

The Investigation of Quantum Polarisation Teleportation Protocols

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Declaration

This thesis is an account of research undertaken between February and November 2002, under the supervision of Dr Ping Koy Lam, Dr Ben Buchler and Dr Timothy Ralph. It is a partial fulfilment of the requirements for the degree of Bachelor of Science with Honours in theoretical physics at the Australian National University, Canberra, Australia.

Except where acknowledged in the customary manner, the material presented in this thesis is, to the best of my knowledge, original and has not been submitted in whole or part for a degree in any university.

Aśka Dolińska
November, 2002



StokeSpunk

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Abstract

The continuous variables regime offers much promise for quantum information and computation protocols. In particular, the continuous variable polarisation teleportation is of great interest, both theoretically and experimentally, at the moment.

In this thesis three schemes for continuous variable polarisation teleportation are analysed and their performance is rated. The double teleporter setup, the quantum nondemolition teleporter scheme and the biased entanglement teleporter setup are each discussed and evaluated. Two methods are employed for the evaluation of the teleportation success. The TV diagram which stresses the usefulness of the experimental design and the fidelity, which measures the quantum input to output state preservation. It is later shown that these two independent assessments, which consider physically different attributes, yield contradicting conclusions. Further it is shown that it is important to decide whether the objective of the polarisation teleportation is the transfer of information or the quantum state recreation before meaningful analysis using TV or fidelity can be made.

Finally, a study of a special cloning limit for a particular input state is made, related to the two of the above polarisation teleportation schemes. A new cloning fidelity limit is derived for these cases and TV cloning limits of information transfer and correlations are discussed.

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