	7.039	14.087	5.659	5.132	10.791
	(0.007)	0.008)	(0.008)	(0.008)	(0.008)	(0.009)
71	6.549	6.822	13.371	5.263	4.973	10.236
	(0.007)	(0.008)	(0.008)	(0.008)	(0.009)	(0.009)
72	6.076	6.607	12.682	4.892	4.821	9.713
	(0.007)	(0.008)	(0.008)	(0.007)	(0.008)	(0.009)
73	5.628	6.395	12.023	4.545	4.677	9.222
	(0.007)	(0.008)	(0.008)	(0.007)	(0.008)	(0.009)
74	5.206	6.187	11.393	4.221	4.541	8.762
	(0.007)	(0.008)	(0.008)	(0.007)	(0.008)	(0.009)
75	4.809	5.984	10.793	3.918	4.412	8.331
	(0.007)	(0.008)	(0.008)	(0.007)	(0.009)	(0.009)
76	4.436	5.786	10.222	3.635	4.291	7.926
	(0.007)	(0.008)	(0.008)	(0.007)	(0.009)	(0.009)
77	4.086	5.594	9.681	3.369	4.175	7.544
	(0.007)	(0.008)	(0.008)	(0.007)	(0.009)	(0.009)
78	3.759	5.408	9.167	3.117	4.065	7.182
	(0.006)	(0.008)	(0.008)	(0.007)	(0.009)	(0.009)
79	3.452	5.226	8.678	2.879	3.957	6.835
	(0.006)	(0.009)	(0.008)	(0.007)	(0.009)	(0.008)
80	3.165	5.049	8.213	2.649	3.849	6.499
	(0.006)	(0.008)	(0.008)	(0.006)	(0.009)	(0.008)
81	2.894	4.875	7.769	2.427	3.740	6.167
	(0.006)	(0.008)	(0.008)	(0.006)	(0.010)	(0.009)

82	2.638	4.702	7.340	2.208	3.626	5.835
	(0.005)	(0.008)	(0.008)	(0.006)	(0.010)	(0.009)
83	2.395	4.528	6.923	1.990	3.504	5.494
	(0.005)	(0.008)	(0.008)	(0.006)	(0.010)	(0.010)
84	2.162	4.350	6.512	1.769	3.369	5.139
	(0.005)	(0.008)	(0.008)	(0.006)	(0.011)	(0.011)
85	1.936	4.165	6.102	1.543	3.217	4.760
	(0.005)	(0.009)	(0.008)	(0.006)	(0.012)	(0.013)
86	1.716	3.970	5.685	1.305	3.043	4.349
	(0.004)	(0.009)	(0.008)	(0.006)	(0.014)	(0.014)
87	1.497	3.758	5.255	1.076	2.888	3.963
	(0.004)	(0.010)	(0.009)	(0.007)	(0.015)	(0.016)
88	1.278	3.526	4.803	0.870	2.787	3.657
	(0.004)	(0.011)	(0.011)	(0.007)	(0.018)	(0.019)
89	1.054	3.266	4.321	0.689	2.728	3.417
	(0.004)	(0.012)	(0.012)	(0.007)	(0.020)	(0.020)
90	0.823	2.973	3.796	0.531	2.700	3.231
	(0.004)	(0.013)	(0.014)	(0.007)	(0.023)	(0.022)
91	0.606	2.719	3.324	0.395	2.690	3.085
	(0.004)	(0.015)	(0.015)	(0.007)	(0.027)	(0.026)
92	0.423	2.554	2.977	0.283	2.683	2.966
	(0.005)	(0.017)	(0.017)	(0.007)	(0.029)	(0.027)
93	0.278	2.446	2.723	0.194	2.669	2.863
	(0.004)	(0.018)	(0.018)	(0.006)	(0.030)	(0.028)
94	0.168	2.361	2.528	0.127	2.640	2.767
	(0.004)	(0.021)	(0.020)	(0.005)	(0.030)	(0.029)
95	0.092	2.273	2.366	0.079	2.592	2.671
	(0.003)	(0.022)	(0.021)	(0.004)	(0.031)	(0.030)
96	0.046	2.171	2.217	0.047	2.528	2.575
	(0.002)	(0.021)	(0.020)	(0.003)	(0.031)	(0.030)
97	0.020	2.055	2.076	0.027	2.451	2.478
	(0.001)	(0.021)	(0.021)	(0.002)	(0.031)	(0.030)
98	0.008	1.934	1.942	0.014	2.367	2.382
	(0.001)	(0.019)	(0.019)	(0.001)	(0.030)	(0.030)
99	0.003	1.815	1.818	0.007	2.280	2.287
	(0.000)	(0.019)	(0.019)	(0.001)	(0.029)	(0.029)

Table 3.7 Health expectancies for 1988 survey

(Standard errors are given in parentheses below each estimate.)

Age x	Females $\hat{e}_1(x)$	Females $\hat{e}_2(x)$	Females $\hat{e}_0(x)$	Males $\hat{e}_1(x)$	Males $\hat{e}_2(x)$	Males $\hat{e}_0(x)$
60	11.527	11.421	22.947	8.871	9.725	18.596
	(0.009)	(0.010)	(0.009)	(0.009)	(0.010)	(0.009)
61	10.841	11.190	22.031	8.312	9.431	17.743
	(0.009)	(0.010)	(0.010)	(0.008)	(0.010)	(0.009)
62	10.179	10.973	21.152	7.822	9.121	16.943
	(0.009)	(0.010)	(0.010)	(0.009)	(0.010)	(0.009)
63	9.543	10.772	20.314	7.390	8.805	16.195
	(0.008)	(0.010)	(0.009)	(0.008)	(0.009)	(0.009)
64	8.933	10.583	19.516	7.001	8.492	15.493
	(0.008)	(0.010)	(0.009)	(0.008)	(0.009)	(0.009)
65	8.346	10.400	18.745	6.634	8.185	14.819
	(0.008)	(0.010)	(0.009)	(0.008)	(0.009)	(0.009)
66	7.774	10.208	17.981	6.264	7.883	14.147
	(0.007)	(0.009)	(0.008)	(0.007)	(0.009)	(0.009)
67	7.207	9.989	17.197	5.885	7.594	13.479
	(0.008)	(0.010)	(0.009)	(0.007)	(0.009)	(0.009)
68	6.654	9.748	16.402	5.505	7.328	12.833
	(0.007)	(0.010)	(0.009)	(0.007)	(0.009)	(0.008)
69	6.128	9.501	15.629	5.129	7.076	12.205
	(0.007)	(0.009)	(0.009)	(0.007)	(0.008)	(0.008)
70	5.627	9.249	14.876	4.760	6.827	11.587
	(0.007)	(0.009)	(0.009)	(0.007)	(0.009)	(0.009)
71	5.154	8.992	14.145	4.405	6.579	10.985
	(0.007)	(0.009)	(0.009)	(0.007)	(0.009)	(0.009)
72	4.706	8.731	13.437	4.071	6.341	10.412
	(0.007)	(0.009)	(0.009)	(0.007)	(0.008)	(0.009)
73	4.285	8.466	12.750	3.756	6.112	9.868
	(0.007)	(0.009)	(0.008)	(0.006)	(0.009)	(0.009)
74	3.888	8.198	12.086	3.459	5.893	9.351
	(0.006)	(0.009)	(0.009)	(0.006)	(0.009)	(0.009)
75	3.516	7.926	11.442	3.178	5.682	8.861
	(0.006)	(0.009)	(0.008)	(0.006)	(0.009)	(0.009)
76	3.166	7.651	10.817	2.913	5.481	8.394
	(0.006)	(0.009)	(0.008)	(0.006)	(0.009)	(0.009)
77	2.839	7.372	10.210	2.661	5.287	7.949
	(0.006)	(0.009)	(0.008)	(0.006)	(0.009)	(0.009)
78	2.532	7.088	9.619	2.421	5.100	7.521
	(0.005)	(0.009)	(0.008)	(0.006)	(0.009)	(0.008)
79	2.243	6.798	9.041	2.190	4.918	7.108
	(0.005)	(0.008)	(0.008)	(0.005)	(0.009)	(0.009)
80	1.972	6.500	8.472	1.965	4.739	6.704
	(0.005)	(0.009)	(0.008)	(0.005)	(0.010)	(0.009)
81	1.716	6.193	7.909	1.744	4.559	6.303
	(0.004)	(0.009)	(0.008)	(0.005)	(0.010)	(0.010)

82	1.474	5.873	7.347	1.524	4.377	5.901
	(0.004)	(0.009)	(0.008)	(0.005)	(0.011)	(0.011)
83	1.249	5.559	6.808	1.309	4.204	5.513
	(0.004)	(0.009)	(0.009)	(0.005)	(0.012)	(0.012)
84	1.047	5.272	6.318	1.107	4.049	5.156
	(0.004)	(0.010)	(0.009)	(0.005)	(0.014)	(0.014)
85	0.867	5.009	5.876	0.922	3.913	4.835
	(0.004)	(0.010)	(0.010)	(0.006)	(0.016)	(0.016)
86	0.710	4.769	5.479	0.759	3.800	4.558
	(0.004)	(0.011)	(0.011)	(0.005)	(0.018)	(0.018)
87	0.574	4.551	5.125	0.615	3.706	4.321
	(0.004)	(0.011)	(0.011)	(0.005)	(0.021)	(0.021)
88	0.458	4.353	4.811	0.491	3.626	4.116
	(0.003)	(0.012)	(0.011)	(0.005)	(0.023)	(0.023)
89	0.361	4.171	4.532	0.385	3.554	3.938
	(0.003)	(0.012)	(0.012)	(0.005)	(0.025)	(0.024)
90	0.280	4.005	4.285	0.296	3.485	3.781
	(0.003)	(0.013)	(0.012)	(0.005)	(0.027)	(0.027)
91	0.214	3.851	4.066	0.223	3.417	3.640
	(0.003)	(0.013)	(0.013)	(0.004)	(0.027)	(0.027)
92	0.162	3.708	3.870	0.164	3.345	3.510
	(0.003)	(0.014)	(0.013)	(0.004)	(0.030)	(0.029)
93	0.120	3.574	3.694	0.119	3.270	3.388
	(0.002)	(0.014)	(0.014)	(0.003)	(0.029)	(0.028)
94	0.088	3.447	3.534	0.084	3.190	3.273
	(0.002)	(0.013)	(0.013)	(0.003)	(0.030)	(0.029)
95	0.063	3.326	3.389	0.058	3.105	3.163
	(0.002)	(0.014)	(0.013)	(0.002)	(0.030)	(0.029)
96	0.045	3.210	3.255	0.039	3.018	3.057
	(0.001)	(0.014)	(0.013)	(0.002)	(0.031)	(0.030)
97	0.031	3.099	3.131	0.026	2.929	2.955
	(0.001)	(0.014)	(0.013)	(0.001)	(0.030)	(0.029)
98	0.022	2.992	3.014	0.017	2.838	2.855
	(0.001)	(0.013)	(0.013)	(0.001)	(0.029)	(0.029)
99	0.015	2.888	2.903	0.011	2.747	2.758
	(0.001)	(0.012)	(0.012)	(0.001)	(0.029)	(0.029)

Table 3.8 Health expectancies for 1993 survey

(Standard errors are given in parentheses below each estimate.)

Age x	Females $\hat{e}_1(x)$	Females $\hat{e}_2(x)$	Females $\hat{e}_0(x)$	Males $\hat{e}_1(x)$	Males $\hat{e}_2(x)$	Males $\hat{e}_0(x)$
60	12.203	11.606	23.808	8.781	10.906	19.687
	(0.010)	(0.011)	(0.010)	(0.009)	(0.011)	(0.010)
61	11.487	11.396	22.884	8.231	10.571	18.803
	(0.009)	(0.011)	(0.010)	(0.009)	(0.010)	(0.010)
62	10.805	11.190	21.994	7.706	10.268	17.974
	(0.009)	(0.011)	(0.010)	(0.008)	(0.010)	(0.010)
63	10.156	10.988	21.144	7.210	9.997	17.207
	(0.009)	(0.011)	(0.010)	(0.008)	(0.010)	(0.010)
64	9.541	10.791	20.331	6.743	9.756	16.498
	(0.009)	(0.011)	(0.009)	(0.008)	(0.010)	(0.009)
65	8.952	10.591	19.544	6.299	9.531	15.830
	(0.009)	(0.010)	(0.009)	(0.008)	(0.009)	(0.009)
66	8.381	10.380	18.761	5.866	9.302	15.168
	(0.008)	(0.010)	(0.008)	(0.007)	(0.010)	(0.009)
67	7.816	10.140	17.956	5.432	9.039	14.471
	(0.008)	(0.010)	(0.009)	(0.007)	(0.009)	(0.009)
68	7.262	9.879	17.141	5.005	8.750	13.755
	(0.008)	(0.009)	(0.009)	(0.007)	(0.010)	(0.009)
69	6.732	9.612	16.344	4.602	8.463	13.065
	(0.008)	(0.009)	(0.009)	(0.007)	(0.009)	(0.009)
70	6.227	9.340	15.567	4.223	8.178	12.401
	(0.007)	(0.010)	(0.009)	(0.007)	(0.010)	(0.009)
71	5.746	9.064	14.810	3.868	7.898	11.766
	(0.007)	(0.010)	(0.009)	(0.007)	(0.010)	(0.010)
72	5.290	8.785	14.075	3.535	7.622	11.157
	(0.007)	(0.010)	(0.009)	(0.007)	(0.009)	(0.009)
73	4.857	8.502	13.360	3.224	7.351	10.575
	(0.007)	(0.009)	(0.009)	(0.007)	(0.010)	(0.009)
74	4.449	8.217	12.666	2.934	7.085	10.019
	(0.007)	(0.009)	(0.009)	(0.006)	(0.009)	(0.009)
75	4.064	7.930	11.994	2.663	6.824	9.487
	(0.006)	(0.009)	(0.009)	(0.006)	(0.010)	(0.009)
76	3.700	7.641	11.341	2.410	6.568	8.978
	(0.007)	(0.009)	(0.009)	(0.006)	(0.009)	(0.009)
77	3.359	7.349	10.707	2.174	6.315	8.488
	(0.006)	(0.009)	(0.008)	(0.005)	(0.009)	(0.009)
78	3.037	7.054	10.091	1.952	6.063	8.015
	(0.006)	(0.009)	(0.009)	(0.005)	(0.009)	(0.009)
79	2.735	6.755	9.490	1.743	5.812	7.554
	(0.006)	(0.009)	(0.008)	(0.005)	(0.009)	(0.009)
80	2.450	6.452	8.902	1.544	5.557	7.102
	(0.005)	(0.009)	(0.008)	(0.005)	(0.010)	(0.009)
81	2.181	6.142	8.324	1.355	5.297	6.653
	(0.005)	(0.009)	(0.008)	(0.005)	(0.011)	(0.010)

82	1.927	5.824	7.751	1.173	5.027	6.200
	(0.005)	(0.009)	(0.009)	(0.004)	(0.011)	(0.011)
83	1.690	5.510	7.200	1.002	4.767	5.769
	(0.005)	(0.009)	(0.009)	(0.004)	(0.012)	(0.012)
84	1.475	5.215	6.690	0.847	4.537	5.384
	(0.005)	(0.010)	(0.010)	(0.004)	(0.013)	(0.013)
85	1.281	4.939	6.221	0.709	4.335	5.044
	(0.005)	(0.010)	(0.010)	(0.005)	(0.014)	(0.014)
86	1.108	4.684	5.792	0.586	4.158	4.744
	(0.005)	(0.011)	(0.011)	(0.005)	(0.015)	(0.014)
87	0.954	4.450	5.405	0.479	4.003	4.481
	(0.005)	(0.011)	(0.011)	(0.005)	(0.016)	(0.016)
88	0.819	4.237	5.056	0.386	3.864	4.251
	(0.005)	(0.012)	(0.012)	(0.005)	(0.017)	(0.017)
89	0.700	4.044	4.744	0.307	3.741	4.048
	(0.005)	(0.012)	(0.012)	(0.005)	(0.018)	(0.017)
90	0.596	3.871	4.467	0.241	3.627	3.868
	(0.005)	(0.013)	(0.013)	(0.004)	(0.019)	(0.018)
91	0.506	3.715	4.221	0.186	3.522	3.708
	(0.005)	(0.014)	(0.013)	(0.004)	(0.019)	(0.018)
92	0.428	3.576	4.004	0.141	3.422	3.564
	(0.005)	(0.014)	(0.014)	(0.004)	(0.020)	(0.019)
93	0.360	3.451	3.811	0.106	3.326	3.432
	(0.005)	(0.015)	(0.014)	(0.003)	(0.020)	(0.019)
94	0.303	3.338	3.640	0.078	3.232	3.310
	(0.005)	(0.015)	(0.014)	(0.003)	(0.020)	(0.020)
95	0.253	3.234	3.487	0.056	3.140	3.197
	(0.004)	(0.015)	(0.014)	(0.002)	(0.020)	(0.019)
96	0.210	3.139	3.349	0.040	3.050	3.090
	(0.004)	(0.015)	(0.014)	(0.002)	(0.020)	(0.020)
97	0.174	3.049	3.223	0.028	2.960	2.988
	(0.004)	(0.014)	(0.013)	(0.001)	(0.019)	(0.019)
98	0.144	2.962	3.106	0.019	2.871	2.890
	(0.004)	(0.015)	(0.014)	(0.001)	(0.019)	(0.019)
99	0.118	2.877	2.995	0.013	2.782	2.795
	(0.003)	(0.014)	(0.013)	(0.001)	(0.019)	(0.019)

Table 3.9 Health expectancies for 1998 survey

(Standard errors are given in parentheses below each estimate.)

Age x	Females			Males		
	$\hat{e}_1(x)$	$\hat{e}_2(x)$	$\hat{e}_0(x)$	$\hat{e}_1(x)$	$\hat{e}_2(x)$	$\hat{e}_0(x)$
60	12.015	12.322	24.337	8.876	11.472	20.347
	(0.009)	(0.010)	(0.009)	(0.008)	(0.010)	(0.009)
61	11.298	12.113	23.410	8.373	11.082	19.455
	(0.009)	(0.010)	(0.009)	(0.008)	(0.010)	(0.010)
62	10.614	11.910	22.524	7.885	10.751	18.636
	(0.009)	(0.010)	(0.009)	(0.008)	(0.010)	(0.010)
63	9.966	11.714	21.680	7.419	10.479	17.899
	(0.008)	(0.010)	(0.009)	(0.008)	(0.010)	(0.010)
64	9.348	11.522	20.870	6.975	10.252	17.227
	(0.008)	(0.010)	(0.008)	(0.008)	(0.009)	(0.009)
65	8.750	11.319	20.068	6.538	10.033	16.570
	(0.008)	(0.010)	(0.008)	(0.007)	(0.009)	(0.008)
66	8.155	11.085	19.240	6.084	9.770	15.854
	(0.007)	(0.009)	(0.008)	(0.007)	(0.009)	(0.008)
67	7.570	10.824	18.395	5.626	9.468	15.094
	(0.007)	(0.009)	(0.008)	(0.007)	(0.009)	(0.009)
68	7.010	10.557	17.567	5.191	9.165	14.357
	(0.007)	(0.009)	(0.008)	(0.007)	(0.009)	(0.009)
69	6.474	10.284	16.757	4.780	8.864	13.643
	(0.007)	(0.009)	(0.008)	(0.007)	(0.009)	(0.009)
70	5.962	10.005	15.967	4.391	8.564	12.955
	(0.007)	(0.009)	(0.008)	(0.007)	(0.009)	(0.009)
71	5.475	9.721	15.196	4.024	8.267	12.291
	(0.007)	(0.009)	(0.008)	(0.006)	(0.009)	(0.009)
72	5.013	9.433	14.446	3.679	7.973	11.653
	(0.007)	(0.009)	(0.008)	(0.006)	(0.009)	(0.008)
73	4.576	9.142	13.718	3.355	7.684	11.039
	(0.006)	(0.009)	(0.008)	(0.006)	(0.009)	(0.009)
74	4.162	8.848	13.011	3.050	7.399	10.449
	(0.006)	(0.009)	(0.009)	(0.006)	(0.009)	(0.009)
75	3.772	8.553	12.325	2.764	7.118	9.881
	(0.006)	(0.009)	(0.008)	(0.006)	(0.009)	(0.008)
76	3.404	8.255	11.659	2.495	6.840	9.335
	(0.006)	(0.009)	(0.008)	(0.005)	(0.009)	(0.009)
77	3.058	7.956	11.013	2.241	6.566	8.807
	(0.005)	(0.008)	(0.008)	(0.005)	(0.009)	(0.008)
78	2.732	7.654	10.386	2.002	6.293	8.295
	(0.005)	(0.009)	(0.009)	(0.005)	(0.009)	(0.009)
79	2.424	7.351	9.775	1.775	6.020	7.794
	(0.005)	(0.009)	(0.008)	(0.004)	(0.010)	(0.009)
80	2.136	7.048	9.184	1.559	5.751	7.310
	(0.005)	(0.008)	(0.008)	(0.004)	(0.010)	(0.010)
81	1.869	6.749	8.618	1.354	5.492	6.847
	(0.004)	(0.008)	(0.008)	(0.004)	(0.011)	(0.011)

82	1.624	6.449	8.073	1.157	5.246	6.403
	(0.004)	(0.008)	(0.008)	(0.004)	(0.011)	(0.011)
83	1.401	6.143	7.545	0.968	5.031	5.999
	(0.004)	(0.008)	(0.008)	(0.004)	(0.012)	(0.013)
84	1.200	5.826	7.027	0.793	4.863	5.656
	(0.004)	(0.008)	(0.008)	(0.004)	(0.013)	(0.013)
85	1.023	5.506	6.529	0.636	4.734	5.370
	(0.004)	(0.009)	(0.009)	(0.004)	(0.014)	(0.014)
86	0.870	5.196	6.066	0.498	4.637	5.135
	(0.004)	(0.009)	(0.010)	(0.004)	(0.016)	(0.016)
87	0.739	4.901	5.639	0.379	4.563	4.942
	(0.004)	(0.010)	(0.010)	(0.004)	(0.016)	(0.016)
88	0.627	4.621	5.248	0.280	4.503	4.783
	(0.004)	(0.011)	(0.011)	(0.003)	(0.019)	(0.018)
89	0.532	4.360	4.892	0.200	4.446	4.646
	(0.004)	(0.011)	(0.011)	(0.003)	(0.019)	(0.018)
90	0.451	4.119	4.570	0.138	4.385	4.523
	(0.004)	(0.012)	(0.012)	(0.003)	(0.019)	(0.019)
91	0.384	3.898	4.282	0.092	4.315	4.407
	(0.003)	(0.013)	(0.013)	(0.002)	(0.020)	(0.020)
92	0.327	3.697	4.023	0.059	4.235	4.294
	(0.003)	(0.012)	(0.012)	(0.002)	(0.021)	(0.020)
93	0.279	3.514	3.793	0.036	4.145	4.182
	(0.003)	(0.013)	(0.013)	(0.001)	(0.020)	(0.020)
94	0.239	3.349	3.588	0.022	4.048	4.069
	(0.003)	(0.014)	(0.013)	(0.001)	(0.021)	(0.020)
95	0.205	3.200	3.405	0.012	3.945	3.957
	(0.003)	(0.013)	(0.013)	(0.001)	(0.020)	(0.020)
96	0.176	3.065	3.242	0.007	3.839	3.846
	(0.003)	(0.013)	(0.013)	(0.000)	(0.020)	(0.020)
97	0.152	2.942	3.094	0.004	3.731	3.734
	(0.003)	(0.013)	(0.012)	(0.000)	(0.020)	(0.020)
98	0.132	2.829	2.960	0.002	3.620	3.622
	(0.003)	(0.013)	(0.013)	(0.000)	(0.020)	(0.020)
99	0.114	2.722	2.837	0.001	3.508	3.509
	(0.003)	(0.013)	(0.012)	(0.000)	(0.020)	(0.020)

3.2 Some comparisons

A feature that is apparent immediately on comparing results from the four surveys is the difference between 1981 and the three later years. This is illustrated in Figures 3.4-3.6 and Tables 3.10 and 3.11, and reference should also be made to the discussion in ABS (2001), Mathers (1996) and Otis and Howe (1991). It seems that the health of elderly Australians declined between 1981 and 1988, a conclusion that is difficult to sustain on grounds other than the given figures. The conventional explanation is that there was no decline in real terms but that there were changes in perceptions and attitudes concerning disability. Whatever the correct explanation may be it would be prudent to use only results from the surveys of 1988, 1993 and 1998 for purposes of forecasting.

Over the period 1981-1998 there has been an increase in age-specific life expectancy of up to about two years, depending on age, accompanied by an increase in the proportion expected to be spent in the Disabled state. This is illustrated in Tables 3.10 and 3.11. The generalisation that the extra years of life tend to be years of disability is compelling on the basis of comparing results for 1981 with estimates from one of the later surveys. The situation is more complex if consideration is restricted to the surveys of 1988, 1993 and 1998. From Tables 3.7, 3.9 and Figures 3.4-3.6, over this decade expectancies for sixty year old females changed as follows:

$$\hat{e}_1(60)^{(1998)} - \hat{e}_1(60)^{(1988)} = 12.02 - 11.53 = 0.49$$

$$\hat{e}_2(60)^{(1998)} - \hat{e}_2(60)^{(1988)} = 12.32 - 11.42 = 0.90$$

$$\hat{e}_0(60)^{(1998)} - \hat{e}_0(60)^{(1988)} = 24.34 - 22.95 = 1.39\ .$$

Thus of the estimated increase in life expectancy some $(0.90/1.39)\times 100 = 65\%$, that is about two thirds, is expected to be in the state of Disability. The proportion tends to increase with age, for example at age eighty it is 77%. For males the picture seems similar but more pessimistic and is muddied by the retirement effect. For the very old, those over 90, females seem less disabled, but that is not the case with males.

Also, from Figure 3.2, there is no support for inferring that there is a trend towards rectangularisation of the survival curve or Disability-free survival curve (see Fries, 1983). There is, however, a curious already noted ‘retirement’ effect around age 65 in the male results. It is not clear why the male probabilities of states 1 and 2 behave in this way and there appear to be questions that require further investigation.

Confidence intervals and other inferential procedures can be implemented since covariances and standard errors of estimates can be calculated. Numerical values of standard errors of probabilities and health expectancies are determined by the covariance matrix of the estimated regression coefficients $\hat{\beta}^{(1)}$ and $\hat{\beta}^{(2)}$. As it happens, the use of previously graduated prevalences meant that the estimated regression curves gave extraordinarily good fits, as can be seen in Figure 3.1, and the result is that the standard errors of estimated probabilities and health expectancies are small. For example, from Tables 3.6-3.9, the health expectancies generally have standard errors less than 0.02. Then if $e_j(x)^{(S,t)}$ denotes the health expectancy of state j at age x for sex S and year t, the standard error of the difference of two expectancies satisfies the inequality

$$SE\left\{\hat{e}_j(x)^{(S_1,t_1)} - \hat{e}_i(y)^{(S_2,t_2)}\right\} \leq 2\sqrt{(0.02)^2} = 0.04\,.$$

Accepting this as a reasonable conservative estimate, a glance at Tables 3.6-3.9 indicates that most expectancies differ by more than 0.08, namely by more than two standard errors, and a plausible conclusion is that expectancies differing by more than this amount are significantly different.

Figure 3.4 Health expectancy of the Disability-free state at the survey years for selected ages

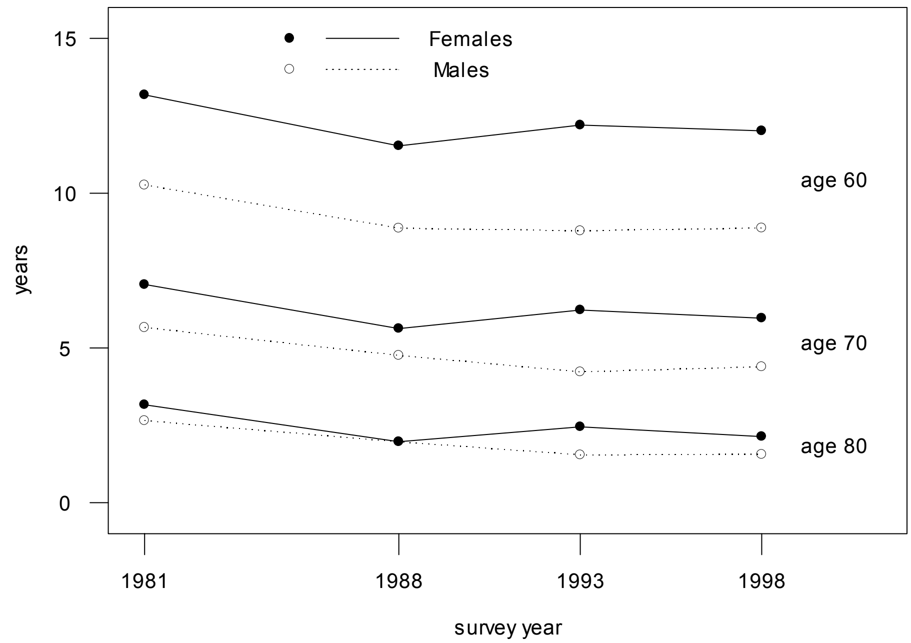


Figure 3.5 Health expectancy of the Disabled state at the survey years for selected ages

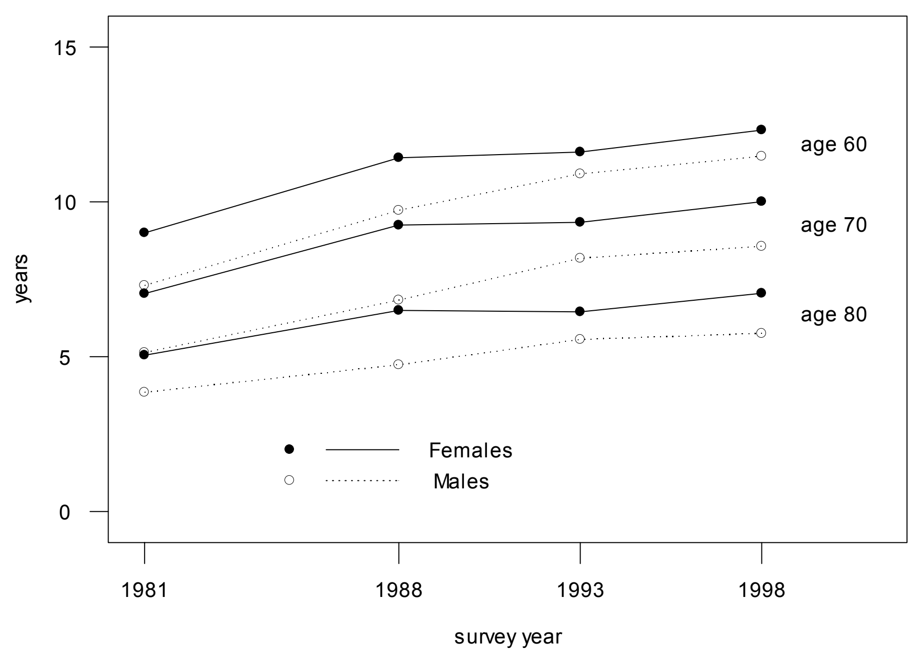


Figure 3.6 Current life expectancy at the survey years for selected ages

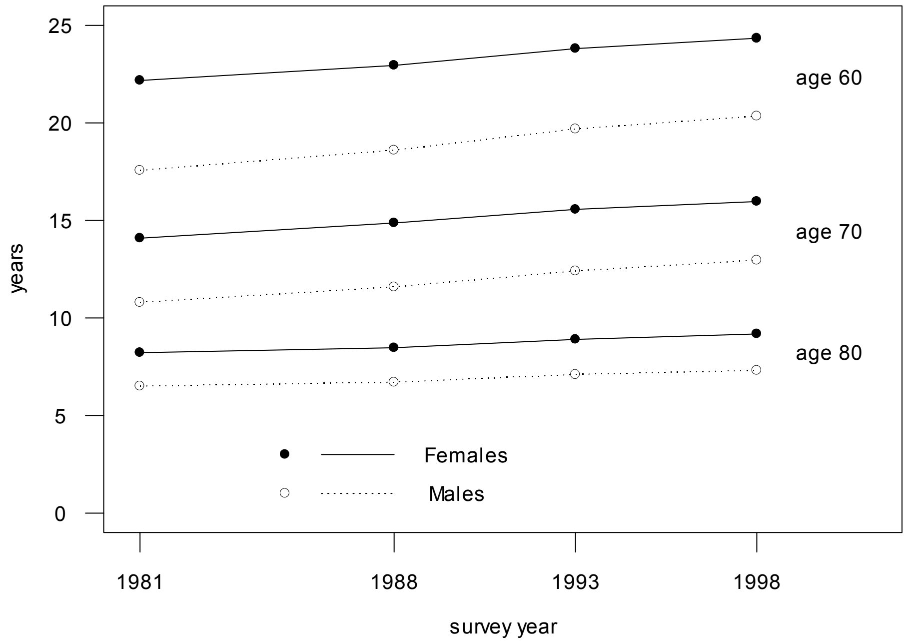


Table 3.10 Health expectancies current at the survey years for selected ages

		1981		1988		1993		1998	
		$\hat{e}_1(x)$	$\hat{e}_2(x)$	$\hat{e}_1(x)$	$\hat{e}_2(x)$	$\hat{e}_1(x)$	$\hat{e}_2(x)$	$\hat{e}_1(x)$	$\hat{e}_2(x)$
		Females							
Age	60	13.2	9.0	11.5	11.4	12.2	11.6	12.0	12.3
	65	9.9	8.1	8.3	10.4	9.0	10.6	8.7	11.3
	70	7.0	7.0	5.6	9.2	6.2	9.3	6.0	10.0
	75	4.8	6.0	3.5	7.9	4.1	7.9	3.8	8.6
	80	3.2	5.0	2.0	6.5	2.5	6.5	2.1	7.0
	85	1.9	4.2	0.9	5.0	1.3	4.9	1.0	5.5
	90	0.8	3.0	0.3	4.0	0.6	3.9	0.5	4.1
		Males							
Age	60	10.3	7.3	8.9	9.7	8.8	10.9	8.9	11.5
	65	7.8	6.0	6.6	8.2	6.3	9.5	6.5	10.0
	70	5.7	5.1	4.8	6.8	4.2	8.2	4.4	8.6
	75	3.9	4.4	3.2	5.7	2.7	6.8	2.8	7.1
	80	2.6	3.8	2.0	4.7	1.5	5.6	1.6	5.8
	85	1.5	3.2	0.9	3.9	0.7	4.3	0.6	4.7
	90	0.5	2.7	0.3	3.5	0.2	3.6	0.1	4.4

Table 3.11 Percentage of future life estimated to be in the state of Disability at the survey years for selected ages

		1981		1988		1993		1998	
		F	M	F	M	F	M	F	M
	60	41	42	50	52	49	55	51	56
	65	45	44	55	55	54	60	56	61
	70	50	48	62	59	60	66	63	66
Age	75	55	53	69	64	66	72	69	72
	80	61	59	77	71	72	78	77	79
	85	68	68	85	81	79	86	84	88
	90	78	84	93	92	87	94	90	97

3.3 Cohort health expectancies

Recall that when considering cohorts in Section 2.3, state 1 was taken as reference and the first task was the estimation of the log odds $\eta_j(x,y)$ of (2.10). Plots of the twenty realised log odds per cohort led to the fitting of polynomials such as those in (2.12), and a graphical representation of the results is shown in Figure 3.7 for selected cohorts, namely males and females aged 60, 65, 70 and 75 in 1980. With

$$\eta(x,y;\beta)=\begin{pmatrix}\eta_2(x,y;\beta^{(2)})\\ \eta_3(x,y;\beta^{(3)})\end{pmatrix}=Z_1'\beta$$

the parameterised vector of the log odds for a particular cohort, the regression analogous to (2.5) is

$$\tilde{\eta}(x,y;\beta)=Z_1'\beta+\varepsilon_1(x,y)\,,$$

in which the error vector $\varepsilon_1(x,y)$ has zero expectation and the covariance matrix $U_1(x,y)$ of (2.11). For a fixed age x in 1980 the twenty values of y are: $x+1,...,x+5$ from the 1981 survey, $x+8,...,x+12$ from the 1988 survey, $x+13,...,x+17$ from the 1993 survey, and $x+18,...,x+22$ from the 1998 survey.

Minimising the loss function

$$L_1(\beta)=\sum_y\{\tilde{\eta}(x,y)-Z_1'\beta\}'U_1(x,y)^{-1}\{\tilde{\eta}(x,y)-Z_1'\beta\}$$

led to the results shown below.

These results also include Figure 3.8 which shows plots of estimated cohort probabilities $\hat{p}_j(x,y)$, j = 1,2,0, ages x = 60,65,70,75, and Figure 3.9 which shows the corresponding ratios $\hat{p}_1(x,y)/\hat{p}_2(x,y)$.

Table 3.12 shows fitted log odds of the form:

$$\begin{aligned}\hat{\eta}_2(x,y) &= \hat{\beta}_1 + \hat{\beta}_2 y + \hat{\beta}_3 g_3(y) + \hat{\beta}_4 g_4(y) \\ \hat{\eta}_3(x,y) &= \hat{\beta}_5 + \hat{\beta}_6 y + \hat{\beta}_7 g_7(y) + \hat{\beta}_8 g_8(y),\end{aligned} \tag{3.2}$$

with $g_k(y)$ being quadratics similar to those in Sections 2.2 and 2.3. Where no parameter estimate is given in the table it is to be understood that the quadratic is not needed and the parameter in question has been set to zero in the regression.

Tables 3.13-3.16 list the estimated probabilities shown in Figure 3.8, and Table 3.17 gives the estimated health expectancies $\hat{e}_j(x)$ for all cohorts aged 60 to 77 in 1980. Note that the standard errors of Table 3.17 have numerical values that seem plausible when compared to those for current expectations shown in Tables 3.6-3.9. They can be used for inferential purposes. Under the assumption of independent cohorts the standard error of the difference between two expectations is

$$s = SE\left\{\hat{e}_j(x_1) - \hat{e}_i(x_2)\right\} = \sqrt{[SE(\hat{e}_j(x_1))]^2 + [SE(\hat{e}_i(x_2))]^2}\ .$$

Thus, for example, a 95% confidence interval for the difference is

$$\hat{e}_j(x_1) - \hat{e}_i(x_2) \pm 1.96s\ .$$

Figure 3.7 Cohort log odds for selected ages in 1980

(The legend and axes are the same as in Figure 2.4:
solid line = $\hat{\eta}_2(60, y)$, dashed line = $\hat{\eta}_3(60, y)$.)

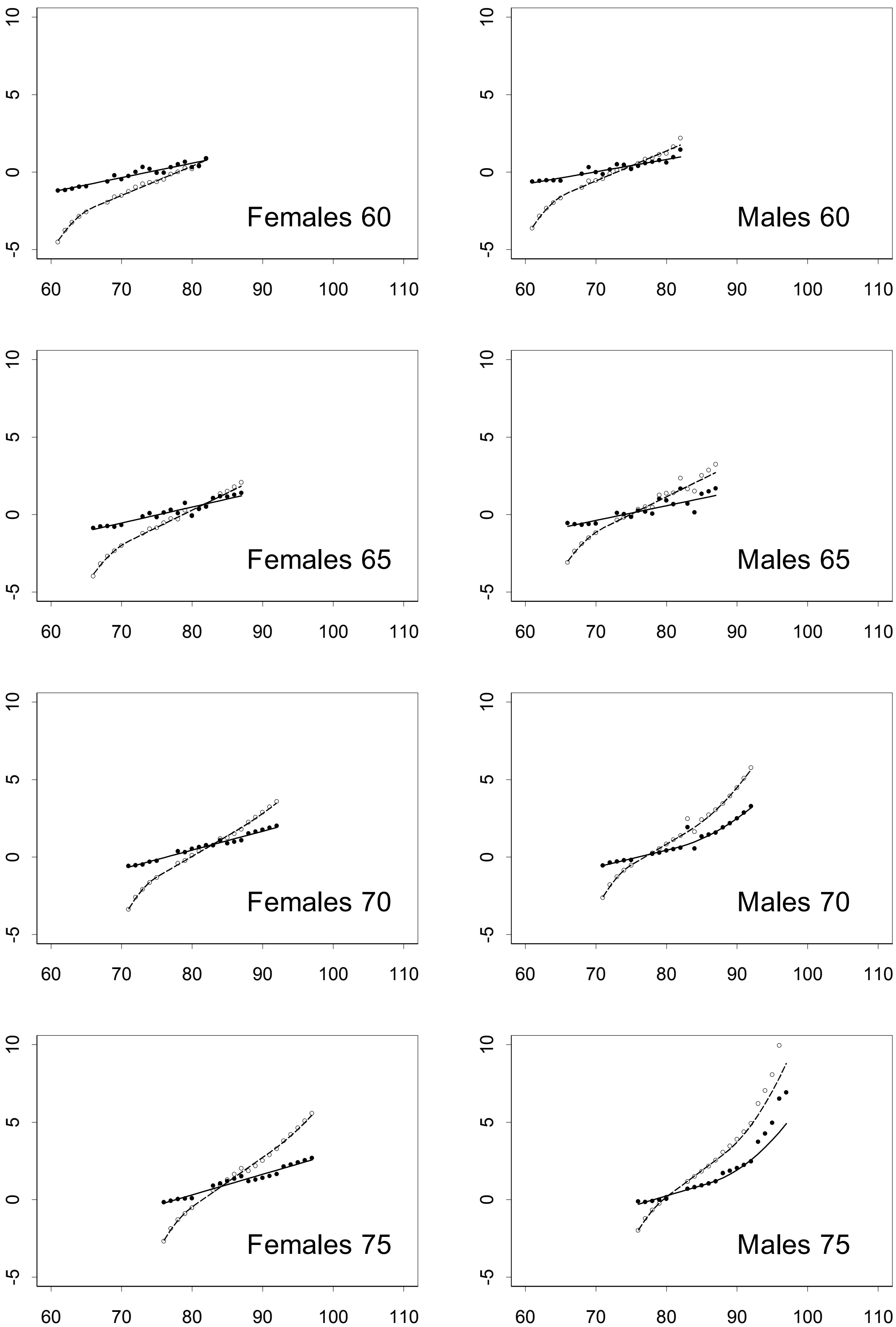


Figure 3.8 Cohort probabilities for selected ages in 1980

(The legend and axes are the same as in Figure 2.5:
solid line = $\hat{p}_1(60, y)$, dashed line = $\hat{p}_2(60, y)$, dotted line = $\hat{p}_0(60, y)$.)

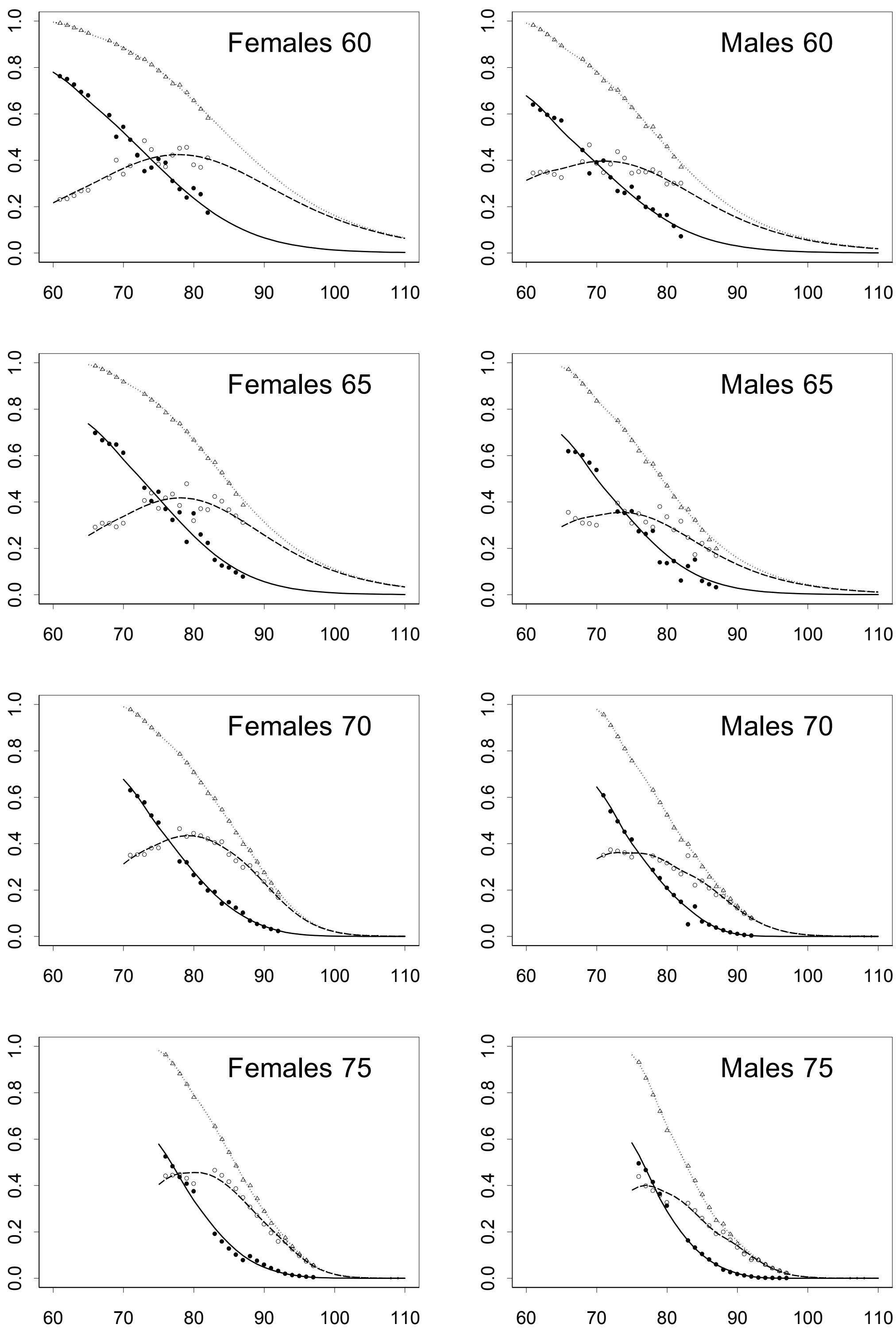


Figure 3.9 Cohort ratios of probabilities for selected ages in 1980

(The legend and axes are the same as in Figure 2.6:
solid line = $\hat{p}_1(60, y) / \hat{p}_2(60, y)$.)

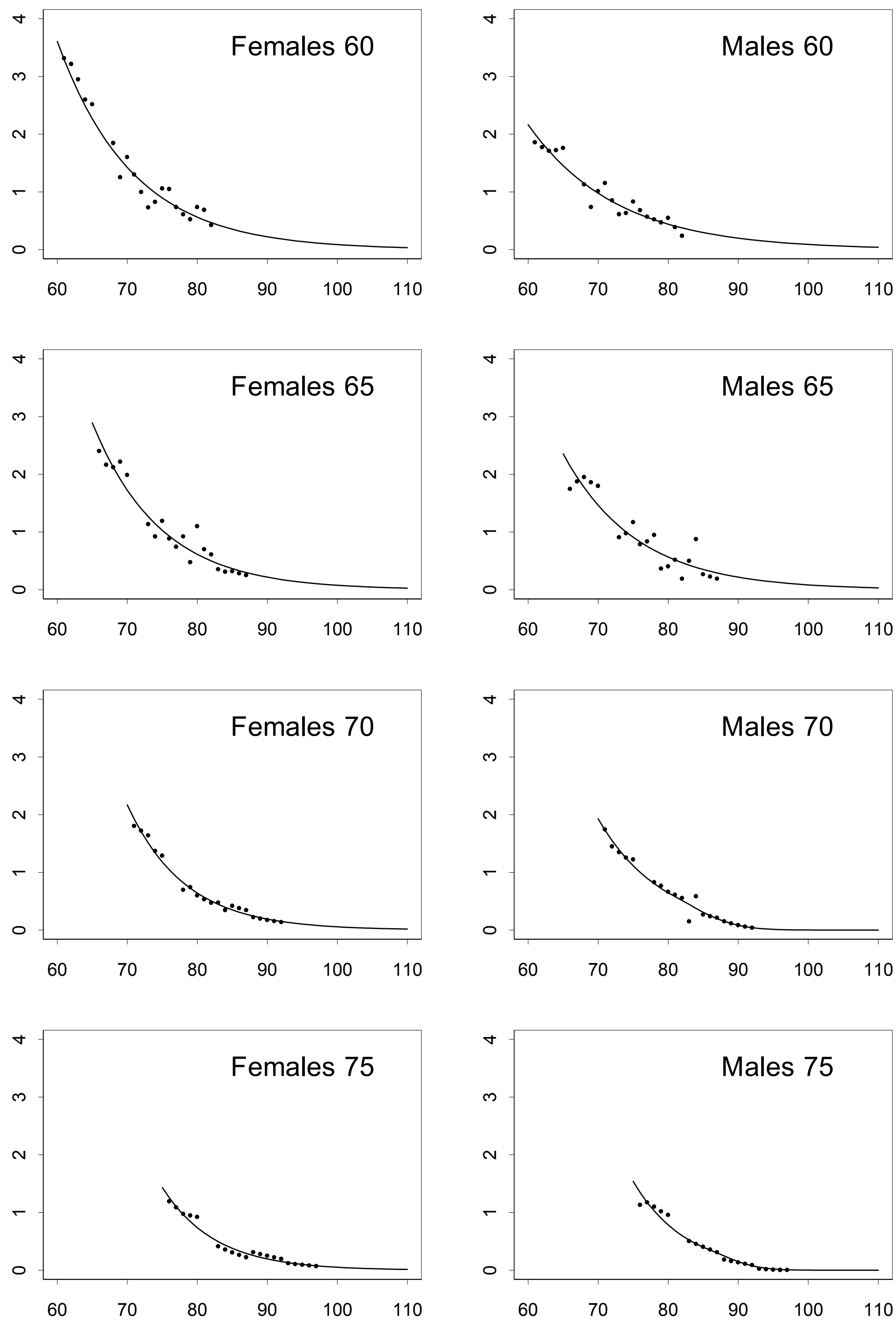


Table 3.12 Fitted log odds as given by (3.2) for selected ages in 1980

(Standard errors are given in parentheses below each parameter estimate. No parameter estimate indicates that the parameter has been set to zero.)

$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	$\hat{\beta}_5$	$\hat{\beta}_6$	$\hat{\beta}_7$	$\hat{\beta}_8$
Females 60							
–1.2832	0.0930			–3.4008	0.1892	–0.0487	
(0.0250)	(0.0057)			(0.0486)	(0.0055)	(0.0025)	
$g_7(y) = (y-66)^2 I(y<66)$,							
Females 65							
–1.0621	0.1034			–3.0553	0.2224	–0.0409	
(0.0420)	(0.0040)			(0.0335)	(0.0048)	(0.0035)	
$g_7(y) = (y-71)^2 I(y<71)$							
Females 70							
–0.7740	0.1219			–2.6102	0.2651	–0.0635	0.0073
(0.0341)	(0.0023)			(0.0175)	(0.0016)	(0.0027)	(0.0008)
$g_7(y) = (y-75)^2 I(y<75)$, $g_8(y) = (y-86)^2 I(y>86)$							
Females 75							
–0.3585	0.1330			–2.0678	0.3190	–0.0558	0.0141
(0.0479)	(0.0036)			(0.0368)	(0.0043)	(0.0037)	(0.0012)
$g_7(y) = (y-80)^2 I(y<80)$, $g_8(y) = (y-91)^2 I(y>91)$							
Males 60							
–0.7729	0.0796			–2.5067	0.1937	–0.0501	
(0.0692)	(0.0051)			(0.0455)	(0.0050)	(0.0020)	
$g_7(y) = (y-66)^2 I(y<66)$							
Males 65							
–0.8564	0.0951			–2.2219	0.2245	–0.0411	
(0.1238)	(0.0129)			(0.0878)	(0.0108)	(0.0042)	
$g_7(y) = (y-71)^2 I(y<71)$							
Males 70							
–0.6558	0.1095		0.0118	–1.8589	0.2688	–0.0618	0.0197
(0.0648)	(0.0150)		(0.0032)	(0.0430)	(0.0086)	(0.0024)	(0.0029)
$g_4(y) = (y-81)^2 I(y>81)$							
$g_7(y) = (y-75)^2 I(y<75)$, $g_8(y) = (y-83)^2 I(y>83)$							
Males 75							
–0.4301	0.1350		0.0196	–1.5165	0.3386	–0.0501	0.0356
(0.0741)	(0.0057)		(0.0037)	(0.0433)	(0.0046)	(0.0055)	(0.0059)
$g_4(y) = (y-86)^2 I(y>86)$							
$g_7(y) = (y-80)^2 I(y<80)$, $g_8(y) = (y-88)^2 I(y>88)$							

Table 3.13 Estimated probabilities for persons aged 60 in 1980

	Females			Males		
Age y	$\hat{p}_1(60,y)$	$\hat{p}_2(60,y)$	$\hat{p}_0(60,y)$	$\hat{p}_1(60,y)$	$\hat{p}_2(60,y)$	$\hat{p}_0(60,y)$
60	0.77947	0.21604	0.99550	0.67792	0.31297	0.99089
61	0.75983	0.23112	0.99094	0.65436	0.32713	0.98148
62	0.73738	0.24615	0.98354	0.62686	0.33935	0.96621
63	0.71210	0.26088	0.97298	0.59556	0.34913	0.94468
64	0.68467	0.27528	0.95996	0.56192	0.35671	0.91863
65	0.65658	0.28972	0.94630	0.52864	0.36340	0.89203
66	0.62972	0.30495	0.93466	0.49875	0.37127	0.87002
67	0.60357	0.32077	0.92434	0.47117	0.37981	0.85097
68	0.57646	0.33622	0.91268	0.44311	0.38679	0.82990
69	0.54851	0.35110	0.89961	0.41474	0.39204	0.80677
70	0.51985	0.36519	0.88504	0.38624	0.39536	0.78160
71	0.49064	0.37827	0.86891	0.35782	0.39662	0.75444
72	0.46106	0.39010	0.85116	0.32968	0.39572	0.72541
73	0.43129	0.40048	0.83177	0.30205	0.39261	0.69466
74	0.40155	0.40921	0.81075	0.27515	0.38728	0.66243
75	0.37204	0.41610	0.78814	0.24918	0.37979	0.62897
76	0.34300	0.42100	0.76400	0.22433	0.37026	0.59459
77	0.31462	0.42381	0.73844	0.20078	0.35885	0.55963
78	0.28712	0.42447	0.71159	0.17865	0.34577	0.52442
79	0.26068	0.42294	0.68362	0.15806	0.33127	0.48933
80	0.23546	0.41925	0.65471	0.13907	0.31562	0.45469
81	0.21160	0.41348	0.62508	0.12170	0.29911	0.42081
82	0.18920	0.40575	0.59495	0.10596	0.28201	0.38797
83	0.16834	0.39621	0.56455	0.09181	0.26459	0.35640
84	0.14907	0.38504	0.53410	0.07918	0.24712	0.32630
85	0.13139	0.37245	0.50384	0.06800	0.22981	0.29782
86	0.11529	0.35867	0.47396	0.05817	0.21287	0.27103
87	0.10073	0.34393	0.44466	0.04957	0.19645	0.24602
88	0.08766	0.32845	0.41610	0.04210	0.18068	0.22278
89	0.07598	0.31246	0.38845	0.03565	0.16566	0.20131
90	0.06563	0.29618	0.36180	0.03010	0.15146	0.18156
91	0.05649	0.27978	0.33627	0.02535	0.13813	0.16347
92	0.04847	0.26345	0.31191	0.02130	0.12567	0.14697
93	0.04146	0.24733	0.28879	0.01786	0.11410	0.13196
94	0.03537	0.23156	0.26693	0.01494	0.10340	0.11835
95	0.03010	0.21623	0.24633	0.01248	0.09355	0.10603
96	0.02555	0.20145	0.22699	0.01041	0.08450	0.09492
97	0.02164	0.18726	0.20890	0.00868	0.07623	0.08491
98	0.01829	0.17372	0.19201	0.00722	0.06869	0.07590
99	0.01543	0.16086	0.17629	0.00600	0.06182	0.06782
100	0.01300	0.14869	0.16169	0.00498	0.05559	0.06057
101	0.01093	0.13724	0.14817	0.00413	0.04994	0.05407
102	0.00918	0.12648	0.13566	0.00343	0.04483	0.04826
103	0.00770	0.11642	0.12412	0.00284	0.04022	0.04306
104	0.00645	0.10703	0.11348	0.00235	0.03606	0.03841
105	0.00540	0.09829	0.10369	0.00194	0.03231	0.03426
106	0.00451	0.09017	0.09468	0.00161	0.02894	0.03055
107	0.00377	0.08265	0.08642	0.00133	0.02591	0.02724
108	0.00314	0.07570	0.07884	0.00110	0.02319	0.02429
109	0.00262	0.06927	0.07189	0.00091	0.02074	0.02165
110	0.00219	0.06335	0.06554	0.00075	0.01855	0.01930

Table 3.14 Estimated probabilities for persons aged 65 in 1980

Age y	Females $\hat{p}_1(65,y)$	Females $\hat{p}_2(65,y)$	Females $\hat{p}_0(65,y)$	Males $\hat{p}_1(65,y)$	Males $\hat{p}_2(65,y)$	Males $\hat{p}_0(65,y)$
65	0.73717	0.25486	0.99203	0.68996	0.29302	0.98299
66	0.71197	0.27295	0.98492	0.65981	0.30817	0.96798
67	0.68336	0.29052	0.97389	0.62438	0.32071	0.94509
68	0.65147	0.30713	0.95860	0.58421	0.33002	0.91423
69	0.61722	0.32267	0.93989	0.54143	0.33636	0.87778
70	0.58233	0.33759	0.91992	0.49928	0.34112	0.84040
71	0.54891	0.35287	0.90178	0.46119	0.34653	0.80772
72	0.51643	0.36815	0.88458	0.42586	0.35190	0.77776
73	0.48317	0.38195	0.86512	0.39032	0.35471	0.74503
74	0.44938	0.39393	0.84330	0.35498	0.35477	0.70975
75	0.41534	0.40375	0.81909	0.32025	0.35199	0.67224
76	0.38139	0.41112	0.79250	0.28655	0.34636	0.63291
77	0.34784	0.41578	0.76362	0.25427	0.33800	0.59227
78	0.31502	0.41757	0.73259	0.22376	0.32712	0.55088
79	0.28327	0.41638	0.69965	0.19531	0.31401	0.50931
80	0.25288	0.41220	0.66508	0.16912	0.29903	0.46815
81	0.22412	0.40511	0.62923	0.14533	0.28260	0.42793
82	0.19721	0.39528	0.59249	0.12398	0.26513	0.38912
83	0.17230	0.38297	0.55528	0.10505	0.24706	0.35211
84	0.14951	0.36849	0.51800	0.08845	0.22876	0.31721
85	0.12886	0.35221	0.48107	0.07403	0.21058	0.28462
86	0.11036	0.33450	0.44486	0.06164	0.19282	0.25446
87	0.09395	0.31576	0.40971	0.05107	0.17570	0.22677
88	0.07952	0.29638	0.37590	0.04213	0.15941	0.20154
89	0.06695	0.27671	0.34366	0.03462	0.14406	0.17869
90	0.05609	0.25707	0.31317	0.02835	0.12975	0.15810
91	0.04678	0.23774	0.28452	0.02315	0.11650	0.13964
92	0.03885	0.21895	0.25779	0.01885	0.10431	0.12316
93	0.03214	0.20087	0.23301	0.01531	0.09318	0.10849
94	0.02650	0.18364	0.21014	0.01241	0.08307	0.09548
95	0.02178	0.16737	0.18914	0.01004	0.07391	0.08395
96	0.01785	0.15210	0.16995	0.00811	0.06566	0.07377
97	0.01459	0.13788	0.15247	0.00654	0.05825	0.06479
98	0.01190	0.12469	0.13659	0.00527	0.05161	0.05688
99	0.00968	0.11254	0.12223	0.00424	0.04568	0.04992
100	0.00787	0.10139	0.10926	0.00341	0.04039	0.04380
101	0.00638	0.09119	0.09757	0.00274	0.03569	0.03843
102	0.00517	0.08190	0.08707	0.00220	0.03151	0.03371
103	0.00418	0.07346	0.07764	0.00177	0.02780	0.02957
104	0.00338	0.06581	0.06919	0.00142	0.02452	0.02593
105	0.00273	0.05890	0.06163	0.00113	0.02161	0.02275
106	0.00220	0.05267	0.05487	0.00091	0.01904	0.01995
107	0.00177	0.04706	0.04883	0.00073	0.01677	0.01750
108	0.00143	0.04201	0.04344	0.00058	0.01477	0.01535
109	0.00115	0.03749	0.03863	0.00047	0.01300	0.01347
110	0.00092	0.03343	0.03435	0.00037	0.01144	0.01182

Table 3.15 Estimated probabilities for persons aged 70 in 1980

	Females			Males		
Age y	$\hat{p}_1(70,y)$	$\hat{p}_2(70,y)$	$\hat{p}_0(70,y)$	$\hat{p}_1(70,y)$	$\hat{p}_2(70,y)$	$\hat{p}_0(70,y)$
70	0.67740	0.31241	0.98981	0.64424	0.33437	0.97861
71	0.64280	0.33488	0.97768	0.60428	0.34992	0.95420
72	0.60275	0.35472	0.95747	0.55588	0.35913	0.91501
73	0.55831	0.37115	0.92946	0.50167	0.36161	0.86328
74	0.51277	0.38507	0.89784	0.44773	0.36007	0.80780
75	0.47058	0.39919	0.86977	0.40084	0.35967	0.76051
76	0.43122	0.41322	0.84444	0.35935	0.35975	0.71909
77	0.39174	0.42405	0.81579	0.31850	0.35575	0.67425
78	0.35264	0.43120	0.78384	0.27899	0.34768	0.62667
79	0.31443	0.43432	0.74875	0.24148	0.33575	0.57723
80	0.27763	0.43319	0.71082	0.20653	0.32039	0.52692
81	0.24269	0.42777	0.67047	0.17459	0.30218	0.47677
82	0.21004	0.41820	0.62824	0.14545	0.28422	0.42968
83	0.17997	0.40479	0.58476	0.11920	0.26928	0.38848
84	0.15271	0.38799	0.54070	0.09506	0.25420	0.34926
85	0.12836	0.36840	0.49675	0.07276	0.23584	0.30860
86	0.10692	0.34664	0.45356	0.05335	0.21462	0.26797
87	0.08792	0.32201	0.40993	0.03742	0.19132	0.22874
88	0.07102	0.29383	0.36485	0.02508	0.16692	0.19201
89	0.05629	0.26305	0.31934	0.01607	0.14250	0.15857
90	0.04372	0.23081	0.27453	0.00984	0.11905	0.12889
91	0.03326	0.19833	0.23159	0.00576	0.09736	0.10312
92	0.02476	0.16682	0.19158	0.00322	0.07798	0.08121
93	0.01804	0.13731	0.15535	0.00173	0.06121	0.06294
94	0.01287	0.11062	0.12349	0.00089	0.04711	0.04800
95	0.00899	0.08727	0.09625	0.00044	0.03558	0.03601
96	0.00615	0.06744	0.07359	0.00021	0.02637	0.02657
97	0.00412	0.05110	0.05523	0.00009	0.01920	0.01929
98	0.00271	0.03799	0.04070	0.00004	0.01373	0.01377
99	0.00175	0.02773	0.02948	0.00002	0.00965	0.00967
100	0.00111	0.01989	0.02100	0.00001	0.00667	0.00668
101	0.00070	0.01402	0.01472	0.00000	0.00454	0.00454
102	0.00043	0.00973	0.01016	0.00000	0.00303	0.00303
103	0.00026	0.00664	0.00690	0.00000	0.00200	0.00200
104	0.00015	0.00446	0.00462	0.00000	0.00129	0.00129
105	0.00009	0.00295	0.00304	0.00000	0.00082	0.00082
106	0.00005	0.00193	0.00198	0.00000	0.00052	0.00052
107	0.00003	0.00124	0.00127	0.00000	0.00032	0.00032
108	0.00002	0.00078	0.00080	0.00000	0.00019	0.00019
109	0.00001	0.00049	0.00050	0.00000	0.00012	0.00012
110	0.00000	0.00030	0.00030	0.00000	0.00007	0.00007

Table 3.16 Estimated probabilities for persons aged 75 in 1980

Age y	Females $\hat{p}_1(75,y)$	Females $\hat{p}_2(75,y)$	Females $\hat{p}_0(75,y)$	Males $\hat{p}_1(75,y)$	Males $\hat{p}_2(75,y)$	Males $\hat{p}_0(75,y)$
75	0.57799	0.40388	0.98187	0.58371	0.37967	0.96338
76	0.53493	0.42694	0.96187	0.53118	0.39544	0.92662
77	0.48625	0.44329	0.92955	0.47009	0.40053	0.87062
78	0.43389	0.45181	0.88570	0.40466	0.39462	0.79928
79	0.38199	0.45435	0.83634	0.34190	0.38160	0.72349
80	0.33537	0.45563	0.79100	0.28816	0.36810	0.65626
81	0.29333	0.45519	0.74851	0.24180	0.35353	0.59533
82	0.25303	0.44851	0.70154	0.19915	0.33325	0.53241
83	0.21518	0.43565	0.65083	0.16105	0.30845	0.46950
84	0.18034	0.41706	0.59740	0.12799	0.28055	0.40854
85	0.14897	0.39351	0.54248	0.10008	0.25108	0.35116
86	0.12132	0.36606	0.48738	0.07712	0.22144	0.29855
87	0.09747	0.33593	0.43340	0.05844	0.19585	0.25429
88	0.07732	0.30437	0.38169	0.04353	0.17709	0.22063
89	0.06061	0.27254	0.33315	0.03109	0.15966	0.19076
90	0.04701	0.24145	0.28846	0.02068	0.13937	0.16004
91	0.03612	0.21187	0.24798	0.01277	0.11754	0.13031
92	0.02721	0.18232	0.20953	0.00732	0.09564	0.10296
93	0.01986	0.15203	0.17189	0.00389	0.07504	0.07893
94	0.01403	0.12265	0.13668	0.00192	0.05675	0.05867
95	0.00958	0.09564	0.10522	0.00088	0.04139	0.04226
96	0.00632	0.07206	0.07838	0.00037	0.02911	0.02948
97	0.00403	0.05247	0.05649	0.00015	0.01977	0.01991
98	0.00248	0.03694	0.03942	0.00005	0.01296	0.01302
99	0.00148	0.02516	0.02664	0.00002	0.00822	0.00823
100	0.00086	0.01660	0.01746	0.00001	0.00504	0.00504
101	0.00048	0.01062	0.01110	0.00000	0.00299	0.00299
102	0.00026	0.00659	0.00685	0.00000	0.00171	0.00171
103	0.00014	0.00397	0.00411	0.00000	0.00095	0.00095
104	0.00007	0.00232	0.00239	0.00000	0.00051	0.00051
105	0.00003	0.00132	0.00135	0.00000	0.00027	0.00027
106	0.00002	0.00073	0.00075	0.00000	0.00013	0.00013
107	0.00001	0.00039	0.00040	0.00000	0.00007	0.00007
108	0.00000	0.00020	0.00021	0.00000	0.00003	0.00003
109	0.00000	0.00010	0.00011	0.00000	0.00001	0.00001
110	0.00000	0.00005	0.00005	0.00000	0.00001	0.00001

Table 3.17 Cohort health expectancies for persons aged 60 to 77 in 1980

(Standard errors are given in parentheses below each estimate.)

	Females			Males		
Age x	$\hat{e}_1(x)$	$\hat{e}_2(x)$	$\hat{e}_0(x)$	$\hat{e}_1(x)$	$\hat{e}_2(x)$	$\hat{e}_0(x)$
60	11.960	11.994	23.955	8.781	10.345	19.127
	(0.310)	(0.379)	(0.135)	(0.184)	(0.251)	(0.109)
61	11.454	11.469	22.923	8.533	9.703	18.236
	(0.270)	(0.307)	(0.101)	(0.250)	(0.358)	(0.123)
62	10.789	11.165	21.954	8.352	9.065	17.418
	(0.247)	(0.198)	(0.085)	(0.191)	(0.302)	(0.116)
63	10.223	10.768	20.991	7.967	8.655	16.622
	(0.284)	(0.342)	(0.093)	(0.215)	(0.316)	(0.119)
64	9.575	10.484	20.058	7.584	8.237	15.821
	(0.283)	(0.350)	(0.096)	(0.211)	(0.303)	(0.130)
65	9.114	10.024	19.138	7.287	7.771	15.058
	(0.248)	(0.292)	(0.096)	(0.038)	(0.080)	(0.107)
66	8.518	9.721	18.239	6.803	7.550	14.352
	(0.253)	(0.285)	(0.090)	(0.032)	(0.108)	(0.122)
67	8.136	9.223	17.359	6.374	7.228	13.602
	(0.112)	(0.076)	(0.087)	(0.093)	(0.173)	(0.094)
68	7.455	9.042	16.497	5.919	6.982	12.901
	(0.042)	(0.108)	(0.083)	(0.101)	(0.184)	(0.100)
69	6.877	8.784	15.661	5.508	6.726	12.234
	(0.046)	(0.120)	(0.082)	(0.106)	(0.195)	(0.110)
70	6.329	8.346	14.675	5.104	6.295	11.399
	(0.062)	(0.089)	(0.030)	(0.062)	(0.107)	(0.056)
71	5.865	8.059	13.924	4.745	6.046	10.791
	(0.039)	(0.073)	(0.038)	(0.051)	(0.066)	(0.053)
72	5.395	7.786	13.181	4.359	5.865	10.224
	(0.024)	(0.065)	(0.046)	(0.074)	(0.124)	(0.052)
73	4.914	7.547	12.461	4.075	5.615	9.690
	(0.029)	(0.076)	(0.051)	(0.081)	(0.136)	(0.056)
74	4.506	7.266	11.772	3.808	5.364	9.172
	(0.031)	(0.080)	(0.049)	(0.079)	(0.136)	(0.059)
75	4.130	6.951	11.081	3.541	5.106	8.646
	(0.051)	(0.107)	(0.057)	(0.077)	(0.143)	(0.069)
76	3.793	6.632	10.425	3.331	4.857	8.188
	(0.073)	(0.124)	(0.052)	(0.074)	(0.124)	(0.055)
77	3.470	6.331	9.800	3.089	4.656	7.744
	(0.078)	(0.123)	(0.047)	(0.081)	(0.123)	(0.047)

4 STATISTICAL METHODS

There is no difficulty in generalising the discussion of Section 2 to the case of $c = a + b$ states, comprising a alive states, for example a states of health, and b absorbing states, such as death by b causes. Again we consider only marginal health expectancies $e_j(x)$, $j = 1,...,a$, for the alive states and give an account of the method introduced by Davis et al. (2001). Proofs given here are more general but also simpler than those in that paper. The case of conditional health expectancies, based on longitudinal data, will be treated elsewhere. A general theory from the present point of view, under a Markov assumption, can be based on Davis et al. (2002a), and a particular case is considered by Nurminen et al. (2002).

4.1 Log odds for large samples

As in Section 2 consider a birth cohort of $l_0(0)$ individuals for which the number of survivors $l_0(x)$ at age x is known. At this stage there is no need to assume a multinomial distribution of frequencies or that individuals are stochastically independent. However throughout the following are assumed to hold:

Assumption 1 For a fixed age x and $y = x+1, x+2,...$ the random variables $\tilde{l}_j(y)$, $j = 1,2,...,c$, denoting the number of the $l_0(x)$ in state j at age y, have expectations

$$E\tilde{l}_j(y) = l_0(x) p_j(x, y) = l_j(y),$$

where as before $p_j(x, y)$ is the conditional probability that an individual alive at age x is in state j at age y. The size $l_0(x)$ of the cohort at age x is known but the expected numbers of survivors $l_j(y)$ are not known.

Let $\tilde{l}(y)$ be the $c \times 1$ random vector of the number of survivors at y and $l(y)$ its expectation. Thus:

$$\tilde{l}(y) = (\tilde{l}_1(y),...,\tilde{l}_c(y))'$$

$$E\tilde{l}(y) = l(y) = (l_1(y),...,l_c(y))'.$$

Assumption 2 As $l_0(x)$ tends to infinity, $l_0(x)^{-1/2}\{\tilde{l}(y)-l(y)\}$ is asymptotically normally distributed with zero mean vector and some $c \times c$ covariance matrix $B(x,y)$ of rank $c-1$, whose *ij*th element is $B_{ij}(x,y)=Cov\{\tilde{l}_i(y),\tilde{l}_j(y)\}$ for $i,j\in\{1,...,c\}$.

Further, $l_0(x)^{-1/2}\{\tilde{l}(y)-l(y)\}$ and $l_0(x)^{-1/2}\{\tilde{l}(y+u)-l(y+u)\}$ are asymptotically jointly normally distributed with some $c \times c$ cross-covariance matrix $C(x,y,y+u)$, whose *ij*th element is $C_{ij}(x,y,y+u)=Cov\{\tilde{l}_i(y),\tilde{l}_j(y+u)\}$.
Thus $B(x,y)=C(x,y,y)$.

Assumption 3 Distinct birth cohorts are stochastically independent.

The third assumption is only required when discussing current health expectancies or the joint variation of the population with year and age.

Suppose state 1 is taken as reference and for $y > x$ write

$$\xi_j(x,y)=\log\left\{\frac{p_j(x,y)}{p_1(x,y)}\right\}=\log\left\{\frac{l_j(y)}{l_1(y)}\right\},\quad j=2,3,...,c.$$

This is estimated by

$$\tilde{\xi}_j(x,y)=\log\left\{\frac{\tilde{p}_j(x,y)}{\tilde{p}_1(x,y)}\right\}=\log\left\{\frac{\tilde{l}_j(y)}{\tilde{l}_1(y)}\right\}.$$

Rearranging terms and letting $l_0(x)$ tend to infinity we find, using the Taylor expansion $\log(1+z)=z-z^2/2+z^3/3-...$ with $z\approx 0$, that

$$\tilde{\xi}_j(x,y)-\xi_j(x,y)=\log\left\{\frac{\tilde{l}_j(y)}{\tilde{l}_1(y)}\right\}-\log\left\{\frac{l_j(y)}{l_1(y)}\right\}$$

$$=\log\left\{\frac{\tilde{l}_j(y)}{l_j(y)}\right\}-\log\left\{\frac{\tilde{l}_1(y)}{l_1(y)}\right\}$$

$$= \log\left\{1 + \frac{\tilde{l}_j(y) - l_j(y)}{l_j(y)}\right\} - \log\left\{1 + \frac{\tilde{l}_1(y) - l_1(y)}{l_1(y)}\right\}$$

$$= \left\{\left[\frac{\tilde{l}_j(y) - l_j(y)}{l_j(y)}\right] - \frac{1}{2}\left[\frac{\tilde{l}_j(y) - l_j(y)}{l_j(y)}\right]^2 + \frac{1}{3}\left[\frac{\tilde{l}_j(y) - l_j(y)}{l_j(y)}\right]^3 - \ldots\right\}$$

$$- \left\{\left[\frac{\tilde{l}_1(y) - l_1(y)}{l_1(y)}\right] - \frac{1}{2}\left[\frac{\tilde{l}_1(y) - l_1(y)}{l_1(y)}\right]^2 + \frac{1}{3}\left[\frac{\tilde{l}_1(y) - l_1(y)}{l_1(y)}\right]^3 - \ldots\right\} \quad (4.1)$$

$$= \frac{\tilde{l}_j(y) - l_j(y)}{l_j(y)} - \frac{\tilde{l}_1(y) - l_1(y)}{l_1(y)} + o_p\{l_0(x)^{-1/2}\}. \quad (4.2)$$

The notation $o_p(m)$ here denotes a random variable which when divided by m converges in probability to zero as m tends to zero.

As regards the step from (4.1) to (4.2), observe that

$$\left[\frac{\tilde{l}_j(y) - l_j(y)}{l_j(y)}\right]^2 = \left[\frac{\tilde{l}_j(y) - l_j(y)}{l_0(x)p_j(x,y)}\right]^2 \quad \text{by Assumption 1}$$

$$= \frac{B_{jj}(x,y)}{l_0(x)p_j(x,y)^2}\left[\frac{l_0(x)^{-1/2}\{\tilde{l}_j(y) - l_j(y)\}}{B_{jj}(x,y)^{1/2}}\right]^2$$

$$= \frac{B_{jj}(x,y)}{l_0(x)p_j(x,y)^2}U^2$$

where, by Assumption 2, $U \xrightarrow{p} N(0,1)$ and hence $U^2 \xrightarrow{p} \chi^2(1)$ as $l_0(x) \to \infty$.

Thus

$$\frac{1}{l_0(x)^{-1/2}}\left[\frac{\tilde{l}_j(y) - l_j(y)}{l_j(y)}\right]^2 = \frac{1}{l_0(x)^{1/2}}\frac{B_{jj}(x,y)}{p_j(x,y)^2}U^2 \xrightarrow{p} 0$$

as $l_0(x) \to \infty$, since $B_{jj}(x,y)$ and $p_j(x,y)$ are constants.

That is,

$$\left[\frac{\tilde{l}_j(y)-l_j(y)}{l_j(y)}\right]^2 = o_p\{l(x)^{-1/2}\}.$$

A similar argument applies for the higher powers of $\{\tilde{l}_j(y)-l_j(y)\}/l_j(y)$ in (4.1).

It follows by (4.2) and Assumption 1 that

$$l_0(x)^{1/2}\{\tilde{\xi}_j(x,y)-\xi_j(x,y)\}$$

$$= l_0(x)^{1/2}\left[\frac{\tilde{l}_j(y)-l_j(y)}{l_0(x)p_j(x,y)} - \frac{\tilde{l}_1(y)-l_1(y)}{l_0(x)p_1(x,y)} + o_p\{l_0(x)^{-1/2}\}\right]$$

$$= l_0(x)^{-1/2}\left[\frac{\tilde{l}_j(y)-l_j(y)}{p_j(x,y)} - \frac{\tilde{l}_1(y)-l_1(y)}{p_1(x,y)}\right] + o_p(1)$$

since $o_p(m)/m = o_p(1)$

$$= l_0(x)^{-1/2}\left(-\frac{1}{p_1(x,y)},0,...,0,\frac{1}{p_j(x,y)},0,...,0\right)\{\tilde{l}(y)-l(y)\} + o_p(1).$$

With $\xi(x,y) = (\xi_2(x,y),\ldots,\xi_c(x,y))'$ and $\tilde{\xi}(x,y) = (\tilde{\xi}_2(x,y),\ldots,\tilde{\xi}_c(x,y))'$, it follows that as $l_0(x)$ tends to infinity,

$$l_0(x)^{1/2}\{\tilde{\xi}(x,y)-\xi(x,y)\} = A(x,y)l_0(x)^{-1/2}\{\tilde{l}(y)-l(y)\} + o_p(1), \quad (4.3)$$

where $A(x,y)$ is the $(c-1)\times c$ matrix

$$A(x,y) = \begin{pmatrix} -p_1(x,y)^{-1} & p_2(x,y)^{-1} & 0 & 0 & \cdots & 0 \\ -p_1(x,y)^{-1} & 0 & p_3(x,y)^{-1} & 0 & \cdots & 0 \\ -p_1(x,y)^{-1} & 0 & 0 & p_4(x,y)^{-1} & & 0 \\ \vdots & \vdots & \vdots & & \ddots & \\ -p_1(x,y)^{-1} & 0 & 0 & 0 & & p_c(x,y)^{-1} \end{pmatrix}.$$

Then (4.3) and the first part of Assumption 2 ensure the following:

Result 1 As $l_0(x)$ tends to infinity, $l_0(x)^{1/2}\{\tilde{\xi}(x,y)-\xi(x,y)\}$ is asymptotically normally distributed with zero mean vector and covariance matrix

$$V(x,y)=A(x,y)B(x,y)A(x,y)'.$$

Note that this result does not require independence of the individuals in the cohort since asymptotic normality can hold in a variety of circumstances. With a suitable interpretation of the $p_j(x,y)$ neither is the identity of survival distributions over the cohort a strict requirement. The classical case of independence and identically distributed survival distributions leads to the multinomial distribution of frequencies with *B*(*x*,*y*) a multinomial covariance matrix and is treated below. For the moment we make the point that asymptotic normality of the log odds holds for most processes that are likely to be of interest.

Use of the second part of Assumption 2 yields

Result 2 As $l_0(x)$ tends to infinity, the $2p\times 1$ vector

$$\begin{pmatrix} l_0(x)^{1/2}\{\tilde{\xi}(x,y)-\xi(x,y)\} \\ l_0(x)^{1/2}\{\tilde{\xi}(x,y+u)-\xi(x,y+u)\} \end{pmatrix}, \quad u>0,$$

is asymptotically normally distributed with zero mean vector and covariance matrix

$$\begin{pmatrix} A(x,y)B(x,y)A(x,y)' & A(x,y)C(x,y,y+u)A(x,y+u)' \\ A(x,y+u)C(x,y,y+u)'A(x,y)' & A(x,y+u)B(x,y+u)A(x,y+u)' \end{pmatrix}.$$

These two results are phrased to take account of the fact that state 1 has been used as reference. If, more generally, state *r* is taken as reference then the log odds under study would be

$$\tilde{\xi}_j(x,y)=\log\left\{\frac{\tilde{p}_j(x,y)}{\tilde{p}_r(x,y)}\right\},$$

and the matrix $A(x,y)$ would require adjustment so that it had *r*th column all of whose terms are $-p_r(x,y)^{-1}$, namely

$$\underset{(c-1)\times c}{A_r(x,y)} = \begin{pmatrix} p_1^{-1} & 0 & \cdots & 0 & -p_r^{-1} & 0 & \cdots & 0 \\ 0 & p_2^{-1} & & 0 & -p_r^{-1} & 0 & \cdots & 0 \\ \vdots & & \ddots & & \vdots & \vdots & & \vdots \\ 0 & 0 & & p_{r-1}^{-1} & -p_r^{-1} & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & -p_r^{-1} & p_{r+1}^{-1} & & 0 \\ \vdots & \vdots & & \vdots & \vdots & & \ddots & \\ 0 & 0 & \cdots & 0 & -p_r^{-1} & 0 & & p_c^{-1} \end{pmatrix},$$

where $p_j = p_j(x,y)$.

4.2 Estimating the log odds

Suppose $\xi(x,y)$ is parameterised as $\xi(x,y;\beta)$. Then a consequence of Result 1 is that for large $l_0(x)$ we can specify the vector regression model

$$\tilde{\xi}(x,y) = \xi(x,y;\beta) + \varepsilon(x,y), \quad y = x+1, x+2, \ldots, \tag{4.4}$$

with the asymptotically normally distributed errors $\varepsilon(x,y)$ having zero mean vector and covariance matrix

$$Var\varepsilon(x,y) = l_0(x)^{-1} A(x,y)B(x,y)A(x,y)' = l_0(x)^{-1} V(x,y).$$

By Assumption 3, the sequence of $\varepsilon(x,y)$ is assumed independent for a current year but will generally be autocorrelated with autocorrelations given by Result 2 when (4.4) is applied to a given cohort. Even for a current year in which formulae for probabilities at successive ages may be connected as in a Markov model, it may seem paradoxical that the errors can be taken as stochastically independent. However, a distinction must be drawn between the mathematical model and the way in which data has been collected. Provided Assumption 3 holds it is the latter that ensures the desired independence in the case of a current year.

Consistent estimates of the parameter vector β can be obtained by minimising the weighted least squares loss function

$$L(\beta) = \sum_y \{\tilde{\xi}(x,y) - \xi(x,y;\beta)\}' V(x,y)^{-1} \{\tilde{\xi}(x,y) - \xi(x,y;\beta)\}. \tag{4.5}$$

As in Section 2, the parameterisation of the log odds will typically be linear in β, say

$$\xi_j(x,y) = \xi_j(x,y;\beta^{(j)}) = z^{(j)}(x,y)'\beta^{(j)} = \sum_{r=1}^{k(j)} z_r^{(j)}(x,y)\beta_r^{(j)} ,$$

where the known coefficients $z_r^{(j)}(x,y)$ are functions of ages x and y or indicators of quantities of interest. In this case, and always in what follows with reference state 1,

$$\xi(x,y) = \xi(x,y;\beta) = \begin{pmatrix} \xi_2(x,y;\beta^{(2)}) \\ \vdots \\ \xi_c(x,y;\beta^{(c)}) \end{pmatrix} = \begin{pmatrix} z^{(2)}(x,y)'\beta^{(2)} \\ \vdots \\ z^{(c)}(x,y)'\beta^{(c)} \end{pmatrix} = Z'\beta , \qquad (4.6)$$

where:

$$\underset{q\times(c-1)}{Z = Z(x,y)} = \begin{pmatrix} z^{(2)} & 0 & \cdots & 0 \\ 0 & z^{(3)} & & 0 \\ \vdots & & \ddots & \\ 0 & 0 & & z^{(c)} \end{pmatrix}, \quad z^{(j)} = z^{(j)}(x,y)$$

$$\beta = \begin{pmatrix} \beta^{(2)} \\ \vdots \\ \beta^{(c)} \end{pmatrix} = \begin{pmatrix} (\beta_1^{(2)},\ldots,\beta_{k(2)}^{(2)})' \\ \\ (\beta_1^{(c)},\ldots,\beta_{k(c)}^{(c)})' \end{pmatrix} \equiv \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_q \end{pmatrix}, \quad q = k(2)+k(3)+\ldots+k(c) .$$

For the linear model (4.6) the regression takes the familiar form

$$\tilde{\xi}(x,y) = Z(x,y)'\beta + \varepsilon(x,y), \quad y = x+1, x+2, \ldots , \qquad (4.7)$$

and minimisation of (4.5) leads to the weighted least squares estimator of β given by:

$$\hat{\beta} = (\hat{\beta}_1,\ldots,\hat{\beta}_q)' = S_1^{-1}S_2$$

$$S_1 = l_0(x)\sum_y Z(x,y)V(x,y)^{-1}Z(x,y)' \qquad (4.8)$$

$$S_2 = l_0(x)\sum_y Z(x,y)V(x,y)^{-1}\tilde{\xi}(x,y) .$$

The estimator $\hat{\beta}$ is consistent for β and by Result 1 asymptotically normally distributed. Equation (4.8) was used to calculate the quantities from which the numerical results given earlier follow.

Standard errors of the $\hat{\beta}_i$, found by usual arguments from the minimisation of (4.5), require independence of the errors $\varepsilon(x,y)$. By Assumption 3, this is the case when considering (4.7) for a current year, in which case the estimated covariance matrix of $\hat{\beta}$ is $\hat{V}ar\hat{\beta} = S_1^{-1}$.

As mentioned, by Result 2 independence of the errors generally fails for cohorts. Under these circumstances, the results of Liang and Zeger (1986) can be used to find a consistent estimator of the variance of $\hat{\beta}$. This estimator is detailed in Section 4.4.

A consequence of these arguments is that it is possible to model and estimate the log odds by vector regression methods and estimate the probabilities $p_j(x,y)$ and hence the health expectancies. Confining attention to the linear model (4.7), we have:

$$\hat{\xi}_j(x,y) = \xi_j(x,y;\hat{\beta}^{(j)}) = z^{(j)}(x,y)'\hat{\beta}^{(j)}$$

$$\hat{p}_1(x,y) = \left\{1 + \sum_{j=2}^{c} e^{\hat{\xi}_j(x,y)}\right\}^{-1}$$

$$\hat{p}_j(x,y) = \hat{p}_1(x,y)e^{\hat{\xi}_j(x,y)}, \quad j = 2,3,...,c.$$

Then, given an estimate $\hat{V}ar\hat{\beta}$ of the covariance matrix of $\hat{\beta}$, and using the delta method, the variances and covariances of the $\hat{p}_j(x,y)$ are estimated by:

$$\hat{V}ar\hat{p}_j(x,y) = D_j(x,y;\hat{\beta})'\left(\hat{V}ar\hat{\beta}\right)D_j(x,y;\hat{\beta})$$

$$\hat{C}ov\{\hat{p}_i(x,y),\hat{p}_j(x,y+u)\} = D_i(x,y;\hat{\beta})'\left(\hat{V}ar\hat{\beta}\right)D_j(x,y+u;\hat{\beta}), \quad u > 0,$$

where

$$D_j(x,y;\beta) = \frac{\partial p_j(x,y)}{\partial \beta} = \left(\frac{\partial p_j(x,y)}{\partial \beta_1}, \ldots, \frac{\partial p_j(x,y)}{\partial \beta_q}\right)'.$$

Note that the derivatives here are easy to calculate. For example, if

$$\xi_2(x,y) = \beta_1^{(2)} + \beta_2^{(2)} y = \beta_1 + \beta_2 y ,$$

so that

$$p_1(x,y) = \left\{ 1 + e^{\beta_1 + \beta_2 y} + \sum_{j=3}^{c} e^{\xi_j(x,y;\beta^{(j)})} \right\}^{-1} ,$$

then the second element of $D_1(x,y;\beta)$ is

$$\frac{\partial p_1(x,y)}{\partial \beta_2} = -y p_1(x,y)^2 e^{\beta_1 + \beta_2 y} = -y p_1(x,y) p_2(x,y) .$$

Estimates of health expectancies follow directly from (2.8). The numerical values of Sections 2 and 3 were obtained using the S-PLUS function `integrate`. In the case of the current analysis, standard errors of the $\hat{e}_j(x)$ were calculated by Monte Carlo using 1000 samplings from the estimated distribution of the regression coefficients. This distribution was taken as multivariate normal with mean vector $\hat{\beta}$ and covariance matrix $\hat{V}ar\hat{\beta} = S_1^{-1}$, as given by (4.8). In the case of the cohort analysis, standard errors of the $\hat{e}_j(x)$ were based on (1.7) and computed according to:

$$SE(\hat{e}_j(x)) = \sqrt{\psi_{jj}} \; , \quad j = 1,2$$

$$SE(\hat{e}_0(x)) = \sqrt{\psi_{11} + \psi_{22} + 2\psi_{12}}$$

$$\psi_{ij} = \hat{C}ov\{\hat{e}_i(x), \hat{e}_j(x)\} = \frac{1}{\hat{p}_0(x,x)^2} \sum_{y=x}^{95} \sum_{t=x}^{95} \frac{\hat{C}ov\{\hat{p}_i(x,y), \hat{p}_j(x,t)\}}{2^{I(y=x)+I(y=95)+I(t=x)+I(t=95)}} ,$$

where $\hat{p}_0(x,x)$ is a term very close to unity, $I(.)$ is the standard indicator function, and $\psi_{jj} = \hat{V}ar\hat{e}_j(x)$. Further details of these calculations can be found in Section 4.5.

4.3 Multinomial frequencies

When all $l_0(x)$ individuals in the cohort of interest have the same probabilistic structure and are stochastically independent the distribution of the frequencies $\tilde{l}_1(y), \tilde{l}_2(y), ..., \tilde{l}_c(y)$ given $l_0(x)$ is multinomial (Chiang, 1968, p248). In that case Assumptions 1 and 2 are automatically fulfilled with the following covariance matrices:

$$\underset{c\times c}{B(x,y)} = \begin{pmatrix} p_1(1-p_1) & -p_1p_2 & -p_1p_3 & \cdots & -p_1p_c \\ -p_2p_1 & p_2(1-p_2) & -p_2p_3 & \cdots & -p_2p_c \\ -p_3p_1 & -p_3p_2 & p_3(1-p_3) & & -p_3p_c \\ \vdots & \vdots & & \ddots & \\ -p_cp_1 & -p_cp_2 & -p_cp_3 & & p_c(1-p_c) \end{pmatrix},$$

$$\underset{c\times c}{C(x,y,y+u)} =$$

$$\begin{pmatrix} p_1\{p_{11}(y,y+u)-p_1(x,y+u)\} & \cdots & p_1\{p_{1c}(y,y+u)-p_c(x,y+u)\} \\ \vdots & \ddots & \vdots \\ p_c\{p_{c1}(y,y+u)-p_1(x,y+u)\} & \cdots & p_c\{p_{cc}(y,y+u)-p_c(x,y+u)\} \end{pmatrix},$$

where $p_j = p_j(x,y)$.

As is well known, $B(x,y)$ is of rank $c-1$. Concerning $C(x,y,y+u)$, the transition probability $p_{ij}(y,y+u)$ of being in state *j* at age *y* + *u* conditional on having been in *i* at *y*, is generally non-zero for *i*, *j* = 1,2,...,*a*. However, for *i* an absorbing state, that is $i = a+1,\ldots,c$, then it is equal to 1 if *j* = *i* and zero otherwise.

The important consequence of multinomial frequencies is that multiplying out the matrices given in Result 1 yields

$$\underset{(c-1)\times(c-1)}{V(x,y)} = A(x,y)B(x,y)A(x,y)'$$

$$= \begin{pmatrix} p_2^{-1}+p_1^{-1} & p_1^{-1} & p_1^{-1} & \cdots & p_1^{-1} \\ p_1^{-1} & p_3^{-1}+p_1^{-1} & p_1^{-1} & \cdots & p_1^{-1} \\ p_1^{-1} & p_1^{-1} & p_4^{-1}+p_1^{-1} & & p_1^{-1} \\ \vdots & \vdots & & \ddots & \\ p_1^{-1} & p_1^{-1} & p_1^{-1} & & p_c^{-1}+p_1^{-1} \end{pmatrix} \qquad (4.9)$$

$$= \text{diag}(p_2, \ldots, p_c)^{-1} + p_1^{-1} \underset{\sim}{1} \underset{\sim}{1}',$$

where $p_j = p_j(x, y)$ and $\underset{\sim}{1} = (1, \ldots, 1)'$ is a column vector of $(c-1)$ 1's.

The inverse of $V(x, y)$, required for weighted least squares minimisation, is easily checked to be the multinomial covariance matrix

$$\underset{(c-1)\times(c-1)}{V(x,y)^{-1}} = \begin{pmatrix} p_2(1-p_2) & -p_2p_3 & -p_2p_4 & \cdots & -p_2p_c \\ -p_3p_2 & p_3(1-p_3) & -p_3p_4 & \cdots & -p_3p_c \\ -p_4p_2 & -p_4p_3 & p_4(1-p_4) & & -p_4p_c \\ \vdots & \vdots & & \ddots & \\ -p_cp_2 & -p_cp_3 & -p_cp_4 & & p_c(1-p_c) \end{pmatrix}.$$

Unfortunately, the matrix $A(x, y)C(x, y, y+u)A(x, y+u)'$ of autocovariances turns out to involve the transition probabilities $p_{ij}(y, y+u)$ which cannot be estimated from cross-sectional data. For this reason the Liang-Zeger approach is used to calculate standard errors of the vector $\hat{\beta}$.

In the special three-state case discussed in earlier sections the off-diagonal matrices in Result 2 can easily be calculated. We find that

$$A(x,y)C(x,y,y+u)A(x,y+u)' = \begin{pmatrix} \frac{p_{11}-p_{21}}{p_1} - \frac{p_{12}-p_{22}}{p_2} & \frac{p_{11}-p_{21}}{p_1} - \frac{p_{13}-p_{23}}{p_3} \\ \frac{p_{11}}{p_1} - \frac{p_{12}}{p_2} & \frac{p_{11}}{p_1} - \frac{1-p_{13}}{p_3} \end{pmatrix},$$

where $p_j = p_j(x, y)$ and $p_{ij} = p_{ij}(y, y+u)$.

Thus, for example, the asymptotic covariance of $l_0(x)^{1/2}\{\tilde{\xi}_2(x,y) - \xi_2(x,y)\}$ and $l_0(x)^{1/2}\{\tilde{\xi}_2(x,y+u) - \xi_2(x,y+u)\}$ is $p_1^{-1}(p_{11}-p_{21}) - p_2^{-1}(p_{12}-p_{22})$, which generally will be non-zero. More importantly, the transition probabilities $p_{ij}(y, y+u)$ cannot be estimated from marginal frequencies alone.

If state *r* is taken as reference, the weight matrix $V(x, y)^{-1}$ becomes the multinomial covariance matrix

$$\underset{(c-1)\times(c-1)}{V_r(x,y)^{-1}} =$$

$$\begin{pmatrix} p_1(1-p_1) & -p_1p_2 & \cdots & -p_1p_{r-1} & -p_1p_{r+1} & \cdots & -p_1p_c \\ -p_2p_1 & p_2(1-p_2) & & -p_2p_{r-1} & -p_2p_{r+1} & \cdots & -p_2p_c \\ \vdots & & \ddots & & \vdots & & \vdots \\ -p_{r-1}p_1 & -p_{r-1}p_2 & & p_{r-1}(1-p_{r-1}) & -p_{r-1}p_{r+1} & & -p_{r-1}p_c \\ -p_{r+1}p_1 & -p_{r+1}p_2 & & -p_{r+1}p_{r-1} & p_{r+1}(1-p_{r+1}) & & -p_{r+1}p_c \\ \vdots & \vdots & & \vdots & & \ddots & \\ -p_cp_1 & -p_cp_2 & \cdots & -p_cp_{r-1} & -p_cp_{r+1} & & p_c(1-p_c) \end{pmatrix}.$$

In practice, for a given set of data, the weight matrix is estimated by substituting the non-parametric maximum likelihood estimators $l_j(y)/l_0(x)$ for the $p_j(x,y)$ and evaluating the $\hat{\beta}$ of (4.8).

4.4 The Liang-Zeger estimator

In this section we provide details of how to construct a consistent estimator of the variance of $\hat{\beta}$, as given by (4.8), in the case of the cohort analysis (see Section 4.2). This estimator was obtained using results in Liang and Zeger (1986), and we focus on the situation where a = 2, b = 1, state 1 is taken as reference, and 5 cohorts are involved. All the numerical results for the cohort analysis in this paper, and also in Davis et al. (2001), were obtained with these parameters. Notes in square brackets below are provided for purposes of clarifying the S-PLUS function `LIANG.COH` in Part B of Section 4.5.

The Liang-Zeger estimator of $Var\hat{\beta}$ is given by

$$\underset{q\times q}{\hat{V}ar\hat{\beta}} = Q_1^{-1}Q_2Q_1^{-1} \qquad [Q_1^{-1} = \texttt{solvnew1},\ \ Q_2 = \texttt{varnew2}]$$

where:

$$\underset{q\times q}{Q_1} = \sum_{h=1}^{5} \underset{q\times 8}{Z(h)} \underset{8\times 8}{W(h)^{-1}} \underset{8\times q}{Z(h)'} \qquad \text{(a symmetric matrix)}$$

$$[Q_1 = \texttt{varnew1},\ h = \texttt{j},\ \ 5 = \texttt{ncoh}]$$

$$\underset{q\times 8}{Z(h)} = \big(Z(x,x+h), Z(x,x+h+7), Z(x,x+h+12), Z(x,x+h+17)\big)$$

[$Z(h)'$ = `dmat`]

$$\underset{q\times 2}{Z(x,y)} = \begin{pmatrix} z^{(2)}(x,y) & 0 \\ 0 & z^{(3)}(x,y) \end{pmatrix}$$ as in Section 4.2 [$Z(x,x+h)$ = `dmat1`]

$$\underset{8\times 8}{W(h)} = \begin{pmatrix} W(x,x+h) & 0 & 0 & 0 \\ 0 & W(x,x+h+7) & 0 & 0 \\ 0 & 0 & W(x,x+h+12) & 0 \\ 0 & 0 & 0 & V(x,x+h+17) \end{pmatrix}$$

where 0 stands for a 2×2 matrix of zeros [$W(h)$ = `vmat`]

$$\underset{2\times 2}{W(x,y)} = \begin{pmatrix} \frac{1}{l_2(y)} + \frac{1}{l_1(y)} & \frac{1}{l_1(y)} \\ \frac{1}{l_1(y)} & \frac{1}{l_3(y)} + \frac{1}{l_1(y)} \end{pmatrix}$$ [$W(x,x+h)$ = `vmat1`]

$$\underset{q\times q}{Q_2} = \sum_{h=1}^{5} \underset{q\times 8}{Z(h)} \underset{8\times 8}{W(h)^{-1}} \underset{8\times 8}{M(h)} \underset{8\times 8}{W(h)^{-1}} \underset{8\times q}{Z(h)'}$$ (a symmetric matrix)

[Q_2 = `varnew2`]

$$\underset{8\times 8}{M(h)} = \{\underset{8\times 1}{\tilde{\xi}(h)} - \underset{8\times q}{Z(h)'} \underset{q\times 1}{\hat{\beta}}\}\{\underset{8\times 1}{\tilde{\xi}(h)} - \underset{8\times q}{Z(h)'} \underset{q\times 1}{\hat{\beta}}\}'$$ [$M(h)$ = `smat`, $\hat{\beta}$ = `prm`]

$$\underset{8\times 1}{\tilde{\xi}(h)} = \begin{pmatrix} \tilde{\xi}(x,x+h) \\ \tilde{\xi}(x,x+h+7) \\ \tilde{\xi}(x,x+h+12) \\ \tilde{\xi}(x,x+h+17) \end{pmatrix}$$ [$\tilde{\xi}(h)$ = `svecob`]

$$\tilde{\xi}(x,y) \equiv \tilde{\eta}(x,y) = \begin{pmatrix} \tilde{\eta}_2(x,y) \\ \tilde{\eta}_3(x,y) \end{pmatrix}$$ as in Section 2.3 [$\tilde{\xi}(x,x+h)$ = `svecob1`].

Note that $W(x,y) = V(x,y)/l_0(x)$ as per (4.9) with $c = 3$. Thus the matrix Q_1 is identical to S_1 as given by (4.8) with $c = 3$ and y ranging over the set $\{x+1, x+2, ..., x+5, x+8, x+9, ..., x+22\}$.

Also note that, for reasons of computational stability, Q_1^{-1} is calculated in the S-PLUS code as $UR^{-1}U'$, where URU' is the singular value decomposition of Q_1 (thus R is a $q \times q$ diagonal matrix whose diagonal elements are the singular values of Q_1).

4.5 Computations

In this section we present S-PLUS code (version 6.1) and data which can be used to reproduce all results in the body of the paper. The S-PLUS computer language provides a convenient environment for these types of calculation. Two good references are Venables and Ripley (1999, 2000). With the aid of the examples provided here the reader who is moderately familiar with S-PLUS should be able to modify the given code so as at analyse other disability and related types of data.

This section consists of three parts. Part A provides the code for producing current results, with a focus on females aged 60 and 61 in 1981. Part B provides the code for producing cohort results, with a focus on females aged 60 and males aged 70 in 1980. The code is laid out in such a way that if it is executed in the given order, the indicated output will be produced. Code for selected figures in the paper is also included. Part C provides a listing of all the data used in this paper, together with a guide on how to import that data into S-PLUS.

Part A: S-PLUS code for analysis of current data

```
1. Set up data for females in 1981
----------------------------------

L0 <- 75412     # estimated no. of Australian females aged 60 in 1981

rvec <- c(   0.2318, 0.2373, 0.2533, 0.2779, 0.2843, 0.2941, 0.3160,
             0.3205, 0.3109, 0.3347, 0.3569, 0.3678, 0.3792, 0.4221,
             0.4374, 0.4565, 0.4793, 0.5066, 0.5138, 0.5208, 0.5342,
             0.5620, 0.5739, 0.5892, 0.6143, 0.5959, 0.5957, 0.6137,
             0.6328, 0.6572, 0.6915, 0.7142, 0.7621, 0.8249, 0.8911,
             0.9546, 0.9901, 1.0000, 1.0000)
   # disability rates for females from the 1981 survey, ages 61-99
   # (see Table 1.2)

qx <- c(     0.00798, 0.00878, 0.00966, 0.01063, 0.01169, 0.01285,
             0.01411, 0.01549, 0.01703, 0.01875, 0.02072, 0.02295,
             0.02548, 0.02833, 0.03150, 0.03501, 0.03892, 0.04327,
             0.04817, 0.05374, 0.06010, 0.06728, 0.07531, 0.08408,
             0.09351, 0.10351, 0.11566, 0.12923, 0.14440, 0.16134,
             0.18027, 0.19451, 0.20883, 0.22309, 0.23710, 0.25072,
             0.26383, 0.27627, 0.28795, 0.29232)
    # hazards from Australian life tables, females 1981, ages 60-99

Lx <- c(L0,rep(NA,39))  # ages 60 to 99
px <- 1-qx
for(i in 2:40) Lx[i] <- Lx[i-1] * px[i-1]
dx <- Lx*qx   # deaths age 60 to 99
cdx <- cumsum(dx)  # cumulative deaths age 60 to 99
L <- Lx[-1]
L3 <- cdx[-40];     L2 <- rvec*L;       L1 <- L-L2

# The following produces the data for females in Table 3.1.

round(cbind(age=61:99,L1=L1,L2=L2,L3=L3),digits=1)
```

```
#       age      L1      L2      L3
#  [1,]  61 57469.2 17341.0   601.8
#  [2,]  62 56556.8 17596.6  1258.6
#  [3,]  63 54835.5 18601.6  1974.9
#  [4,]  64 52465.2 20191.2  2755.6
#  ..............................
#  [36,]  96 163.4 3436.2 71812.4
#  [37,]  97  26.2 2623.7 72762.1
#  [38,]  98   0.0 1917.8 73494.2
#  [39,]  99   0.0 1365.6 74046.4
```

```
2. Declare functions needed for current analysis
------------------------------------------------
```

```
INTEG.SPLINE <-
function(xvec,yvec,a=min(xvec),b=max(xvec)){
# Integrates under a spline through a set of points.
# Inputs:    xvec = vector of x values
#            yvec = vector of y values
#            a = lower bound for the integration
#            b = upper bound for the integration.
# Outputs:   area under a spline from a to b.
       fit <- smooth.spline(xvec, yvec)
       assign("fit",fit,frame=1)
       spline.f <- function(x){ predict.smooth.spline(fit,x)$y }
       spline.ff <- function(f,fun){ fun(t) }
       integrate(spline.f,a,b)$integral
}

REG.CUR <-
function(L1,L2,L3,xmat1,xmat2,nobs=length(L1)){
# Performs a regression for analysing current disability data.
# Inputs:    L1 = vector of numbers in state 1 (disability-free)
#            L2 = vector of numbers in state 2 (disabled)
#            L3 = vector of numbers in state 3 (dead)
#            xmat1 = design matrix for xi1, where
#                    nrow=51 (ages 60-110), ncol=variable
#             eg:   col1 = 1,1,1,...
#                   col2 = 0,1,2,...
#                   col3 = 36,25,16,9,4,1,0,0,0,...
#                   col4 = 0,0,...,1,4,9,...
#            xmat2 = design matrix for xi2
#            nobs = number of observations used for the regression.
# Outputs:   $param = vector of parameter estimates
#            $paramvr = estimated covariance  matrix of estimates.
       xmatreg1 <- xmat1[2:(nobs+1),]
       xmatreg2 <- xmat2[2:(nobs+1),]
       np1 <- ncol(xmat1)   # number of parameters for xi1
       np2 <- ncol(xmat2)
       np <- np1+np2      # total number of parameters
       L1 <- L1[1:nobs]
       L2 <- L2[1:nobs]
       L3 <- L3[1:nobs]
       sum1 <- matrix(0,np,np)
       sum2 <- matrix(0,np,1)
       xi1vec <- log(L1/L3)
       xi2vec <- log(L2/L3)
       w11vec <- 1/L1+1/L3
       w12vec <- 1/L3
       w22vec <- 1/L2+1/L3
       for(i in 1:nobs) {
              dv <- matrix(c(xmatreg1[i,],rep(0,np),
                     xmatreg2[i,]),nrow=2,ncol=np,byrow=T)
              varmat <- matrix(c(w11vec[i],w12vec[i],
                            w12vec[i],w22vec[i]),2,2)
              wtvec <- solve(varmat)
              dvt <- t(dv)
```

```
            prod1 <- dvt %*% wtvec %*% dv
            sum1 <- sum1 + prod1
            xi1 <- xi1vec[i]
            xi2 <- xi2vec[i]
            xivec <- matrix(c(xi1,xi2),2,1)
            prod2 <- dvt %*% wtvec %*% xivec
            sum2 <- sum2+prod2
            }
      inv1 <- solve(sum1)
      estprm <- inv1 %*% sum2
      list(param=estprm,paramvr=inv1)
}

EST.CUR <-
function(b,xmat1,xmat2){
# Estimates current probabilities and log odds.
# Inputs:     b = vector of parameter estimates (from call to REG.CUR)
#             xmat1, xmat2 = design matrices for xi1, xi2.
# Outputs:    $p1vec = vector of probabilities for state 1
#             $p2vec = vector of probabilities for state 2
#             $p0vec = vector of probabilities for state 0
#             $xi1vec = vector of log odds for state 1 (rel. to 3)
#             $xi2vec = vector of log odds for state 2 (rel. to 3).
      np1 <- ncol(xmat1)   # number of parameters for xi1
      np2 <- ncol(xmat2)
      np <- np1+np2
      p1 <- rep(NA,nrow(xmat1))
      p2 <- p1
      xi1 <- xmat1 %*% b[1:np1]
      xi2 <- xmat2 %*% b[(np1+1):np]
      t1 <- exp(xi1)
      t2 <- exp(xi2)
      p3 <- 1/(1+t1+t2)
      p1 <- p3*t1
      p2 <- p3*t2
      p0 <- p1+p2
      list(p1vec=p1,p2vec=p2,p0vec=p0,xi1vec=xi1,xi2vec=xi2)
}

EXP.CUR <-
function(str=60,fin=110,p1vec,p2vec,xmat1,xmat2){
# Computes estimates of current health expectancies via integration.
# Inputs:     str = starting age
#             fin = finishing age
#             p1vec, p2vec = vectors from call to EST.CUR
#             xmat1, xmat2 = design matrices for xi1, xi2.
# Ouptuts:    $e1 = estimate of expected years in state 1
#             $e2 = estimate of expected years in state 2
#             $e0 = estimate of expected years in state 0.
      i <- str-59   # index for starting age
      den <- p1vec[i] + p2vec[i]
      e1 <- INTEG.SPLINE(x=60:110,y=p1vec,a=str,b=fin)/den
      e2 <- INTEG.SPLINE(x=60:110,y=p2vec,a=str,b=fin)/den
      e0 <- e1+e2
      list(e1=e1,e2=e2,e0=e0)
}

SES.CUR <-
function(m=10,b,v,str=60,fin=110,xmat1,xmat2,alp=0.05){
# Computes standard errors for current health expectancies via
# Monte Carlo, and also confidence intervals for these SE's.
# Inputs:     m = number of simulations
#             b = vector of parameter estimates
#             v = covariance matrix for parameter estimates
#             str, fin = starting, finishing ages
#             xmat1, xmat2 = design matrices for xi1, xi2
#             alp = level for confidence intervals.
# Outputs:    $s1 = standard error for e1
```

```
#             $s2 = standard error for e2
#             $s0 = standard error for e0
#             $ci.s1 = 1-alp confidence interval for s1
#             $ci.s2 = 1-alp confidence interval for s2
#             $ci.s0 = 1-alp confidence interval for s0.
      e1vec <- rep(NA,m)
      e2vec <- rep(NA,m)
      e0vec <- rep(NA,m)
      np <- length(b)
      for(j in 1:m){
            br <- rmvnorm(n=1,d=np,mean=as.vector(b),cov=v)
            estr <- EST.CUR(b=br,xmat1=xmat1,xmat2=xmat2)
            expr <- EXP.CUR(str=str,fin=fin,p1vec=estr$p1vec,
                            p2vec=estr$p2vec,xmat1=xmat1,xmat2=xmat2)
            e1vec[j] <- expr$e1
            e2vec[j] <- expr$e2
            e0vec[j] <- expr$e0
            }
      s1 <- sqrt(var(e1vec))
      s2 <- sqrt(var(e2vec))
      s0 <- sqrt(var(e0vec))
      quants <- qchisq(c(1-alp/2,alp/2),m-1)
      ci.s1 <- sqrt(((m-1)*(s1^2))/quants)
      ci.s2 <- sqrt(((m-1)*(s2^2))/quants)
      ci.s0 <- sqrt(((m-1)*(s0^2))/quants)
      list(s1=s1,s2=s2,s0=s0,ci.s1=ci.s1,ci.s2=ci.s2,ci.s0=ci.s0)
}

3. Apply above functions to females aged 60 and 61 in 1981
----------------------------------------------------------

nobs <- 37   # This is the number of observations available for the
             # regression, since the proportion of females disabled is
             # 1 for females aged 98 and 99 in 1981 (see Table 1.2).

c11 <- rep(1,51);                        c12 <- 0:50
c13 <- c((6:1)^2,rep(0,45));             c14 <- c(rep(0,31),(1:20)^2)
xmat1 <- cbind(c11,c12,c13,c14)
c21 <- c11;                              c22 <- c12
c23 <- c13;                              c24 <- c14
xmat2 <- cbind(c21,c22,c23,c24)

# Columns c13, c14, c23 and c24 here correspond to functions g3, g4,
# g7 and g8, respectively, in (3.1) and Table 3.1 (Females 1981).

cbind(age=60:110,xmat1,xmat2)

#       age c11 c12 c13 c14 c21 c22 c23 c24
#  [1,]  60   1   0  36   0   1   0  36   0
#  [2,]  61   1   1  25   0   1   1  25   0
#  [3,]  62   1   2  16   0   1   2  16   0
#  [4,]  63   1   3   9   0   1   3   9   0
#  [5,]  64   1   4   4   0   1   4   4   0
#  [6,]  65   1   5   1   0   1   5   1   0
#  [7,]  66   1   6   0   0   1   6   0   0
#  [8,]  67   1   7   0   0   1   7   0   0
# .........................................
# [30,]  89   1  29   0   0   1  29   0   0
# [31,]  90   1  30   0   0   1  30   0   0
# [32,]  91   1  31   0   1   1  31   0   1
# [33,]  92   1  32   0   4   1  32   0   4
# [34,]  93   1  33   0   9   1  33   0   9
# .........................................
# [50,] 109   1  49   0 361   1  49   0 361
# [51,] 110   1  50   0 400   1  50   0 400

# The following produces the 8 parameter estimates and associated
# standard errors in rows 1 and 2 of Table 3.1 (Females 1981).
```

```
reg <-REG.CUR(L1,L2,L3,xmat1,xmat2,nobs)
b <- reg$param;     v <- reg$paramvr

rbind(t(round(b,4)),round(sqrt(diag(v)),4))

#        [,1]    [,2]   [,3]    [,4]   [,5]    [,6]   [,7]    [,8]
# [1,] 3.6409 -0.1981 0.0377 -0.0702 2.3883 -0.1333 0.0367 -0.0196
# [2,] 0.0056  0.0003 0.0011  0.0010 0.0057  0.0003 0.0011  0.0003

# The following produces the estimated probabilities
# for females in Table 3.2.

est <- EST.CUR(b=b,xmat1,xmat2)

round(cbind(age=60:110,p1= est$p1vec, est$p2vec, est$p0vec),5)

#      age
# [1,]  60  0.78001 0.21472 0.99473
# [2,]  61  0.76362 0.22686 0.99049
# [3,]  62  0.74556 0.23854 0.98410
# ................................
# [51,] 110 0.00000 0.00001 0.00001

# The following produces the expectancies which appear
# in rows 1 and 3 of Table 3.6.

expmat <- matrix(0,nrow=2,ncol=4)
for(age in 60:61){
       res <- EXP.CUR(str=age,fin=110,p1vec=est$p1vec,p2vec=est$p2vec,
                      xmat1=xmat1,xmat2=xmat2)
       expmat[age-59,] <- c(age=age,e1=res$e1,e2=res$e2,e0=res$e0)
       }

matrix(round(expmat,3),nrow=2,ncol=4,
    dimnames=list(rep(" ",2),rep(" ",4)))

#  60 13.175 9.000 22.176
#  61 12.453 8.816 21.269

# The following produces standard errors that are similar
# to those which appear in rows 2 and 4 of Table 3.6, i.e.
#       0.010, 0.010, 0.009
#       0.009, 0.010, 0.010.

m <- 10

# A

sesmat <- matrix(0,nrow=2,ncol=4)
for(age in 60:61){
       res <- SES.CUR(m=m,b=b,v=v,str=age,fin=110,
                     xmat1=xmat1,xmat2=xmat2,alp=0.05)
       sesmat[age-59,] <- c(age,res$s1,res$s2,res$s0)
       }

matrix(round(sesmat,3),nrow=2,ncol=4,
    dimnames=list(rep(" ",2),rep(" ",4)))

# B

#  60 0.010 0.010 0.010
#  61 0.009 0.005 0.011

# Increasing the number of Monte Carlo simulations m
# from 10 to 1000 and repeating the routine
# from A to B above, leads to more precise standard errors,
# but takes a few minutes of computing time.
```

```
4. Create Figures 2.1-2.3
-------------------------

obsinc <- 1:nobs
yall <- 60:110
yobs <- 61:(60+nobs)
hatinc <- yobs - 59

xi1obs <- log(L1[obsinc]/L3[obsinc])
xi2obs <- log(L2[obsinc]/L3[obsinc])

p3obs <- 1/(1+exp(xi1obs)+exp(xi2obs))
p1obs <- p3obs*exp(xi1obs)
p2obs <- p3obs*exp(xi2obs)
p0obs <- p1obs+p2obs

xi1hat <- est$xi1vec
xi2hat <- est$xi2vec

p1hat <- est$p1vec
p2hat <- est$p2vec
p0hat <- est$p0vec

# Figure 2.1

par(mfrow=c(1,1))
plot(c(60,110),c(-9,6),type=”n”,xlab=”age”,ylab=”log odds”)
points(yobs,xi1obs,pch=16)
lines(yobs,xi1hat[hatinc],lty=1)
points(yobs,xi2obs,pch=1)
lines(yobs,xi2hat[hatinc],lty=4)
lines(c(60,63),c(-4,-4),lty=1)
lines(c(60,63),c(-6,-6),lty=4)
points(75,-4,pch=16)
points(75,-6,pch=1)

# Guide to transferring the graph into a Word document:

# In S-PLUS, R-click on any dot, Convert to Objects,
# then select (outer frame), Copy, and Paste into Word document.
# Select graph in Word, R-click, Format Picture, Layout, Behind text,
# OK. This allows equations and text to be overlaid on the graph.

# Figure 2.2

par(mfrow=c(1,1))
plot(c(60,110),c(0,1),type=”n”,xlab=”age”,ylab=”probability”)
points(yobs,p1obs,pch=16); lines(yall,p1hat,lty=1)
points(yobs,p2obs,pch=1);  lines(yall,p2hat,lty=4)
points(yobs,p0obs,pch=2);  lines(yall,p0hat,lty=2)
lines(c(85,90),c(0.9,0.9),lty=1)
lines(c(85,90),c(0.75,0.75),lty=4)
lines(c(85,90),c(0.6,0.6),lty=2)
points(102,0.9,pch=16)
points(102,0.75,pch=1)
points(102,0.6,pch=2)

# Figure 2.3

plot(c(60,110),c(0,4),type=”n”,xlab=”age”,ylab=”ratio”)
points(yobs,p1obs/p2obs,pch=16)
lines(yall,p1hat/p2hat,lty=1)
lines(c(80,85),c(3,3),lty=1)
points(98,3,pch=16)
```

```
5. Recreate the data in Step 1 using a more general procedure
-------------------------------------------------------------

(a) Follow the instructions in Part C so as to create the following
    six objects in S-PLUS:

       RatsF      =   99 by 4 matrix, where the ith row consists of the
                      disability rates for females aged i years in
                      1981, 1988, 1993, 1998

       RatsM      =   99 by 4 matrix, where the ith row consists of the
                      disability rates for males aged i years in
                      1981, 1988, 1993, 1998

       PopsF      =   100 by 100 matrix, where the (i,j)th entry is the
                      number of females aged i in year (1900 + j)

       PopsM      =   100 by 100 matrix, where the (i,j)th entry is the
                      number of males aged i in year (1900 + j)

       HazsF      =   99 by 99 matrix, where the (i,j)th entry is the
                      hazard for females aged i in year (1900 + j)

       HazsM      =   99 by 99 matrix, where the (i,j)th entry is the
                      hazard for males aged i in year (1900 + j).

(b) Declare the following function.

DATA.CUR <-
function(fem=T,yr=81){
# Creates current data.
# Inputs:     fem = T for females and F for males
#             yr = 81,88,93 or 98.
# Outputs:    $L = number alive aged 60 in the selected year
#             $L0 = vector of 39 numbers in state 0 (age 61-99)
#             $L1 = vector of 39 numbers in state 1 (age 61-99)
#             $L2 = vector of 39 numbers in state 2 (age 61-99)
#             $L3 = vector of 39 numbers in state 3 (age 61-99)
#             $qvec = vector of 39 hazards (age 60-98)
#             $dvec = vector of 39 deaths (age 60-98)
#             $rvec = vector of 39 disability rates (age 61-99).
      if(yr==81) per <- 1
      if(yr==88) per <- 2
      if(yr==93) per <- 3
      if(yr==98) per <- 4
      rat <- RatsM; pop <- PopsM; qxm <- HazsM
      if(fem==T){
             rat <- RatsF; pop <- PopsF; qxm <- HazsF
             }
      Lx <- c(pop[60,yr],rep(NA,39))  # age 60-99
      qx <- qxm[60:99,yr]
      px <- 1 - qx
      for(i in 2:40)
             Lx[i] <- Lx[i-1] * px[i-1]
      dx <- Lx * qx      # deaths age 60-99
      cdx <- cumsum(dx)  # cumulative deaths age 60-99
      L <- Lx[1]
      L0 <- Lx[-1]
      L3 <- cdx[-40]  # NB: L0 + L3 = L
      rvec <- rat[61:99,per]
      L2 <- rvec * L0
      L1 <- L0 - L2
      dvec <- dx[-40]
      qvec <- qx[-40]
      list(L=L,dvec=dvec,qvec=qvec,rvec=rvec,
             L0=L0,L1=L1,L2=L2,L3=L3)
}
```

(c) Test the above function, as in the following examples.

```
data <- DATA.CUR(fem=T,yr=81)
cbind(data$L1,data$L2,data$L3)

#           [,1]     [,2]       [,3]
#  [1,] 57469.21 17341.01   601.7878
#  [2,] 56556.78 17596.60  1258.6214
#  ................................ (same as in Step 1 above)

data <- DATA.CUR(fem=F,yr=88)
cbind(data$L1,data$L2,data$L3)

#           [,1]     [,2]      [,3]
#  [1,] 45133.30 29405.24  1051.457
#  [2,] 42355.24 31031.68  2203.077
#  ............................... (see Table 1.4, males)
```

6. Calculate all the data in Tables 3.6-3.9

```
m <- 10
maxage <- 62

# NB: With m=1000 and maxage=99, the job takes 16 hours.

# Females 1981
# ============

data <- DATA.CUR(fem=T,yr=81);   nobs <- 37
c11 <- rep(1,51);                       c12 <- 0:50
c13 <- c((6:1)^2,rep(0,45));            c14 <- c(rep(0,31),(1:20)^2)
xmat1 <- cbind(c11,c12,c13,c14);        xmat2 <- xmat1

# A

resmat <- matrix(NA,nrow=80,ncol=4)
L1 <- data$L1;      L2 <- data$L2;  L3 <- data$L3
reg <- REG.CUR(L1,L2,L3,xmat1,xmat2,nobs)
b <- reg$param;     v <- reg$paramvr
est <- EST.CUR(b=b,xmat1,xmat2)
for(age in 60:maxage){
       exp <- EXP.CUR(str=age,fin=110,
                      p1vec=est$p1vec,p2vec=est$p2vec,xmat1,xmat2)
       ses <- SES.CUR(m=m,b=b,v=v,str=age,fin=110,
                       xmat1,xmat2,alp=0.05)
       resmat[(age-59)*2-1,] <- c(age,exp$e1,exp$e2,exp$e0)
       resmat[(age-59)*2-0,] <- c(age,ses$s1,ses$s2,ses$s0)
       }

# B

RESULTS.F.1981 <- resmat

# Females 1988
# ============

data <- DATA.CUR(fem=T,yr=88);   nobs <- 39

c11 <- rep(1,51);                c12 <- 0:50
c13 <- c((7:1)^2,rep(0,44));     c14 <- c(rep(0,23),(1:28)^2)
xmat1 <- cbind(c11,c12,c13,c14); xmat2 <- xmat1

# Execute the commands from A to B above (as for Females 1981).

RESULTS.F.1988 <- resmat
```

```
# Females 1993
# ============

data <- DATA.CUR(fem=T,yr=93);   nobs <- 39
c11 <- rep(1,51);                       c12 <- 0:50
c13 <- c((7:1)^2,rep(0,44));            c14 <- c(rep(0,23),(1:28)^2)
xmat1 <- cbind(c11,c12,c13,c14);  xmat2 <- xmat1

# Execute the commands from A to B above (as for Females 1981).

RESULTS.F.1993 <- resmat

# Females 1998
# ============

data <- DATA.CUR(fem=T,yr=98);   nobs <- 39
c11 <- rep(1,51);                 c12 <- 0:50
c13 <- c((6:1)^2,rep(0,45));      c14 <- c(rep(0,20),(1:31)^2)
xmat1 <- cbind(c11,c12,c13,c14)
c24 <- c(rep(0,25),(1:26)^2)
xmat2 <- cbind(c11,c12,c13,c24)

# Execute the commands from A to B above (as for Females 1981).

RESULTS.F.1998 <- resmat

# Males 1981
# ==========

data <- DATA.CUR(fem=F,yr=81);   nobs <- 36
c11 <- rep(1,51);                        c12 <- 0:50
c13 <- c((7:1)^2,rep(0,44));             c14 <- c(rep(0,27),(1:24)^2)
xmat1 <- cbind(c11,c12,c13,c14)
c23 <- c((10:1)^2,rep(0,41))
xmat2 <- cbind(c11,c12,c23,c14)

# Execute the commands from A to B above (as for Females 1981).

RESULTS.M.1981 <- resmat

# Males 1988
# ==========

data <- DATA.CUR(fem=F,yr=88);   nobs <- 35
c11 <- rep(1,51);                       c12 <- 0:50
c13 <- c((6:1)^2,rep(0,45));            c14 <- c(rep(0,23),(1:28)^2)
xmat1 <- cbind(c11,c12,c13,c14)
c23 <- c((10:1)^2,rep(0,41))
c24 <- c(rep(0,25),(1:26)^2)
xmat2 <- cbind(c11,c12,c23,c24)

# Execute the commands from A to B above (as for Females 1981).

RESULTS.M.1988 <- resmat

# Males 1993
# ==========

data <- DATA.CUR(fem=F,yr=93);   nobs <- 36
c11 <- rep(1,51);                       c12 <- 0:50
c13 <- c((7:1)^2,rep(0,44));            c14 <- c(rep(0,23),(1:28)^2)
xmat1 <- cbind(c11,c12,c13,c14);  xmat2 <- xmat1

# Execute the commands from A to B above (as for Females 1981).

RESULTS.M.1993 <- resmat
```

```
# Males 1998
# ==========

data <- DATA.CUR(fem=F,yr=98);    nobs <- 36
c11 <- rep(1,51);                 c12 <- 0:50
c13 <- c((6:1)^2,rep(0,45));      c14 <- c(rep(0,23),(1:28)^2)
xmat1 <- cbind(c11,c12,c13,c14)
c23 <- c((6:1)^2,rep(0,45));      c24 <- c(rep(0,20),(1:31)^2)
xmat2 <- cbind(c11,c12,c23,c24)

RESULTS.M.1998 <- resmat
```

```
7. Display all the data in Tables 3.6-3.9
-----------------------------------------

round(cbind(RESULTS.F.1981,RESULTS.M.1981[,-1]),3)

      [,1]   [,2]  [,3]   [,4]   [,5]  [,6]   [,7]
 [1,]   60 13.175 9.000 22.176 10.265 7.298 17.564
 [2,]   60  0.007 0.008  0.008  0.010 0.008  0.010
 [3,]   61 12.453 8.816 21.269  9.695 7.017 16.712
 [4,]   61  0.006 0.013  0.012  0.012 0.008  0.007
 [5,]   62 11.766 8.637 20.403  9.164 6.747 15.911
 [6,]   62  0.009 0.010  0.013  0.010 0.009  0.011
 ...................................................  (etc)
```

Part B: S-PLUS code for analysis of cohort data

```
1. Set up the data for the cohort of females aged 60 in 1980
------------------------------------------------------------

(a) Make sure the 6 objects in Step 5 (a) of Part A exist (see Part C).

(b) Declare the following function.

DATA.COH <-
function(fem=T,age=60){
# Creates cohort data.
# Inputs:     fem = T for females and F for males
#             age = 60,61,..., or 77.
# Outputs:    $L = number alive aged age in 1980
#             $L0 = vector of 20 numbers in state 0
#             $L1 = vector of 20 numbers in state 1
#             $L2 = vector of 20 numbers in state 2
#             $L3 = vector of 20 numbers in state 3
#             $rvec = vector of 20 disability rates.
       rat <- RatsM; pop <- PopsM; qxm <- HazsM
       if(fem==T) {
              rat <- RatsF; pop <- PopsF; qxm <- HazsF
              }
       L0 <- rep(NA, 20);  L1 <- rep(NA, 20)
       L2 <- rep(NA, 20);  L3 <- rep(NA, 20)
       rvec <- rep(NA, 20)
       L <- pop[age,80]
       for(coh in 1:5) {
              qxvec <- diag(qxm[age:(age+17+coh),(81-coh):98])
              pxvec <- 1 - qxvec
              pxcum <- cumprod(pxvec)
              L0val <- pop[age,81-coh]
              L0vec <- c(L0val,(pxcum*L0val))
              L0coh <- L0vec[c(1,8,13,18)+coh]
              rcoh <- diag(rat[age+c(0,7,12,17)+coh,1:4])
              rcoh[rcoh==1] <- 0.999
              L2coh <- L0coh * rcoh
              L1coh <- L0coh - L2coh
              L3coh <- L0val - L0coh
              inc <- c(0,5,10,15) + coh
```

```
            L0[inc] <- L0coh
            L1[inc] <- L1coh
            L2[inc] <- L2coh
            L3[inc] <- L3coh
            rvec[inc] <- rcoh
      }
      list(L=L,L0=L0,L1=L1,L2=L2,L3=L3,rvec=rvec)
}
```

```
(c) Test the above function, as follows.

data <- DATA.COH(fem=T,age=60)
cbind(data$L1,data$L2,data$L3)

#           [,1]     [,2]       [,3]
# [1,] 50433.26 15217.95   546.7955
# [2,] 45516.69 14161.68  1057.6328
# ................................ (see Table 1.7, females)
```

```
2. Declare all other functions needed for cohort analysis
----------------------------------------------------------

REG.COH <-
function(L1,L2,L3,xmat2,xmat3){
# Performs a regression for analysing cohort disability data.
# Inputs:     L1 = vector of 20 no.'s in state 1 (disability-free)
#             L2 = vector of 20 no.'s in state 2 (disabled)
#             L3 = vector of 20 no.'s in state 3 (dead)
#             xmat2 = design matrix for eta2, where
#                     row 1 is for age in 1980 & last row is for age 110
#                     eg if age=63 in 1980, xmat2 has 48 rows
#                     the number of columns is variable
#             eg:    col1 = 1,1,1,...
#                    col2 = 0,1,2,...
#                    col3 = 36,25,16,9,4,1,0,0,0,...
#                    col4 = 0,0,...,1,4,9,...
#             xmat3 = design matrix for eta3.
# Outputs:    $param = vector of parameter estimates
#             $paramvr = estimated covariance  matrix of estimates.
      inc <- c(1:5,8:22) + 1   # rows of design matrices to use
      xmatreg2 <- xmat2[inc,]
      xmatreg3 <- xmat3[inc,]
      np2 <- ncol(xmat2)
      np3 <- ncol(xmat3)
      np <- np2 + np3
      sum1 <- matrix(0,np,np)
      sum2 <- matrix(0,np,1)
      eta2vec <- log(L2/L1)
      eta3vec <- log(L3/L1)
      w22vec <- 1/L2 + 1/L1
      w23vec <- 1/L1
      w33vec <- 1/L3 + 1/L1
      for(i in 1:20){
            dv <- matrix(c(xmatreg2[i,],rep(0,np),xmatreg3[i,]),
                         nrow=2,ncol=np,byrow=T)
            varmat <- matrix(c(w22vec[i],w23vec[i],w23vec[i],
                         w33vec[i]),2,2)
            wtvec <- solve(varmat)
            dvt <- t(dv)
            prod1 <- dvt %*% wtvec %*% dv
            sum1 <- sum1 + prod1
            eta2 <- eta2vec[i]
            eta3 <- eta3vec[i]
            eta <- matrix(c(eta2,eta3),2,1)
            prod2 <- dvt %*% wtvec %*% eta
            sum2 <- sum2 + prod2
            }
      inv1 <- solve(sum1)
```

```
       estprm <- inv1 %*% sum2
       list(param = estprm, paramvr = inv1)
}

EST.COH <-
function(b,xmat2,xmat3){
# Estimates cohort probabilities and log odds.
# Inputs:    b = vector of parameter estimates (from call to REG.COH)
#            xmat2, xmat3 = design matrices for eta2, eta3.
# Outputs:   $p1vec = vector of probabilities for state 1
#            $p2vec = vector of probabilities for state 2
#            $p0vec = vector of probabilities for state 0
#            $eta2vec = vector of log odds for state 2 (rel. to 1)
#            $eta3vec = vector of log odds for state 3 (rel. to 1).
       np2 <- ncol(xmat2)
       np3 <- ncol(xmat3)
       np <- np2 + np3
       eta2 <- xmat2 %*% b[1:np2]
       eta3 <- xmat3 %*% b[(np2+1):np]
       t2 <- exp(eta2)
       t3 <- exp(eta3)
       p1 <- 1/(1+t2+t3)
       p2 <- p1 * t2
       p3 <- p1 * t3
       p0 <- 1 - p3
       list(p1vec=p1,p2vec=p2,p0vec=p0,eta2vec=eta2,eta3vec=eta3)
}

EXP.COH <-
function(age1980=60,fin=95,p1vec,p2vec){
# Computes estimates of cohort health expectancies via integration.
# Inputs:    age1980 = age in 1980 = starting age
#            fin = finishing age
#            p1vec, p2vec = vectors from call to EST.COH
#            xmat2, xmat3 = design matrices for eta2, eta3.
# Ouptuts:   $e1 = estimate of expected years in state 1
#            $e2 = estimate of expected years in state 2
#            $e0 = estimate of expected years in state 0.
       xvec <- age1980:(age1980+length(p1vec)-1)
       den <- p1vec[1] + p2vec[1]   # denominator
       e1 <- INTEG.SPLINE(x=xvec,y=p1vec,a=age1980,b=fin)/den
       e2 <- INTEG.SPLINE(x=xvec,y=p2vec,a=age1980,b=fin)/den
       list(e1=e1,e2=e2,e0=e1+e2)
}

LIANG.COH <-
function(prm,ncoh=5,L1,L2,L3,xmat2,xmat3){
# Computes the covariance matrix for the vector of cohort parameter
# estimates using the Liang-Zeger technique.
# Inputs:    prm = vector of parameter estimates
#            ncoh = number of cohorts
#            L1,L2,L3 = vectors of numbers in states 1,2,3
#            xmat2, xmat3 = design matrices for eta2, eta3.
# Outputs:   varfnl = required covariance matrix.
       np2 <- ncol(xmat2)
       np3 <- ncol(xmat3)
       np <- np2 + np3
       yvec <- c(1:5,8:22)
       mvec <- rbind(L1,L2,L3,yvec)
       xmat2reg <- xmat2[yvec+1,]
       xmat3reg <- xmat3[yvec+1,]
       varnew1 <- matrix(0,np,np)
       varnew2 <- matrix(0,np,np)
       for(j in 1:ncoh){
              w11 <- 1/mvec[2,j] + 1/mvec[1,j]
              w22 <- 1/mvec[3,j] + 1/mvec[1,j]
              w12 <- 1/mvec[1,j]
              vmat1 <- matrix(c(w11,w12,w12,w22),2,2)
```

```
            w11 <- 1/mvec[2,j+ncoh] + 1/mvec[1,j+ncoh]
            w22 <- 1/mvec[3,j+ncoh] + 1/mvec[1,j+ncoh]
            w12 <- 1/mvec[1,j+ncoh]
            vmat2 <- matrix(c(w11,w12,w12,w22),2,2)
            ncoh1 <- ncoh * 2
            w11 <- 1/mvec[2,j+ncoh1] + 1/mvec[1,j+ncoh1]
            w22 <- 1/mvec[3,j+ncoh1] + 1/mvec[1,j+ncoh1]
            w12 <- 1/mvec[1,j+ncoh1]
            vmat3 <- matrix(c(w11,w12,w12,w22),2,2)
            ncoh2 <- ncoh * 3
            w11 <- 1/mvec[2,j+ncoh2] + 1/mvec[1,j+ncoh2]
            w22 <- 1/mvec[3,j+ncoh2] + 1/mvec[1,j+ncoh2]
            w12 <- 1/mvec[1,j+ncoh2]
            vmat4 <- matrix(c(w11,w12,w12,w22),2,2)
            vmat1a <- cbind(vmat1,matrix(0,2,6))
            vmat2a <- cbind(matrix(0,2,2),vmat2,matrix(0,2,4))
            vmat3a <- cbind(matrix(0,2,4),vmat3,matrix(0,2,2))
            vmat4a <- cbind(matrix(0,2,6),vmat4)
            vmat <- rbind(vmat1a,vmat2a,vmat3a,vmat4a)
            x <- j
            dmat1 <- matrix(c(xmat2reg[x, ],rep(0,np),
                          xmat3reg[x,]),2,np,byrow=T)
            x <- j + ncoh
            dmat2 <- matrix(c(xmat2reg[x,],rep(0,np),
                          xmat3reg[x,]),2,np,byrow=T)
            x <- j + ncoh1
            dmat3 <- matrix(c(xmat2reg[x,],rep(0,np),
                          xmat3reg[x,]),2,np,byrow=T)
            x <- j + ncoh2
            dmat4 <- matrix(c(xmat2reg[x,],rep(0,np),
                          xmat3reg[x,]),2,np,byrow=T)
            dmat <- rbind(dmat1,dmat2,dmat3,dmat4)
            svecob1 <- log(c(mvec[2,j]/mvec[1,j],
                          mvec[3,j]/mvec[1, j]))
            svecob2 <- log(c(mvec[2,j+ncoh]/mvec[1,j+ncoh],
                          mvec[3,j+ncoh]/mvec[1,j+ncoh]))
            svecob3 <- log(c(mvec[2,j+ncoh1]/mvec[1,j+ncoh1],
                          mvec[3,j+ncoh1]/mvec[1,j+ncoh1]))
            svecob4 <- log(c(mvec[2,j +ncoh2]/mvec[1,j+ncoh2],
                          mvec[3,j+ncoh2]/mvec[1,j+ncoh2]))
            svecob <- c(svecob1,svecob2,svecob3,svecob4)
            svec <- svecob - dmat %*% prm
            smat <- svec %*% t(svec)
            invmat <- solve(vmat)
            vartmp1 <- t(dmat) %*% invmat %*% dmat
            varnew1 <- varnew1 + vartmp1
            vartmp21 <- invmat %*% dmat
            vartmp22 <- t(dmat) %*% invmat
            vartmp2 <- vartmp22 %*% smat %*% vartmp21
            varnew2 <- varnew2 + vartmp2
            }
       sdhm <- svd(varnew1)
       indhm <- 1/sdhm$d
       diaghm <- diag(indhm,np,np)
       rhshm1 <- 1 * sdhm$u
       rhshm2 <- rhshm1 %*% diaghm
       solvnew1 <<- rhshm2 %*% t(sdhm$u)
       solvnew1 %*% varnew2 %*% solvnew1
}

PROB.COH <-
function(pmvec2,xmat2,xmat3,age1980=60,age=67){
# Computes probabilities and derivatives for use in SES.COH.
# Inputs:    pmvec2 = vector of paramater estimates
#            xmat2 = cov matrix for eta2
#            xmat3 = cov matrix for eta3
#            age1980 = age in 1980
#            age = age at which results are required.
```

```
# Output:    punprtpd = vector of estimates p1,p2,p3
#            dmatx = matrix of derivatives.
       np2 <- ncol(xmat2)
       np3 <- ncol(xmat3)
       np <- np2 + np3
       i <- age - age1980 + 1 # row number of xmat2 & xmat3 to be used
       dv <- matrix(c(xmat2[i,],rep(0,np),xmat3[i,]),2,np,byrow=T)
       pthet12 <- sum(dv[1,]*pmvec2)
       pthet13 <- sum(dv[2,]*pmvec2)
       expt12 <- exp(pthet12)
       expt13 <- exp(pthet13)
       p11 <- 1/(1+expt12+expt13)
       p12 <- p11 * expt12
       p13 <- p11 * expt13
       d11 <-  - p11 * p12
       d12 <- p12 * (1 - p12)
       d13 <-  - p13 * p12
       dm <- c(d11,d12,d13)
       e11 <-  - p11 * p13
       e12 <-  - p12 * p13
       e13 <- p13 * (1 - p13)
       em <- c(e11,e12,e13)
       tmp <- numeric(0)  # to become dmatx
       for(j in 1:np2) tmp <- cbind(tmp,dm*xmat2[i,j])
       for(j in 1:np3) tmp <- cbind(tmp,em*xmat3[i,j])
       ps <- c(p11, p12, p13)
       list(punprtpd=ps,dmatx=tmp)
}

SES.COH <-
function(age1980=60,pmvec,cvarmat,fin=95,xmat2,xmat3){
# Computes SE's for e1, e2 and e0 in cohort analysis, by summation.
# Inputs:    age1980 = age in 1980
#            pmvec = vector of parameter estimates
#            cvarmat = output from LIANG.COH
#            fin = finishing age
#            xmat2 = cov matrix for eta2
#            xmat3 = cov matrix for eta3.
# Outputs:   $s1 = SE for expected years healthy (state 1)
#            $s2 = SE for expected years disabled (state 2)
#            $s0 = SE for expected years alive (state 0)
#            $den = denominator in internal calculation.
       xvec <- age1980:fin
       var1sum <- 0
       var2sum <- 0
       var12sum <- 0
       for(age1 in xvec){
              disp <- PROB.COH(pmvec2=pmvec,xmat2=xmat2,xmat3=xmat3,
                            age1980=age1980,age=age1)
              punprtpd1 <- disp$punprtpd
              dmatx1 <- disp$dmatx
              if(age1==age1980){
                     den <- punprtpd1[1] + punprtpd1[2]
                     dmatx1 <- dmatx1/2
                     }
              if(age1==fin) dmatx1 <- dmatx1/2
              cov1sum <- 0
              cov2sum <- 0
              cov12sum <- 0
              for(age2 in xvec){
                     disp <- PROB.COH(pmvec2=pmvec,xmat2=xmat2,
                                   xmat3=xmat3,age1980=age1980,age=age2)
                     punprtpd2 <- disp$punprtpd
                     dmatx2 <- disp$dmatx
                     if(age2==age1980) dmatx2 <- dmatx2/2
                     if(age2==fin) dmatx2 <- dmatx2/2
                     probcov <- dmatx1 %*% cvarmat %*% t(dmatx2)
                     cov1term <- probcov[1,1]
```

```
                    cov2term <- probcov[2,2]
                    cov12term <- probcov[1,2]
                    cov1sum <- cov1sum + cov1term
                    cov2sum <- cov2sum + cov2term
                    cov12sum <- cov12sum + cov12term
              }
              var1sum <- var1sum + cov1sum
              var2sum <- var2sum + cov2sum
              var12sum <- var12sum + cov12sum
       }
       s1 <- sqrt(var1sum)/den
       s2 <- sqrt(var2sum)/den
       s0 <- sqrt(var1sum+var2sum+2*var12sum)/den
       list(s1=s1,s2=s2,s0=s0,den=den)
}
```

3. Apply the above functions to females aged 60 in 1980 (see Step 1)

```
age <- 60
data <- DATA.COH(fem=T,age=age)
L1 <- data$L1; L2 <- data$L2; L3 <- data$L3
y <- (age:110)
nrx <- 51-(age-60)

c21 <- rep(1,nrx);         c22 <- 0:(nrx-1)
xmat2 <- cbind(c21,c22)
c31 <- c21;                       c32 <- c22
c33 <- c((6:1)^2,rep(0,(nrx-6)))
xmat3 <- cbind(c31,c32,c33)

cbind(y,xmat2,xmat3) # see (3.2) & Table 3.12, F60 (c33=g7)

#         y c21 c22 c31 c32 c33
#  [1,] 60   1   0   1   0  36
#  [2,] 61   1   1   1   1  25
#  [3,] 62   1   2   1   2  16
#  [4,] 63   1   3   1   3   9
#  [5,] 64   1   4   1   4   4
#  [6,] 65   1   5   1   5   1
#  [7,] 66   1   6   1   6   0
#  [8,] 67   1   7   1   7   0
#  ...........................
#  [50,] 109   1  49   1  49   0
#  [51,] 110   1  50   1  50   0

reg <- REG.COH(L1,L2,L3,xmat2,xmat3)
b <- reg$param

round(t(b),4)

#          [,1]  [,2]    [,3]   [,4]    [,5]
#  [1,] -1.2832 0.093 -3.4008 0.1892 -0.0487    (see Table 3.12, F60)

est <- EST.COH(b,xmat2,xmat3)

round(cbind(age=60:110,p1=est$p1,p2=est$p2,p0=est$p0),5)

#         age
#   [1,]  60 0.77947 0.21604 0.99550
#   [2,]  61 0.75983 0.23112 0.99094
#   [3,]  62 0.73738 0.24615 0.98354
#   ............................... (see Table 3.13, females)

res.exp <- EXP.COH(age1980=age,fin=95,p1vec=est$p1,p2vec=est$p2)
c(res.exp$e1,res.exp$e2,res.exp$e0)

# [1] 11.96021 11.99434 23.95455    (see Table 3.17, row 1, females)
```

```
res.liang <- LIANG.COH(prm=b,ncoh=5,L1,L2,L3,xmat2,xmat3)

rbind(round(sqrt(diag(res.liang)),4))

#       [,1]   [,2]   [,3]   [,4]   [,5]
# [1,] 0.025 0.0057 0.0486 0.0055 0.0025 (see table 3.12, F60, SE's)

res.ses <- SES.COH(age1980=age,pmvec=b,
       cvarmat=res.liang,fin=95,xmat2=xmat2,xmat3=xmat3)

cbind(s1=res.ses$s1,s2=res.ses$s2,s0=res.ses$s0)

#             s1        s2        s0
# [1,] 0.3099747 0.3790607 0.1348972   (see Table 3.17, F60, SE's)

4. Create Figures 2.4-2.6
-------------------------

yobs <- c((age+1):(age+22))[-c(6,7)]
eta2obs <- log(L2/L1)
eta3obs <- log(L3/L1)
eta2hat <- est$eta2
eta3hat <- est$eta3
p1obs <- 1/(1+exp(eta2obs)+exp(eta3obs))
p2obs <- p1obs*exp(eta2obs); p3obs <- p1obs*exp(eta3obs)
p0obs <- p1obs + p2obs
p1hat <- est$p1;    p2hat <- est$p2;    p0hat <- est$p0

par(mfrow=c(1,1))
plot(c(age,110),c(-9,6),type="n",xlab="age",ylab="log odds")
points(yobs,eta2obs,pch=16)
lines(y[2:23],eta2hat[2:23],lty=1)
points(yobs,eta3obs,pch=1)
lines(y[2:23],eta3hat[2:23],lty=4)
lines(c(77,81),c(-4,-4),lty=1)
lines(c(77,81),c(-6,-6),lty=4)
points(92,-4,pch=16)
points(92,-6,pch=1)

par(mfrow=c(1,1))
plot(c(age,110),c(0,1),type="n",xlab="age",ylab="probability")
points(yobs,p1obs,pch=16);  lines(y,p1hat)
points(yobs,p2obs,pch=1);   lines(y,p2hat,lty=4)
points(yobs,p0obs,pch=2);   lines(y,p0hat,lty=2)
lines(c(85,90),c(0.9,0.9),lty=1)
lines(c(85,90),c(0.75,0.75),lty=4)
lines(c(85,90),c(0.6,0.6),lty=2)
points(102,0.9,pch=16)
points(102,0.75,pch=1)
points(102,0.6,pch=2)

plot(c(age,110),c(0,4),type="n",ylab="ratio",xlab="age")
points(yobs,p1obs/p2obs,pch=16)
lines(y,p1hat/p2hat)
lines(c(80,85),c(3,3),lty=1)
points(98,3,pch=16)

5. Compute health expectancies for males aged 70 in 1980
--------------------------------------------------------

age <- 70
data <- DATA.COH(fem=F,age=age)
L1 <- data$L1; L2 <- data$L2; L3 <- data$L3
y <- (age:110)
nrx <- 51-(age-60)
```

```
c21 <- rep(1,nrx);          c22 <- 0:(nrx-1)
c23 <- c(rep(0,12),(1:(nrx-12))^2)
xmat2 <- cbind(c21,c22,c23)
c31 <- c21;                          c32 <- c22
c33 <- c((5:1)^2,rep(0,(nrx-5)))
c34 <- c(rep(0,14),(1:(nrx-14))^2)
xmat3 <- cbind(c31,c32,c33,c34)

cbind(y,xmat2,xmat3) # see Table 3.12, M70 (c23=g4,c33=g7,c34=g8)

#         y c21 c22 c23 c31 c32 c33 c34
#  [1,] 70   1   0   0   1   0  25   0
#  [2,] 71   1   1   0   1   1  16   0
#  [3,] 72   1   2   0   1   2   9   0
#  [4,] 73   1   3   0   1   3   4   0
#  [5,] 74   1   4   0   1   4   1   0
#  [6,] 75   1   5   0   1   5   0   0
#  [7,] 76   1   6   0   1   6   0   0
#  [8,] 77   1   7   0   1   7   0   0
#  [9,] 78   1   8   0   1   8   0   0
# [10,] 79   1   9   0   1   9   0   0
# [11,] 80   1  10   0   1  10   0   0
# [12,] 81   1  11   0   1  11   0   0
# [13,] 82   1  12   1   1  12   0   0
# [14,] 83   1  13   4   1  13   0   0
# [15,] 84   1  14   9   1  14   0   1
# [16,] 85   1  15  16   1  15   0   4
# [17,] 86   1  16  25   1  16   0   9
# [18,] 87   1  17  36   1  17   0  16
#  ...................................
# [40,] 109   1  39 784   1  39   0 676
# [41,] 110   1  40 841   1  40   0 729

reg <- REG.COH(L1,L2,L3,xmat2,xmat3);  b <- reg$param
est <- EST.COH(b,xmat2,xmat3)
res.exp <- EXP.COH(age1980=age,fin=95,p1vec=est$p1,p2vec=est$p2)
res.liang <- LIANG.COH(prm=b,ncoh=5,L1,L2,L3,xmat2,xmat3)
res.ses <- SES.COH(age1980=age,pmvec=b,
       cvarmat=res.liang,fin=95,xmat2=xmat2,xmat3=xmat3)

exp.vec <- c(res.exp$e1,res.exp$e2,res.exp$e0)
ses.vec <- c(res.ses$s1,res.ses$s2,res.ses$s0)
rbind(exp.vec,ses.vec)

#                 [,1]      [,2]        [,3]
#  exp.vec 5.10400910 6.2954298 11.39943886
#  ses.vec 0.06162257 0.1071253  0.05589387  (see Table 3.17, M70)
```

6. Produce all figures in Table 3.17

```
# Create matrices to store all expectancies and standard errors

RESULTS.F.COH <- matrix(NA,nrow=36,ncol=4)
RESULTS.M.COH <- matrix(NA,nrow=36,ncol=4)

# Females

fem <- T

for(age in 60:77){

       d <- DATA.COH(fem=fem,age=age)
       L1 <- d$L1; L2 <- d$L2; L3 <- d$L3
       nrx <- 51 – (age - 60)

       if(age<70){
              c21 <- rep(1,nrx); c22 <- 0:(nrx-1)
```

```
        xmat2 <- cbind(c21,c22)
        c31 <- c21;   c32 <- c22
        c33 <- c((6:1)^2,rep(0,(nrx-6)))
        xmat3 <- cbind(c31,c32,c33)
        }

    if(age>=70){
        c21 <- rep(1,nrx); c22 <- 0:(nrx-1)
        xmat2 <- cbind(c21,c22)
        c31 <- c21;   c32 <- c22
        c33 <- c((5:1)^2,rep(0,(nrx-5)))
        c34 <- c(rep(0,17),(1:(nrx-17))^2)
        xmat3 <- cbind(c31,c32,c33,c34)
        }

    res.reg <- REG.COH(L1,L2,L3,xmat2,xmat3)
    b <- res.reg $param; v <- res.reg $paramvr
    res.est <- EST.COH(b,xmat2,xmat3)
    p1 <- res.est$p1;   p2 <- res.est$p2
    rese <- EXP.COH(age1980=age,fin=95,p1vec=p1,p2vec=p2)
    res.liang <- LIANG.COH(prm=b,ncoh=5,L1,L2,L3,xmat2,xmat3)
    ress <- SES.COH(age1980=age,pmvec=b,
         cvarmat=res.liang,fin=95,xmat2=xmat2,xmat3=xmat3)

    i <- (age-60)*2 + 1    # 60~1, 61~3, 62~5, ...
    RESULTS.F.COH[i,] <- c(age,rese$e1,rese$e2,rese$e0)
    RESULTS.F.COH[i+1,] <- c(age,ress$s1,ress$s2,ress$s0)
    cat(age); cat(" ")

    }

# Males

fem <- F

for(age in 60:77){

    d <- DATA.COH(fem=fem,age=age)
    L1 <- d$L1; L2 <- d$L2; L3 <- d$L3
    nrx <- 51 – (age - 60)

    if(age<70){
        c21 <- rep(1,nrx); c22 <- 0:(nrx-1)
        xmat2 <- cbind(c21,c22)
        c31 <- c21;   c32 <- c22
        c33 <- c((6:1)^2,rep(0,(nrx-6)))
        xmat3 <- cbind(c31,c32,c33)
        }

    if(age>=70){
        c21 <- rep(1,nrx); c22 <- 0:(nrx-1)
        c23 <- c(rep(0,12),(1:(nrx-12))^2)
        xmat2 <- cbind(c21,c22,c23)
        c31 <- c21;   c32 <- c22
        c33 <- c((5:1)^2,rep(0,(nrx-5)))
        c34 <- c(rep(0,14),(1:(nrx-14))^2)
        xmat3 <- cbind(c31,c32,c33,c34)
        }

    res.reg <- REG.COH(L1,L2,L3,xmat2,xmat3)
    b <- res.reg$param; v <- res.reg$paramvr
    res.est <- EST.COH(b,xmat2,xmat3)
    p1 <- res.est$p1;   p2 <- res.est$p2
    rese <- EXP.COH(age1980=age,fin=95,p1vec=p1,p2vec=p2)
    res.liang <- LIANG.COH(prm=b,ncoh=5,L1,L2,L3,xmat2,xmat3)
    ress <- SES.COH(age1980=age,pmvec=b,
      cvarmat=res.liang,fin=95,xmat2=xmat2,xmat3=xmat3)
```

```
        i <- (age-60)*2 + 1    # 60~1, 61~3, 62~5, ...
        RESULTS.M.COH[i,] <- c(age,rese$e1,rese$e2,rese$e0)
        RESULTS.M.COH[i+1,] <- c(age,ress$s1,ress$s2,ress$s0)
        cat(age); cat(" ")

        }

matrix(round(cbind(RESULTS.F.COH,RESULTS.M.COH[,-1]),3),
  nrow=36,ncol=7, dimnames=list(rep(" ",36),rep(" ",7)))

#  60 11.960 11.994 23.955 8.781 10.345 19.127
#  60  0.310  0.379  0.135 0.184  0.251  0.109
#  61 11.454 11.469 22.923 8.533  9.703 18.236
#  61  0.270  0.307  0.101 0.250  0.358  0.123
#  62 10.789 11.165 21.954 8.352  9.065 17.418
#  62  0.247  0.198  0.085 0.191  0.302  0.116
#  ........................................... (see table 3.17)
```

Part C: Importing and listing of data

The following are instructions on how to create the six data matrices first mentioned in Step 5 of Part A, namely RatsF, RatsM, PopsF, PopsM, HazsF and HazsM. A listing of these matrices is provided below the instructions.

(a) Select all the data from the listing below, from

```
  55 0.1993 0.2055988 0.2249022 0.3135074
                                 (first line of RatsF)
```

to

```
  99 0.29010 0.29010 0.29010 0.31044 0.28309 0.27087 0.26960 0.26029
                                 (last line of HazsM),
```

inclusive, in one large block.

Copy and paste this data into a new Word file (blank document).

(b) In the new file, delete all lines that are non-numeric or have year numbers, but otherwise leave a blank line between blocks of numbers.

(c) Replace all spaces in the new file with commas, as follows:

```
Edit; Replace; Find what: 5 spaces; Replace with: ,; Replace All
Edit; Replace; Find what: 4 spaces; Replace with: ,; Replace All
Edit; Replace; Find what: 3 spaces; Replace with: ,; Replace All
Edit; Replace; Find what: 2 spaces; Replace with: ,; Replace All
Edit; Replace; Find what: 1 space;  Replace with: ,; Replace All
```

(NB: Don't type "5 spaces"; type "   ". Also, type ",".)

The new file should now have exactly 649 lines and look as follows:

```
-----------------------TOP OF FILE------------------------------
,55,0.1993,0.2055988,0.2249022,0.3135074
,56,0.2197,0.2015599,0.2917623,0.3023903
,57,0.2063,0.2059220,0.1795296,0.3077936
........................................
,98,1.0000,0.9832417,0.9452964,0.9442836
,99,1.0000,0.9909090,0.9639437,0.9525335

,55,0.2683,0.2643630,0.2651024,0.2851012
,56,0.2852,0.2767488,0.2448480,0.2839287
,57,0.3049,0.3262657,0.3181993,0.3451357
........................................
,98,1.0000,1.0000000,1.0000000,1.0000000
,99,1.0000,1.0000000,1.0000000,1.0000000

,55,62822,63685,63088,62547,69108,73402,74910,74589,74317,75363
,56,64671,63173,63332,62901,61917,69450,73756,74460,74101,73528
,57,63135,63969,63460,62943,62622,61295,68590,74224,73943,73666
...............................................................
,99,162,140,117,136,252,204,164,206,236,298
,100,146,176,162,116,72,212,244,221,227,279

,55,75880,74695,75573,74249,73766,75299,69890,70536,71017,71758
,56,75687,76357,74500,75301,73697,73826,75618,69933,70274,70911
...............................................................
...............................................................
,98,0.27982,0.27982,0.27982,0.30385,0.27433,0.26247,0.26164,0.25349
,99,0.29010,0.29010,0.29010,0.31044,0.28309,0.27087,0.26960,0.26029
-----------------BOTTOM OF FILE-------------------------------------
```

(d) Save the new Word file as temp.txt (Plain Text), and close it.

(e) In Excel, open temp.txt by specifying:

Delimited, Next, Comma, Next, General, Finish.

Then save the file as temp.xls (Microsoft Excel Workbook).

(f) Import the data in temp.xls into S-PLUS as follows:

File, Import Data, From File, File Name: temp.xls (use Browse), OK, Data set: temp, OK (a window comes up).

(g) In S-PLUS, execute the following commands:

```
temp1 <- as.matrix(temp)

RatsF <- matrix(NA,nrow=99,ncol=4)
RatsF[55:99,] <- temp1[1:45,2:5]

RatsM <- matrix(NA,nrow=99,ncol=4)
RatsM[55:99,] <- temp1[47:91,2:5]

PopsF <- matrix(NA,nrow=100,ncol=100)
PopsF[55:100,71:80] <- temp1[93:138,2:11]
PopsF[55:100,81:90] <- temp1[140:185,2:11]
PopsF[55:100,91:100] <- temp1[187:232,2:11]

PopsM <- matrix(NA,nrow=100,ncol=100)
PopsM[55:100,71:80] <- temp1[234:279,2:11]
PopsM[55:100,81:90] <- temp1[281:326,2:11]
PopsM[55:100,91:100] <- temp1[328:373,2:11]
```

```
HazsF <- matrix(NA,nrow=99,ncol=99)
HazsF[55:99,76:83] <- temp1[375:419,2:9]
HazsF[55:99,84:91] <- temp1[421:465,2:9]
HazsF[55:99,92:99] <- temp1[467:511,2:9]

HazsM <- matrix(NA,nrow=99,ncol=99)
HazsM[55:99,76:83] <- temp1[513:557,2:9]
HazsM[55:99,84:91] <- temp1[559:603,2:9]
HazsM[55:99,92:99] <- temp1[605:649,2:9]
```

(h) Check that the S-PLUS commands in the following listing of the six matrices produce the indicated output.

1. Female disability rates, ages 55-99 (RatsF)

```
matrix(cbind(55:99,RatsF[55:99,]),nrow=45,ncol=5,
   dimnames=list(rep(" ",45),c(" ",1981,1988,1993,1998)) )

       1981      1988      1993      1998
  55 0.1993 0.2055988 0.2249022 0.3135074
  56 0.2197 0.2015599 0.2917623 0.3023903
  57 0.2063 0.2059220 0.1795296 0.3077936
  58 0.2154 0.2278814 0.3169573 0.2393704
  59 0.2140 0.2587509 0.2691653 0.2824268
  60 0.2215 0.2401015 0.2542223 0.3189648
  61 0.2318 0.2716015 0.2920057 0.2780898
  62 0.2373 0.2812965 0.2374669 0.2341846
  63 0.2533 0.3119927 0.2558490 0.2491068
  64 0.2779 0.2916484 0.2814329 0.3655436
  65 0.2843 0.3013091 0.3350347 0.3468378
  66 0.2941 0.3206068 0.3286690 0.3340599
  67 0.3160 0.3997699 0.3614100 0.3101715
  68 0.3205 0.3516279 0.3753843 0.4578704
  69 0.3109 0.4441304 0.3590907 0.3466937
  70 0.3347 0.3844138 0.3961472 0.4350594
  71 0.3569 0.4350549 0.4572598 0.4297257
  72 0.3678 0.5011875 0.4919127 0.4217315
  73 0.3792 0.4683925 0.5783805 0.5112553
  74 0.4221 0.5206096 0.5480405 0.4742339
  75 0.4374 0.4564373 0.4860648 0.4717039
  76 0.4565 0.5303137 0.4883018 0.5958506
  77 0.4793 0.5739028 0.5762965 0.5077879
  78 0.5066 0.5899413 0.5202373 0.6216865
  79 0.5138 0.5744212 0.6781060 0.6565818
  80 0.5208 0.6270672 0.4762757 0.5763959
  81 0.5342 0.6533124 0.5883575 0.5931125
  82 0.5620 0.6807099 0.6220030 0.7027790
  83 0.5739 0.7090065 0.6786195 0.7396995
  84 0.5892 0.7375659 0.7439378 0.7635796
  85 0.6143 0.7657044 0.7059109 0.7586456
  86 0.5959 0.7927384 0.7261047 0.7802117
  87 0.5957 0.8179879 0.7457424 0.8004158
  88 0.6137 0.8410714 0.7648119 0.8191925
  89 0.6328 0.8621195 0.7834051 0.8366012
  90 0.6572 0.8813134 0.8016234 0.8527089
  91 0.6915 0.8988342 0.8195681 0.8675826
  92 0.7142 0.9148606 0.8373404 0.8812906
  93 0.7621 0.9295020 0.8550358 0.8939174
  94 0.8249 0.9427901 0.8727434 0.9055609
  95 0.8911 0.9547522 0.8905519 0.9163195
  96 0.9546 0.9654159 0.9085501 0.9262914
  97 0.9901 0.9748379 0.9268089 0.9355759
  98 1.0000 0.9832417 0.9452964 0.9442836
  99 1.0000 0.9909090 0.9639437 0.9525335
```

```
2. Male disability rates, ages 55-99 (RatsM)
--------------------------------------------

matrix(cbind(55:99,RatsM[55:99,]),nrow=45,ncol=5,
   dimnames=list(rep(" ",45),c(" ",1981,1988,1993,1998)) )

       1981      1988      1993      1998
  55 0.2683 0.2643630 0.2651024 0.2851012
  56 0.2852 0.2767488 0.2448480 0.2839287
  57 0.3049 0.3262657 0.3181993 0.3451357
  58 0.3228 0.3032753 0.4169421 0.3801598
  59 0.3270 0.2745861 0.3218317 0.4446799
  60 0.3393 0.3778626 0.4338061 0.4549682
  61 0.3500 0.3944971 0.3856238 0.4782925
  62 0.3607 0.4228503 0.3947087 0.3855523
  63 0.3692 0.5031659 0.4407543 0.3597551
  64 0.3675 0.4957966 0.4245755 0.4838984
  65 0.3624 0.4615851 0.4592479 0.4566838
  66 0.3642 0.4278536 0.4249114 0.4213251
  67 0.3479 0.4474574 0.4001990 0.5175705
  68 0.3392 0.4702757 0.4987245 0.4160009
  69 0.3498 0.5761743 0.4241290 0.4439737
  70 0.3574 0.4973138 0.5275263 0.5386618
  71 0.3649 0.4651265 0.5756399 0.4904182
  72 0.4087 0.5401888 0.5950985 0.5224612
  73 0.4260 0.5243963 0.6206027 0.6298552
  74 0.4441 0.5058766 0.6131062 0.5851121
  75 0.4501 0.4610929 0.5464130 0.6416261
  76 0.4696 0.5604215 0.5952277 0.6111097
  77 0.4605 0.5447037 0.6377697 0.6703209
  78 0.4765 0.5473407 0.5134454 0.6572498
  79 0.4958 0.5666938 0.7325728 0.6809742
  80 0.5110 0.6019923 0.7126560 0.6460970
  81 0.5106 0.6222325 0.6591785 0.7217931
  82 0.5251 0.6435762 0.8404221 0.8087217
  83 0.5363 0.6657798 0.8715889 0.6678048
  84 0.5402 0.6888661 0.6318902 0.5334359
  85 0.5524 0.7128820 0.7900565 0.7906082
  86 0.5767 0.7378744 0.8085832 0.8156625
  87 0.5688 0.7638849 0.8271993 0.8424911
  88 0.6177 0.7907917 0.8458999 0.8703195
  89 0.6970 0.8182812 0.8646789 0.8977520
  90 0.7353 0.8460293 0.8835296 0.9233546
  91 0.7934 0.8737115 0.9024457 0.9456933
  92 0.8046 0.9010167 0.9214205 0.9634176
  93 0.8429 0.9278610 0.9404443 0.9765058
  94 0.8580 0.9543516 0.9595049 0.9860013
  95 0.9342 0.9806021 0.9785904 0.9929775
  96 0.9657 1.0000000 0.9976885 0.9985077
  97 1.0000 1.0000000 1.0000000 1.0000000
  98 1.0000 1.0000000 1.0000000 1.0000000
  99 1.0000 1.0000000 1.0000000 1.0000000

3. Female populations, ages 55-100+, years 1971-2000 (PopsF)
------------------------------------------------------------

matrix(cbind(55:100,PopsF[55:100,71:80]),nrow=46,ncol=11,
   dimnames=list(rep(" ",46),c(" ",1971:1980)) )

matrix(cbind(55:100,PopsF[55:100,81:90]),nrow=46,ncol=11,
   dimnames=list(rep(" ",46),c(" ",1981:1990)) )

matrix(cbind(55:100,PopsF[55:100,91:100]),nrow=46,ncol=11,
   dimnames=list(rep(" ",46),c(" ",1991:2000)) )

      1971  1972  1973  1974  1975  1976  1977  1978  1979  1980
  55 62822 63685 63088 62547 69108 73402 74910 74589 74317 75363
  56 64671 63173 63332 62901 61917 69450 73756 74460 74101 73528
```

57	63135	63969	63460	62943	62622	61295	68590	74224	73943	73666
58	61148	63226	63232	63738	62464	62246	61119	67832	74571	73456
59	58151	60868	63236	62494	63846	61919	61697	60953	66964	74969
60	61510	58086	60572	63271	61585	63933	61595	61209	60736	66198
61	52458	60664	57990	60301	63190	60751	63180	61313	60635	60586
62	53360	52396	59783	57801	59893	63151	60593	62437	60930	60138
63	51234	52349	52285	58836	57528	59478	62217	60427	61570	60655
64	48481	50690	51352	52177	57692	57228	58603	61187	60171	60846
65	47704	47679	50146	50324	51867	56535	56401	57738	60152	59940
66	43493	47197	46855	49506	49124	51637	55899	55650	56817	59189
67	40795	42787	46647	45941	48768	47937	50778	55281	54783	55916
68	39880	40367	42013	45977	44891	47987	47429	49912	54555	53924
69	37528	38980	39871	41111	45134	43834	46815	46915	48918	53865
70	41179	36708	38003	39334	40093	44336	42725	45693	46297	47925
71	37594	40102	35816	37009	38710	39151	42915	41730	44532	45682
72	32113	36938	38905	34892	35896	38036	38268	41471	40641	43328
73	31065	30967	36130	37667	33779	34693	36476	37302	40019	39475
74	29990	29922	29752	35309	36246	32650	33290	34936	36254	38555
75	29395	28590	28701	28532	34325	34810	31315	31861	33297	35251
76	27155	28117	27134	27465	27171	33305	33193	29968	30439	31718
77	24456	25530	26834	25724	26115	25885	31413	31592	28624	29073
78	23715	23220	23887	25547	24198	24797	24497	29509	30042	27238
79	20991	22172	21859	22137	24141	22728	23247	23071	27590	28387
80	20478	19609	20629	20494	20257	22793	21275	21780	21636	25680
81	16787	18903	18292	19138	19094	18605	21017	19941	20326	20219
82	15525	15618	17300	16932	17606	17757	17287	19362	18524	18789
83	13674	14033	14349	15683	15499	16084	16128	16020	17633	17077
84	11606	12195	12535	13150	14019	14124	14431	14517	14719	15891
85	9609	10171	10682	11120	11897	12453	12557	12838	12922	13477
86	8231	8347	8787	9286	9716	10749	10912	11107	11333	11479
87	6498	7010	7212	7544	7954	8444	9221	9508	9711	9906
88	5213	5510	5939	6218	6322	6830	7201	7879	8244	8464
89	4232	4334	4559	4920	5221	5297	5729	6147	6629	7096
90	3406	3463	3513	3751	4001	4322	4410	4777	5105	5516
91	2489	2751	2821	2907	3004	3212	3533	3658	3923	4277
92	1880	1988	2197	2282	2280	2410	2592	2851	2956	3222
93	1351	1476	1552	1727	1761	1819	1879	2067	2268	2384
94	979	1033	1111	1167	1322	1389	1386	1464	1656	1803
95	706	719	769	824	872	1014	1041	1058	1126	1325
96	469	528	528	562	598	657	744	791	808	854
97	284	321	399	381	390	416	470	568	597	597
98	218	177	215	298	278	258	287	332	415	427
99	162	140	117	136	252	204	164	206	236	298
100	146	176	162	116	72	212	244	221	227	279

	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990
55	75880	74695	75573	74249	73766	75299	69890	70536	71017	71758
56	75687	76357	74500	75301	73697	73826	75618	69933	70274	70911
57	72725	75015	76758	74264	74970	73056	73651	75972	70122	69838
58	73202	72943	74259	77105	74043	74654	73330	73495	76302	70178
59	72962	73126	73090	73505	77461	73867	74721	73593	73277	76452
60	75412	72764	73000	73270	72805	77897	73785	74790	73817	72895
61	65486	74885	72585	72964	73348	72000	77556	73712	74819	73885
62	60471	65258	74218	72311	72971	73456	71980	77108	73501	74697
63	59647	59748	64887	73454	71995	72805	72991	71904	76682	73168
64	60280	59042	58866	64510	72740	71676	72004	72551	71782	76008
65	60108	59661	58285	57991	64042	71995	70952	70995	71986	71442
66	59753	59580	58951	57577	57009	63659	71316	70157	69957	71229
67	58182	58937	58859	58199	56730	56053	63106	70556	69355	68753
68	55009	57752	57963	58131	57415	55866	55654	62444	69822	68313
69	53019	54387	57210	56926	57235	56526	55112	55168	61754	68825
70	53107	52054	53635	56590	55842	56280	55539	54281	54560	60809
71	46892	51858	51027	52817	55759	54679	55308	54446	53408	53798
72	45043	46155	50414	49894	51796	54915	53735	54212	53289	52355
73	42062	43581	45230	48840	48622	50697	53461	52705	53066	51980
74	38327	40874	42076	44260	47179	47282	49160	51875	51488	51696
75	36963	37249	39623	40448	43252	45477	45765	47466	50202	50154
76	34141	35790	36121	38275	38779	42051	43784	44094	45694	48321

77	30020	32687	34460	34857	36855	36887	40350	41993	42340	43757
78	27566	28937	31125	33119	33521	35228	35458	38413	40149	40385
79	25730	26374	27617	29521	31573	32057	33515	33896	36396	38074
80	26758	24398	25069	26327	27767	29995	30378	31765	32153	34240
81	23738	25270	22991	23765	24985	25942	28148	28544	29868	30375
82	18773	22274	23667	21550	22376	23563	24370	26205	26704	27920
83	17230	17284	20793	22027	19972	20848	21808	22565	24220	24740
84	15568	15748	15782	19254	20308	18336	19077	19979	20864	22050
85	14142	14082	14283	14399	17658	18535	16725	17256	18114	18973
86	12240	12633	12573	12850	12854	16065	16468	15116	15514	16222
87	10029	10754	11119	11198	11413	11333	14158	14416	13541	13682
88	8587	8773	9319	9785	9859	9976	9979	12288	12443	11824
89	7228	7374	7577	8031	8473	8579	8632	8670	10581	10385
90	5977	6110	6241	6472	6745	7144	7325	7386	7462	8794
91	4490	4982	5088	5286	5508	5541	5964	6125	6204	6263
92	3506	3697	4065	4215	4389	4598	4560	4889	5055	5003
93	2584	2810	2979	3281	3416	3583	3710	3691	3940	3956
94	1858	2034	2214	2414	2619	2714	2803	2907	2889	3046
95	1366	1447	1572	1726	1937	2032	2110	2141	2255	2175
96	1033	1127	1087	1214	1316	1568	1544	1580	1617	1656
97	620	767	959	838	938	1013	1165	1131	1202	1141
98	449	351	544	831	636	719	734	832	841	869
99	322	329	115	363	774	514	509	513	604	592
100	374	461	529	323	276	675	808	872	946	1028

	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000
55	76361	77989	79914	81599	83040	86498	89699	91913	101006	105081
56	72143	75907	77682	79767	81221	83408	86448	89508	91622	100680
57	70717	72136	75413	77399	79647	80972	83284	86221	89231	91330
58	69326	70730	72045	74878	77163	79618	80876	83055	85994	88824
59	70101	69632	70690	72012	74443	77044	79478	80486	82803	85590
60	76553	69967	69825	70686	71955	74112	76801	79167	80161	82436
61	72468	76083	69792	70019	70664	71977	73851	76436	78818	79789
62	73845	72263	75551	69580	70259	70746	71665	73442	76095	78421
63	74469	73204	71924	75020	69384	70594	70355	71196	73040	75663
64	72754	73753	72511	71630	74524	69227	70187	69882	70738	72507
65	75264	72112	72944	71762	71282	74008	68764	69636	69415	70171
66	71003	74263	71336	72175	71006	71007	73364	68175	69138	68824
67	70364	70075	73235	70526	71340	70308	70355	72622	67543	68491
68	67483	69628	69058	72075	69701	70532	69523	69577	71889	66877
69	67134	66877	68782	67933	70859	68885	69716	68697	68735	71073
70	67739	66173	66171	67963	66754	69641	68007	68748	67796	67851
71	59752	66312	65172	65444	67052	65659	68600	66984	67720	66823
72	52987	58773	64737	64171	64632	66113	64497	67461	65933	66630
73	51194	51414	57714	63152	63037	63771	64846	63260	66217	64685
74	50589	49880	49746	56572	61489	61833	62366	63456	61919	64909
75	50281	49081	48472	47911	55387	59722	60433	60874	62023	60457
76	48729	48521	47452	46925	46133	54118	58138	58838	59313	60492
77	46354	46743	46595	45773	45367	44244	52549	56409	57086	57641
78	41774	44682	44663	44663	43936	43666	42626	50808	54556	55270
79	38364	40053	42848	42527	42577	42049	41982	40994	48907	52712
80	35910	36635	38281	40969	40223	40386	40155	40124	39228	47021
81	32125	34023	34903	36424	38869	37902	38390	38129	38267	37323
82	28610	30405	32083	33044	34395	36804	35697	36306	36090	36248
83	25969	26514	28611	29997	31121	32385	34583	33363	34126	34009
84	22801	23868	24417	26735	27822	29126	30141	32078	30960	31870
85	19975	20833	21813	22322	24772	25583	26914	27624	29587	28455
86	17309	18083	18905	19711	20127	22765	23271	24532	25187	27016
87	14421	15524	16220	16935	17580	17938	20575	20923	22189	22725
88	12018	12831	13772	14347	14927	15481	15883	18291	18669	19828
89	10351	10548	11313	11975	12524	13043	13552	13843	16216	16440
90	8574	8901	9148	9796	10306	10788	11239	11603	11894	14199
91	7228	7259	7598	7837	8323	8714	9131	9434	9830	10014
92	5202	6038	6105	6361	6560	6976	7184	7523	7863	8178
93	4058	4229	4944	5075	5191	5408	5668	5827	6151	6412
94	3111	3254	3396	4027	4125	4196	4327	4546	4676	4894
95	2374	2475	2619	2700	3206	3331	3314	3390	3608	3688
96	1707	1852	1959	2074	2056	2512	2590	2514	2613	2826

```
  97 1230 1323 1415 1509  1562  1510  1916  1978  1939  2018
  98  848  782 1025 1092  1162  1180  1101  1427  1524  1461
  99  590  735  697  753   825   930   886   793  1086  1186
  100 1031  968  917  992  1086  1243  1561  1758  1871  2317
```

4. Male populations, ages 55-100+, years 1971-2000 (PopsM)

```
matrix(cbind(55:100,PopsM[55:100,71:80]),nrow=46,ncol=11,
   dimnames=list(rep(" ",46),c(" ",1971:1980)) )

matrix(cbind(55:100,PopsM[55:100,81:90]),nrow=46,ncol=11,
   dimnames=list(rep(" ",46),c(" ",1981:1990)) )

matrix(cbind(55:100,PopsM[55:100,91:100]),nrow=46,ncol=11,
   dimnames=list(rep(" ",46),c(" ",1991:2000)) )
```

```
      1971  1972  1973  1974  1975  1976  1977  1978  1979  1980
  55 61378 63058 61526 60990 68741 74247 74755 75210 75570 76188
  56 64159 60991 62485 61164 60078 68315 73252 73801 74176 74601
  57 62935 63052 60462 61946 60527 59110 66946 72305 72728 73170
  58 60684 62163 61802 59934 61188 59819 58210 65680 71217 71691
  59 57413 59546 61306 60513 59233 60365 58450 57258 64244 70212
  60 56113 56578 58330 60385 59049 58432 59046 57120 56246 62822
  61 50913 55086 55710 57081 59372 57576 57503 57850 55734 55278
  62 50163 50039 53906 54799 55659 58340 56772 56569 56590 54353
  63 47102 48861 49046 52624 53730 54234 56981 55849 55507 55304
  64 44879 46121 47400 47980 51220 52559 52888 55642 54906 54531
  65 44187 43732 44948 45944 46793 49732 51023 51532 54272 54014
  66 39973 42712 42597 43876 44428 45655 48194 49576 50134 52903
  67 37817 38521 41180 41324 42654 42977 44131 46747 48090 48696
  68 35139 36405 36959 39556 39882 41389 41484 42660 45198 46547
  69 32515 33582 34878 35366 37813 38454 39483 39969 41022 43619
  70 31548 30905 32004 33373 33703 35969 36592 37611 38369 39428
  71 29271 30088 29372 30478 31846 31997 34228 34746 35798 36855
  72 24229 27790 28442 27699 28779 30280 30431 32531 32885 33902
  73 21891 22605 26227 26806 25950 27111 28336 28863 30734 30992
  74 20106 20487 20927 24758 25108 24223 25242 26406 27223 28933
  75 18434 18557 18981 19360 23213 23474 22525 23489 24430 25656
  76 17190 16884 17007 17585 17678 21724 21599 20919 21714 22550
  77 15083 15550 15465 15549 16153 16122 19806 19808 19324 19997
  78 14396 13738 13935 14032 14092 14766 14666 17965 18124 17724
  79 12722 12927 12328 12361 12624 12686 13214 13307 16143 16443
  80 11529 11448 11493 10994 10859 11341 11346 11815 11965 14486
  81 10064 10273 10158 10180  9783  9545 10083 10147 10494 10779
  82  8660  8880  9038  8958  8872  8706  8468  8916  8999  9237
  83  7398  7470  7674  7869  7783  7682  7519  7446  7794  7930
  84  6168  6334  6342  6676  6756  6711  6528  6470  6514  6773
  85  4949  5157  5339  5407  5676  5719  5682  5540  5523  5670
  86  4155  4116  4261  4483  4442  4847  4743  4811  4640  4727
  87  3180  3343  3385  3445  3628  3665  3946  3935  3987  3922
  88  2338  2505  2687  2711  2708  2966  2989  3222  3202  3331
  89  1855  1792  1997  2096  2142  2116  2344  2424  2552  2607
  90 1418 1419 1391 1535 1569 1684 1634 1876 1947 1974
  91  962 1068 1075 1078 1148 1168 1323 1302 1459 1533
  92  649  695  802  826  803  852  906 1030  997 1133
  93  508  474  520  579  602  603  626  684  803  747
  94  345  349  342  373  410  436  424  441  515  591
  95  245  249  231  267  261  299  306  304  324  383
  96  173  179  168  169  188  206  212  216  214  245
  97  100  113  139  119  110  136  148  143  155  166
  98   76   73   66  109   92   75   92  102   92  113
  99   47   48   45   39   97   60   49   62   74   61
 100   57   60   64   67   50   81   92   76   64   84

      1981  1982  1983  1984  1985  1986  1987  1988  1989  1990
  55 77794 77061 79074 78048 77423 78068 73257 73185 73713 73648
  56 76118 77599 76726 78453 77475 77477 77564 72974 72637 73388
```

57	73533	75150	77321	76299	77768	76737	76881	77045	72649	71954
58	72074	72947	74083	76902	75803	77126	76251	76298	76524	72160
59	70618	71369	72276	72816	76618	75426	76418	75799	75638	75779
60	69162	70088	70543	71573	71603	76196	74722	75590	75280	74829
61	61378	68390	69431	69722	70947	70320	75314	74009	74829	74592
62	54339	60652	67488	68723	68792	70226	69607	74365	73238	73865
63	52927	53368	59741	66569	67843	67829	69317	68826	73376	72342
64	54041	51957	52261	58885	65501	67028	66436	68340	68001	72187
65	53450	52905	50936	51167	57953	64560	65567	65038	67280	66937
66	53056	52194	51744	49864	50124	57121	63187	64046	63537	66028
67	51492	51612	50831	50518	48711	48974	56075	61666	62427	61914
68	47216	49981	50027	49456	49162	47588	47842	54888	60099	60661
69	44931	45828	48312	48376	47958	47809	46234	46652	53625	58249
70	42029	43204	44303	46672	46646	46389	46222	44751	45423	52177
71	37817	40469	41391	42747	44905	44908	44838	44498	43323	44030
72	35216	36326	38694	39490	41051	43145	43226	42992	42744	41655
73	31911	33288	34742	36889	37534	39349	41198	41337	41216	40812
74	29056	30239	31375	33161	35005	35553	37417	39093	39495	39214
75	27092	27435	28367	29391	31552	33180	33589	35234	36980	37463
76	24040	25273	25802	26675	27443	29905	31054	31450	33050	34696
77	20691	22265	23422	24184	24933	25519	27875	28825	29402	30785
78	18259	19125	20380	21685	22512	23252	23751	25793	26702	27207
79	16109	16706	17484	18605	19890	20886	21401	21877	23663	24386
80	14802	14599	15205	16002	16812	18136	19058	19457	20103	21499
81	12752	13351	13114	13821	14494	15100	16459	17306	17625	18273
82	9607	11497	11933	11772	12401	13055	13624	14742	15532	15775
83	7999	8460	10174	10568	10388	11018	11619	12108	13116	13730
84	6896	6970	7343	8952	9320	9032	9607	10173	10690	11499
85	5730	5936	5986	6355	7803	8126	7848	8309	8851	9328
86	4897	4864	5061	5151	5401	6791	6967	6832	7151	7573
87	3934	4110	4061	4251	4332	4550	5705	5801	5891	5930
88	3230	3247	3401	3391	3528	3589	3755	4694	4760	4948
89	2690	2646	2647	2769	2774	2893	2953	3072	3839	3761
90	2087	2140	2101	2105	2216	2193	2330	2365	2489	3027
91	1492	1648	1654	1701	1658	1713	1740	1827	1852	1928
92	1187	1156	1286	1315	1338	1256	1325	1335	1425	1429
93	840	881	865	998	967	1054	972	1003	1013	1051
94	549	626	662	683	758	720	799	725	755	737
95	424	413	445	504	539	566	547	574	545	546
96	280	325	305	344	366	441	422	411	419	377
97	175	187	248	260	256	268	322	311	312	271
98	124	95	130	203	209	205	192	233	232	216
99	76	90	32	84	182	178	144	136	170	156
100	69	96	135	83	80	164	233	257	276	288

	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000
55	78211	80411	82506	84805	87223	89994	93326	95346	104588	108663
56	73471	77500	79931	81890	84005	87028	89678	92987	94897	104026
57	72867	72964	76719	79527	81328	83249	86664	89256	92420	94376
58	71128	72488	72383	75891	79057	80825	82831	86214	88689	91823
59	71625	70789	72015	71773	75111	78763	80331	82289	85676	88002
60	74917	71163	70366	71463	71229	74404	78311	79808	81663	84997
61	73906	74346	70613	69928	70990	70726	73821	77693	79163	80912
62	73826	72937	73713	70054	69475	70561	70115	73116	76957	78425
63	72838	72658	71854	73012	69535	69093	69832	69297	72377	76128
64	71292	71604	71481	70793	72276	69043	68303	69023	68478	71566
65	70962	69911	70264	70302	69671	71664	68174	67352	68100	67602
66	65760	69268	68381	68819	68947	68559	70576	67036	66289	67035
67	64647	64154	67572	66865	67355	67652	67314	69299	65859	65194
68	60093	63000	62447	65729	65315	65832	66239	65872	67982	64630
69	58680	58635	61197	60726	63899	63738	64226	64724	64496	66548
70	56313	56982	57113	59336	58915	62016	62097	62626	63100	62902
71	50681	54461	55160	55493	57401	57024	60294	60365	60930	61473
72	42612	48932	52532	53353	53797	55430	55182	58417	58517	59097
73	39967	40703	47073	50554	51400	52076	53473	53209	56456	56602
74	38921	38155	38701	45074	48518	49559	50044	51405	51077	54391
75	37348	36962	36310	36659	43099	46589	47422	47908	49263	48908
76	35469	35191	35027	34417	34576	41123	44373	45232	45714	46998

```
  77 32449 33156 33017 32951 32476 32462 38964 42147  43029  43452
  78 28608 30243 30876 30740 30865 30570 30514 36755  39833  40735
  79 25119 26513 28074 28512 28490 28849 28588 28480  34606  37507
  80 22228 23130 24500 25878 26134 26182 26789 26555  26541  32407
  81 19484 20317 21250 22504 23727 23827 24037 24688  24540  24563
  82 16604 17655 18419 19323 20530 21621 21595 21904  22698  22523
  83 14052 14812 15915 16598 17438 18589 19489 19439  19891  20699
  84 12045 12448 13115 14239 14777 15636 16585 17494  17411  17883
  85 10031 10551 10934 11526 12520 13045 13785 14644  15587  15441
  86  8097  8696  9090  9492  9944 10983 11384 12055  12848  13708
  87  6498  6904  7455  7718  8138  8485  9425  9819  10400  11192
  88  4979  5485  5816  6274  6477  6862  7153  8067   8369   8909
  89  4223  4210  4577  4756  5163  5355  5665  6023   6779   7058
  90 2977 3430 3464 3740 3837 4187 4376 4684 4944 5663
  91 2338 2421 2751 2798 3022 3047 3353 3537 3760 4017
  92 1517 1845 1970 2159 2237 2380 2396 2662 2814 2956
  93 1073 1158 1446 1535 1669 1763 1836 1848 2071 2181
  94  769  824  896 1152 1231 1224 1371 1398 1407 1581
  95  533  612  626  675  913  979  943 1028 1072 1023
  96  400  407  479  489  507  733  747  704  788  827
  97  262  289  298  380  377  367  564  576  542  601
  98  164  146  205  219  324  305  266  449  453  417
  99  122  193  179  193  215  300  252  204  371  371
 100  237  175  163  175  195  286  472  598  696  950
```

```
5. Female hazards, ages 55-99, years 1976-1999 (HazsF)
------------------------------------------------------

# Note: ABS life tables only go back to 1978. So the hazards for 1976
#       and 1977 were estimated using appropriate ratios of ABS
#       population and death statistics.

matrix(cbind(55:99,HazsF[55:99,76:83]),nrow=45,ncol=9,
   dimnames=list(rep(“ “,45),c(“ “,1976:1983)) )

matrix(cbind(55:99,HazsF[55:99,84:91]),nrow=45,ncol=9,
   dimnames=list(rep(“ “,45),c(“ “,1984:1991)) )

matrix(cbind(55:99,HazsF[55:99,92:99]),nrow=45,ncol=9,
   dimnames=list(rep(“ “,45),c(“ “,1992:1999)) )
```

```
        1976     1977    1978    1979    1980    1981    1982    1983
  55 0.006185 0.005433 0.00591 0.00546 0.00530 0.00497 0.00522 0.00507
  56 0.006796 0.006345 0.00647 0.00593 0.00579 0.00546 0.00571 0.00550
  57 0.006917 0.007333 0.00708 0.00646 0.00633 0.00600 0.00628 0.00598
  58 0.008097 0.007837 0.00774 0.00705 0.00692 0.00660 0.00691 0.00652
  59 0.008979 0.008574 0.00845 0.00772 0.00756 0.00726 0.00761 0.00714
  60 0.009307 0.009108 0.00921 0.00846 0.00826 0.00798 0.00835 0.00784
  61 0.010057 0.009370 0.01001 0.00929 0.00903 0.00878 0.00915 0.00862
  62 0.011876 0.011701 0.01086 0.01019 0.00986 0.00966 0.00999 0.00947
  63 0.012761 0.012521 0.01179 0.01118 0.01079 0.01063 0.01091 0.01038
  64 0.013577 0.013583 0.01280 0.01225 0.01183 0.01169 0.01192 0.01137
  65 0.015123 0.014237 0.01394 0.01341 0.01298 0.01285 0.01304 0.01247
  66 0.016364 0.015295 0.01523 0.01468 0.01427 0.01411 0.01428 0.01369
  67 0.018691 0.016287 0.01671 0.01611 0.01575 0.01549 0.01567 0.01508
  68 0.020005 0.019587 0.01839 0.01771 0.01742 0.01703 0.01720 0.01664
  69 0.023087 0.021275 0.02034 0.01948 0.01929 0.01875 0.01892 0.01839
  70 0.023547 0.023101 0.02262 0.02145 0.02134 0.02072 0.02084 0.02031
  71 0.026359 0.023954 0.02523 0.02368 0.02358 0.02295 0.02302 0.02242
  72 0.030734 0.029450 0.02818 0.02620 0.02606 0.02548 0.02551 0.02475
  73 0.034993 0.031939 0.03141 0.02908 0.02884 0.02833 0.02837 0.02735
  74 0.037243 0.036948 0.03486 0.03233 0.03196 0.03150 0.03163 0.03030
  75 0.044872 0.038799 0.03846 0.03599 0.03546 0.03501 0.03536 0.03367
  76 0.048521 0.046245 0.04220 0.04001 0.03941 0.03892 0.03955 0.03755
  77 0.051149 0.049120 0.04622 0.04441 0.04385 0.04327 0.04431 0.04197
  78 0.061298 0.057885 0.05077 0.04924 0.04882 0.04817 0.04966 0.04695
  79 0.065734 0.064912 0.05616 0.05466 0.05445 0.05374 0.05569 0.05248
  80 0.069583 0.066745 0.06262 0.06081 0.06090 0.06010 0.06240 0.05859
```

81	0.083150	0.075130	0.07044	0.06792	0.06828	0.06728	0.06984	0.06531
82	0.090894	0.091282	0.07976	0.07612	0.07668	0.07531	0.07799	0.07266
83	0.104079	0.101935	0.09059	0.08552	0.08612	0.08408	0.08691	0.08063
84	0.117814	0.111912	0.10269	0.09619	0.09667	0.09351	0.09674	0.08921
85	0.129929	0.116270	0.11575	0.10807	0.10843	0.10351	0.10757	0.09838
86	0.152200	0.129857	0.12863	0.12175	0.12004	0.11566	0.11927	0.11104
87	0.158929	0.145537	0.14293	0.13716	0.13288	0.12923	0.13224	0.12534
88	0.172914	0.162339	0.15882	0.15452	0.14710	0.14440	0.14663	0.14149
89	0.189541	0.183278	0.17648	0.17408	0.16284	0.16134	0.16258	0.15970
90	0.215872	0.179365	0.19611	0.19611	0.18027	0.18027	0.18027	0.18027
91	0.226027	0.206057	0.20927	0.20927	0.19451	0.19451	0.19451	0.19451
92	0.255187	0.231481	0.22295	0.22295	0.20883	0.20883	0.20883	0.20883
93	0.278725	0.266631	0.23715	0.23715	0.22309	0.22309	0.22309	0.22309
94	0.299496	0.275613	0.25181	0.25181	0.23710	0.23710	0.23710	0.23710
95	0.318540	0.261287	0.26694	0.26694	0.25072	0.25072	0.25072	0.25072
96	0.334855	0.315860	0.28250	0.28250	0.26383	0.26383	0.26383	0.26383
97	0.406250	0.353191	0.29848	0.29848	0.27627	0.27627	0.27627	0.27627
98	0.418605	0.285714	0.31486	0.31486	0.28795	0.28795	0.28795	0.28795
99	0.352941	0.426829	0.33159	0.33159	0.29232	0.29232	0.29232	0.29232

	1984	1985	1986	1987	1988	1989	1990	1991
55	0.00473	0.00490	0.00470	0.00446	0.00449	0.00425	0.00414	0.00396
56	0.00514	0.00536	0.00513	0.00486	0.00494	0.00470	0.00452	0.00436
57	0.00559	0.00584	0.00560	0.00533	0.00542	0.00522	0.00494	0.00480
58	0.00612	0.00636	0.00611	0.00585	0.00595	0.00580	0.00543	0.00530
59	0.00674	0.00693	0.00668	0.00642	0.00652	0.00644	0.00598	0.00586
60	0.00743	0.00755	0.00731	0.00704	0.00716	0.00712	0.00662	0.00648
61	0.00822	0.00822	0.00802	0.00772	0.00786	0.00783	0.00732	0.00716
62	0.00909	0.00895	0.00883	0.00846	0.00862	0.00857	0.00809	0.00791
63	0.01003	0.00974	0.00976	0.00927	0.00944	0.00937	0.00892	0.00872
64	0.01106	0.01064	0.01081	0.01019	0.01035	0.01025	0.00982	0.00961
65	0.01221	0.01169	0.01201	0.01125	0.01134	0.01126	0.01079	0.01060
66	0.01349	0.01293	0.01335	0.01247	0.01246	0.01241	0.01188	0.01172
67	0.01490	0.01440	0.01486	0.01386	0.01371	0.01375	0.01311	0.01296
68	0.01646	0.01611	0.01652	0.01542	0.01514	0.01527	0.01452	0.01434
69	0.01814	0.01803	0.01831	0.01716	0.01674	0.01700	0.01612	0.01586
70	0.01994	0.02014	0.02023	0.01907	0.01856	0.01891	0.01794	0.01754
71	0.02189	0.02242	0.02228	0.02116	0.02061	0.02101	0.01998	0.01942
72	0.02400	0.02489	0.02448	0.02346	0.02296	0.02330	0.02224	0.02157
73	0.02635	0.02757	0.02688	0.02602	0.02559	0.02586	0.02474	0.02401
74	0.02902	0.03055	0.02953	0.02888	0.02857	0.02871	0.02751	0.02682
75	0.03212	0.03393	0.03253	0.03216	0.03191	0.03195	0.03056	0.03002
76	0.03576	0.03779	0.03599	0.03593	0.03570	0.03565	0.03395	0.03364
77	0.04007	0.04224	0.04004	0.04029	0.04000	0.03990	0.03774	0.03767
78	0.04513	0.04738	0.04478	0.04523	0.04489	0.04474	0.04204	0.04212
79	0.05097	0.05332	0.05034	0.05076	0.05038	0.05030	0.04694	0.04701
80	0.05751	0.06013	0.05669	0.05682	0.05647	0.05652	0.05259	0.05232
81	0.06467	0.06786	0.06381	0.06336	0.06307	0.06338	0.05899	0.05796
82	0.07237	0.07641	0.07157	0.07029	0.07011	0.07076	0.06610	0.06391
83	0.08055	0.08566	0.07990	0.07769	0.07745	0.07863	0.07388	0.07037
84	0.08916	0.09552	0.08875	0.08564	0.08518	0.08690	0.08225	0.07780
85	0.09827	0.10590	0.09826	0.09433	0.09344	0.09584	0.09108	0.08681
86	0.10963	0.11639	0.10962	0.10610	0.10530	0.10746	0.10317	0.09928
87	0.12230	0.12792	0.12230	0.11934	0.11866	0.12048	0.11686	0.11354
88	0.13644	0.14059	0.13644	0.13423	0.13372	0.13509	0.13236	0.12985
89	0.15222	0.15451	0.15222	0.15098	0.15069	0.15146	0.14993	0.14849
90	0.16982	0.16982	0.16982	0.16982	0.16982	0.16982	0.16982	0.16982
91	0.18434	0.18434	0.18434	0.18434	0.18434	0.18434	0.18434	0.18434
92	0.19942	0.19942	0.19942	0.19942	0.19942	0.19942	0.19942	0.19942
93	0.21499	0.21499	0.21499	0.21499	0.21499	0.21499	0.21499	0.21499
94	0.23102	0.23102	0.23102	0.23102	0.23102	0.23102	0.23102	0.23102
95	0.24748	0.24748	0.24748	0.24748	0.24748	0.24748	0.24748	0.24748
96	0.26432	0.26432	0.26432	0.26432	0.26432	0.26432	0.26432	0.26432
97	0.28147	0.28147	0.28147	0.28147	0.28147	0.28147	0.28147	0.28147
98	0.29893	0.29893	0.29893	0.29893	0.29893	0.29893	0.29893	0.29893
99	0.31662	0.31662	0.31662	0.31662	0.31662	0.31662	0.31662	0.31662

	1992	1993	1994	1995	1996	1997	1998	1999
55	0.00404	0.00377	0.00354	0.00385	0.00378	0.00371	0.00362	0.00343
56	0.00440	0.00418	0.00386	0.00423	0.00415	0.00408	0.00398	0.00377
57	0.00480	0.00463	0.00423	0.00464	0.00454	0.00448	0.00436	0.00414
58	0.00525	0.00512	0.00462	0.00508	0.00496	0.00492	0.00478	0.00455
59	0.00577	0.00564	0.00505	0.00557	0.00543	0.00539	0.00522	0.00499
60	0.00634	0.00621	0.00551	0.00610	0.00594	0.00590	0.00572	0.00548
61	0.00698	0.00681	0.00601	0.00670	0.00652	0.00647	0.00626	0.00601
62	0.00768	0.00747	0.00657	0.00738	0.00717	0.00709	0.00685	0.00659
63	0.00844	0.00820	0.00723	0.00814	0.00791	0.00778	0.00750	0.00723
64	0.00929	0.00901	0.00798	0.00899	0.00874	0.00854	0.00823	0.00792
65	0.01023	0.00992	0.00883	0.00994	0.00967	0.00939	0.00905	0.00869
66	0.01130	0.01093	0.00979	0.01101	0.01072	0.01035	0.00998	0.00955
67	0.01252	0.01208	0.01087	0.01220	0.01190	0.01144	0.01102	0.01053
68	0.01391	0.01338	0.01211	0.01352	0.01321	0.01265	0.01220	0.01165
69	0.01550	0.01485	0.01353	0.01499	0.01467	0.01402	0.01352	0.01293
70	0.01726	0.01651	0.01519	0.01660	0.01629	0.01556	0.01501	0.01440
71	0.01923	0.01839	0.01707	0.01839	0.01807	0.01727	0.01668	0.01608
72	0.02141	0.02047	0.01916	0.02039	0.02005	0.01922	0.01857	0.01799
73	0.02386	0.02281	0.02142	0.02266	0.02226	0.02144	0.02074	0.02016
74	0.02663	0.02543	0.02387	0.02525	0.02477	0.02398	0.02321	0.02260
75	0.02978	0.02840	0.02657	0.02822	0.02765	0.02688	0.02605	0.02534
76	0.03334	0.03176	0.02964	0.03161	0.03095	0.03019	0.02929	0.02841
77	0.03736	0.03557	0.03323	0.03548	0.03474	0.03396	0.03298	0.03189
78	0.04189	0.03987	0.03749	0.03987	0.03906	0.03822	0.03716	0.03584
79	0.04700	0.04470	0.04253	0.04485	0.04398	0.04302	0.04187	0.04036
80	0.05273	0.05010	0.04840	0.05044	0.04955	0.04839	0.04714	0.04551
81	0.05911	0.05615	0.05512	0.05670	0.05581	0.05438	0.05303	0.05138
82	0.06619	0.06286	0.06279	0.06367	0.06282	0.06110	0.05964	0.05802
83	0.07395	0.07025	0.07141	0.07138	0.07062	0.06864	0.06707	0.06551
84	0.08248	0.07836	0.08107	0.07987	0.07922	0.07710	0.07543	0.07390
85	0.09171	0.08730	0.09181	0.08918	0.08866	0.08659	0.08482	0.08326
86	0.10240	0.09845	0.10250	0.09936	0.09895	0.09718	0.09532	0.09363
87	0.11435	0.11102	0.11443	0.11044	0.11009	0.10890	0.10696	0.10505
88	0.12769	0.12520	0.12775	0.12244	0.12209	0.12160	0.11959	0.11756
89	0.14259	0.14119	0.14262	0.13539	0.13496	0.13510	0.13301	0.13112
90	0.15922	0.15922	0.15922	0.14930	0.14870	0.14920	0.14704	0.14541
91	0.17355	0.17355	0.17355	0.16418	0.16328	0.16373	0.16149	0.16005
92	0.18815	0.18815	0.18815	0.18004	0.17871	0.17852	0.17618	0.17486
93	0.20284	0.20284	0.20284	0.19686	0.19496	0.19341	0.19099	0.18914
94	0.21744	0.21744	0.21744	0.21463	0.21200	0.20824	0.20547	0.20222
95	0.23179	0.23179	0.23179	0.23333	0.22982	0.22279	0.21923	0.21405
96	0.24575	0.24575	0.24575	0.25291	0.24752	0.23685	0.23243	0.22518
97	0.25916	0.25916	0.25916	0.27334	0.26108	0.25021	0.24567	0.23633
98	0.27190	0.27190	0.27190	0.29456	0.27466	0.26268	0.25949	0.24815
99	0.28387	0.28387	0.28387	0.31652	0.28863	0.27830	0.27383	0.26069

6. Male hazards, ages 55-99, years 1976-1999 (HazsM)

```
matrix(cbind(55:99,HazsM[55:99,76:83]),nrow=45,ncol=9,
   dimnames=list(rep(" ",45),c(" ",1976:1983)) )

matrix(cbind(55:99,HazsM[55:99,84:91]),nrow=45,ncol=9,
   dimnames=list(rep(" ",45),c(" ",1984:1991)) )

matrix(cbind(55:99,HazsM[55:99,92:99]),nrow=45,ncol=9,
   dimnames=list(rep(" ",45),c(" ",1992:1999)) )
```

	1976	1977	1978	1979	1980	1981	1982	1983
55	0.011947	0.011370	0.01122	0.01095	0.01061	0.01053	0.01020	0.00987
56	0.013408	0.013133	0.01242	0.01202	0.01162	0.01156	0.01123	0.01088
57	0.014718	0.013757	0.01374	0.01318	0.01273	0.01266	0.01236	0.01197
58	0.016834	0.015135	0.01519	0.01443	0.01398	0.01384	0.01360	0.01314
59	0.018222	0.017451	0.01674	0.01581	0.01540	0.01513	0.01499	0.01443
60	0.020314	0.018731	0.01842	0.01732	0.01700	0.01653	0.01655	0.01585
61	0.021189	0.019269	0.02023	0.01901	0.01878	0.01807	0.01827	0.01743
62	0.023637	0.023515	0.02221	0.02088	0.02077	0.01978	0.02018	0.01915

63	0.026791	0.026360	0.02437	0.02297	0.02293	0.02169	0.02224	0.02099
64	0.029167	0.026112	0.02672	0.02530	0.02529	0.02383	0.02447	0.02295
65	0.032796	0.029967	0.02925	0.02787	0.02783	0.02622	0.02684	0.02503
66	0.033534	0.031726	0.03195	0.03068	0.03055	0.02889	0.02938	0.02727
67	0.037578	0.037207	0.03477	0.03371	0.03346	0.03186	0.03216	0.02973
68	0.040518	0.038617	0.03775	0.03693	0.03657	0.03513	0.03521	0.03251
69	0.046029	0.045311	0.04094	0.04036	0.03988	0.03871	0.03860	0.03565
70	0.049598	0.046103	0.04449	0.04405	0.04340	0.04255	0.04238	0.03924
71	0.053630	0.047885	0.04855	0.04807	0.04724	0.04666	0.04662	0.04326
72	0.057695	0.058526	0.05324	0.05245	0.05149	0.05106	0.05131	0.04770
73	0.065213	0.061335	0.05862	0.05719	0.05830	0.05575	0.05644	0.05251
74	0.069645	0.067665	0.06461	0.06223	0.06165	0.06071	0.06195	0.05763
75	0.082559	0.070855	0.07104	0.06752	0.06760	0.06600	0.06784	0.06300
76	0.081661	0.084541	0.07765	0.07292	0.07399	0.07168	0.07405	0.06866
77	0.095894	0.081036	0.08421	0.07844	0.08077	0.07777	0.08062	0.07468
78	0.104158	0.091368	0.09062	0.08419	0.08771	0.08430	0.08764	0.08118
79	0.112486	0.101862	0.09693	0.09039	0.09477	0.09134	0.09519	0.08831
80	0.119037	0.109730	0.10332	0.09713	0.10175	0.09899	0.10319	0.09616
81	0.124149	0.113557	0.11014	0.10467	0.10870	0.10720	0.11174	0.10475
82	0.140133	0.132145	0.11789	0.11322	0.11550	0.11583	0.12080	0.11402
83	0.165582	0.148690	0.12687	0.12294	0.12254	0.12490	0.13042	0.12381
84	0.163016	0.163450	0.13725	0.13380	0.13028	0.13458	0.14069	0.13397
85	0.184298	0.156635	0.14911	0.14603	0.13980	0.14501	0.15172	0.14453
86	0.194966	0.181320	0.16467	0.16195	0.15383	0.15840	0.16423	0.15798
87	0.214734	0.192347	0.18186	0.17960	0.16926	0.17302	0.17777	0.17267
88	0.223196	0.206089	0.20085	0.19918	0.18624	0.18898	0.19243	0.18873
89	0.264178	0.244454	0.22181	0.22089	0.20492	0.20643	0.20830	0.20629
90	0.257126	0.238678	0.24497	0.24497	0.22548	0.22548	0.22548	0.22548
91	0.275685	0.222222	0.25733	0.25733	0.23784	0.23784	0.23784	0.23784
92	0.338028	0.280353	0.26912	0.26912	0.24928	0.24928	0.24928	0.24928
93	0.353234	0.329073	0.28037	0.28037	0.25957	0.25957	0.25957	0.25957
94	0.339450	0.339623	0.29110	0.29110	0.26844	0.26844	0.26844	0.26844
95	0.344482	0.343137	0.30134	0.30134	0.27578	0.27578	0.27578	0.27578
96	0.334951	0.367925	0.31112	0.31112	0.28191	0.28191	0.28191	0.28191
97	0.389706	0.304054	0.32048	0.32048	0.28721	0.28721	0.28721	0.28721
98	0.333333	0.391304	0.32941	0.32941	0.29206	0.29206	0.29206	0.29206
99	0.300000	0.448980	0.33797	0.33797	0.29682	0.29682	0.29682	0.29682

	1984	1985	1986	1987	1988	1989	1990	1991
55	0.00915	0.00908	0.00859	0.00850	0.00797	0.00780	0.00743	0.00698
56	0.01026	0.01015	0.00961	0.00951	0.00890	0.00866	0.00831	0.00779
57	0.01145	0.01131	0.01074	0.01061	0.00995	0.00962	0.00929	0.00872
58	0.01270	0.01255	0.01199	0.01175	0.01114	0.01072	0.01037	0.00976
59	0.01403	0.01389	0.01333	0.01295	0.01246	0.01197	0.01157	0.01094
60	0.01541	0.01533	0.01477	0.01419	0.01391	0.01337	0.01291	0.01224
61	0.01686	0.01687	0.01630	0.01553	0.01545	0.01490	0.01438	0.01366
62	0.01839	0.01854	0.01895	0.01700	0.01706	0.01657	0.01597	0.01521
63	0.02003	0.02033	0.01972	0.01866	0.01876	0.01836	0.01767	0.01686
64	0.02183	0.02228	0.02164	0.02054	0.02057	0.02024	0.01949	0.01862
65	0.02383	0.02439	0.02373	0.02269	0.02251	0.02225	0.02141	0.02050
66	0.02611	0.02671	0.02601	0.02508	0.02466	0.02444	0.02347	0.02251
67	0.02870	0.02925	0.02849	0.02772	0.02705	0.02685	0.02570	0.02465
68	0.03165	0.03203	0.03120	0.03058	0.02975	0.02956	0.02815	0.02694
69	0.03495	0.03509	0.03414	0.03366	0.03277	0.03263	0.03087	0.02943
70	0.03859	0.03846	0.03737	0.03695	0.03612	0.03608	0.03390	0.03216
71	0.04256	0.04217	0.04097	0.04051	0.03982	0.03992	0.03726	0.03524
72	0.04684	0.04625	0.04496	0.04435	0.04390	0.04410	0.04096	0.03873
73	0.05137	0.05075	0.04936	0.04853	0.04835	0.04862	0.04502	0.04272
74	0.05613	0.05569	0.05416	0.05306	0.05316	0.05342	0.04943	0.04726
75	0.06113	0.06117	0.05934	0.05800	0.05837	0.05852	0.05420	0.05239
76	0.06647	0.06718	0.06491	0.06339	0.06401	0.06397	0.05933	0.05811
77	0.07218	0.07372	0.07086	0.06933	0.07009	0.06988	0.06487	0.06440
78	0.07844	0.08083	0.07720	0.07587	0.07664	0.07635	0.07093	0.07128
79	0.08530	0.08858	0.08405	0.08308	0.08361	0.08363	0.07756	0.07873
80	0.09284	0.09706	0.09154	0.09103	0.09093	0.09181	0.08492	0.08651
81	0.10102	0.10626	0.09972	0.09967	0.09860	0.10087	0.09302	0.09432
82	0.10987	0.11622	0.10871	0.10896	0.10663	0.11070	0.10185	0.10195
83	0.11928	0.12684	0.11849	0.11885	0.11509	0.12110	0.11126	0.10947

84	0.12923	0.13817	0.12896	0.12927	0.12431	0.13172	0.12122	0.11730
85	0.13964	0.14988	0.14000	0.14021	0.13454	0.14250	0.13153	0.12627
86	0.15194	0.16079	0.15225	0.15243	0.14748	0.15442	0.14484	0.14018
87	0.16532	0.17249	0.16558	0.16572	0.16167	0.16734	0.15949	0.15563
88	0.17988	0.18505	0.18007	0.18017	0.17722	0.18134	0.17563	0.17278
89	0.19572	0.19851	0.19582	0.19588	0.19427	0.19652	0.19339	0.19182
90	0.21296	0.21296	0.21296	0.21296	0.21296	0.21296	0.21296	0.21296
91	0.22749	0.22749	0.22749	0.22749	0.22749	0.22749	0.22749	0.22749
92	0.24224	0.24224	0.24224	0.24224	0.24224	0.24224	0.24224	0.24224
93	0.25705	0.25705	0.25705	0.25705	0.25705	0.25705	0.25705	0.25705
94	0.27181	0.27181	0.27181	0.27181	0.27181	0.27181	0.27181	0.27181
95	0.28638	0.28638	0.28638	0.28638	0.28638	0.28638	0.28638	0.28638
96	0.30063	0.30063	0.30063	0.30063	0.30063	0.30063	0.30063	0.30063
97	0.31448	0.31448	0.31448	0.31448	0.31448	0.31448	0.31448	0.31448
98	0.32781	0.32781	0.32781	0.32781	0.32781	0.32781	0.32781	0.32781
99	0.34055	0.34055	0.34055	0.34055	0.34055	0.34055	0.34055	0.34055

	1992	1993	1994	1995	1996	1997	1998	1999
55	0.00686	0.00660	0.00626	0.00643	0.00618	0.00597	0.00581	0.00559
56	0.00769	0.00738	0.00703	0.00717	0.00690	0.00666	0.00648	0.00623
57	0.00863	0.00825	0.00790	0.00801	0.00771	0.00745	0.00724	0.00694
58	0.00969	0.00921	0.00888	0.00895	0.00862	0.00832	0.00808	0.00774
59	0.01086	0.01027	0.00997	0.01001	0.00964	0.00930	0.00901	0.00862
60	0.01215	0.01143	0.01116	0.01119	0.01079	0.01039	0.01004	0.00959
61	0.01354	0.01269	0.01246	0.01250	0.01207	0.01159	0.01117	0.01067
62	0.01504	0.01407	0.01389	0.01394	0.01348	0.01292	0.01243	0.01185
63	0.01667	0.01558	0.01544	0.01554	0.01504	0.01438	0.01381	0.01316
64	0.01843	0.01724	0.01714	0.01729	0.01676	0.01600	0.01535	0.01462
65	0.02034	0.01907	0.01896	0.01921	0.01864	0.01778	0.01704	0.01623
66	0.02241	0.02107	0.02095	0.02130	0.02070	0.01973	0.01891	0.01803
67	0.02464	0.02327	0.02311	0.02356	0.02293	0.02186	0.02097	0.02001
68	0.02707	0.02566	0.02548	0.02602	0.02535	0.02419	0.02323	0.02220
69	0.02970	0.02826	0.02809	0.02867	0.02797	0.02673	0.02571	0.02461
70	0.03258	0.03107	0.03098	0.03152	0.03078	0.02948	0.02843	0.02726
71	0.03577	0.03414	0.03419	0.03459	0.03381	0.03246	0.03138	0.03015
72	0.03935	0.03749	0.03776	0.03787	0.03706	0.03568	0.03459	0.03331
73	0.04339	0.04118	0.04171	0.04139	0.04055	0.03916	0.03808	0.03676
74	0.04795	0.04530	0.04608	0.04530	0.04442	0.04298	0.04191	0.04054
75	0.05302	0.04993	0.05090	0.04976	0.04880	0.04724	0.04616	0.04471
76	0.05863	0.05508	0.05618	0.05488	0.05383	0.05202	0.05090	0.04932
77	0.06471	0.06083	0.06197	0.06068	0.05953	0.05741	0.05619	0.05443
78	0.07125	0.06716	0.06831	0.06716	0.06591	0.06349	0.06211	0.06009
79	0.07817	0.07411	0.07525	0.07431	0.07297	0.07034	0.06870	0.06634
80	0.08553	0.08162	0.08285	0.08211	0.08073	0.07794	0.07599	0.07323
81	0.09331	0.08977	0.09113	0.09054	0.08919	0.08627	0.08399	0.08080
82	0.10158	0.09838	0.10022	0.09960	0.09834	0.09533	0.09270	0.08908
83	0.11025	0.10748	0.10999	0.10926	0.10819	0.10507	0.10211	0.09811
84	0.11930	0.11701	0.12041	0.11950	0.11872	0.11548	0.11224	0.10792
85	0.12870	0.12706	0.13134	0.13031	0.12994	0.12653	0.12308	0.11853
86	0.14104	0.13960	0.14335	0.14166	0.14183	0.13819	0.13463	0.12995
87	0.15457	0.15338	0.15646	0.15353	0.15438	0.15044	0.14687	0.14222
88	0.16939	0.16852	0.17077	0.16590	0.16757	0.16323	0.15980	0.15533
89	0.18563	0.18515	0.18639	0.17874	0.18130	0.17645	0.17329	0.16918
90	0.20343	0.20343	0.20343	0.19203	0.19511	0.18960	0.18683	0.18323
91	0.21423	0.21423	0.21423	0.20573	0.20849	0.20212	0.19984	0.19687
92	0.22405	0.22405	0.22405	0.21983	0.22097	0.21351	0.21173	0.20977
93	0.23268	0.23268	0.23268	0.23430	0.23212	0.22326	0.22229	0.22042
94	0.24155	0.24155	0.24155	0.24909	0.24150	0.23092	0.23064	0.22810
95	0.25069	0.25069	0.25069	0.26419	0.24844	0.23769	0.23770	0.23383
96	0.26012	0.26012	0.26012	0.27928	0.25721	0.24606	0.24543	0.23963
97	0.26982	0.26982	0.26982	0.29300	0.26570	0.25420	0.25383	0.24669
98	0.27982	0.27982	0.27982	0.30385	0.27433	0.26247	0.26164	0.25349
99	0.29010	0.29010	0.29010	0.31044	0.28309	0.27087	0.26960	0.26029

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