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Endogenous Transactions in Multi-Seller Model

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Abstract

We examine the problem of a buyer who wishes to purchase and combine n objects owned by n individual owners to realize a higher value. The owners are able to delay their entry into the sale process: They can either sell now or sell later. Among other assumptions, the simple assumptions of *competition* –that the presence of more owners at point of sale reduces their surplus– and *discounting* lead to interesting results: There is costly delay in equilibrium. Moreover, with sufficiently strong competition, the probability of delay increases with n . Thus, buyers who discount the future will face increased costs as the number of owners increases. The source of transactions costs is the owners' desire to dis-coordinate in the presence of competition. These costs are unrelated to transactions costs currently identified in the literature, specifically those due to asymmetric information, or public goods problems where players impose negative externalities on each other by under-contributing.

1 Introduction

Consider a situation where a buyer wishes to combine many objects that are separately owned by different sellers. These objects could be patents, land, property rights over pollution, labor contracts, or any objects that can be combined to yield greater value to a buyer than the sum of values of the individual sellers. Such surplus value presents a potential problem from the buyer’s perspective: Owners may perceive a strategic advantage from delaying entry into the sale process. For example, if owners wait until their object is one of the few remaining to be sold, then they might perceive an opportunity to extract a greater share of the buyer’s surplus. We call this situation the *holdout problem*, and the purpose of our paper is to analyze this problem.

We show that the relatively mild assumption of competition –that the payoff to a player from sale is decreasing in the number of fellow players who are present– (along with a weak additional assumption) is sufficient to lead to a positive probability of equilibrium delay (see Proposition 3). Moreover, we demonstrate that if competition at point of sale becomes severe with sufficiently large numbers of players, then an increase in the number of players leads to an increase in the probability that parties will delay sale (see Theorem 8). Our theory therefore derives a new source of transactions costs as an endogenous phenomena: With discounting, the buyer faces costly and increasing delay as the number of sellers increases.

Competition at point of sale is captured in a simple way in our model. Consider Figure 1, which represents what might be called a ‘dis-coordination’ game.¹ Players 1 and 2 independently make a binary choice; either 1 – to sell now, or 0 – to sell later. If both players end up selling now, they receive 4 each, thus undermining their payoff relative to the situation where they make separate deals with the buyer.

¹Binmore (1992) refers to a similar kind of game as the “Australian Battle of the Sexes”. In this game the cooperative and noncooperative payoff regions are “upside-down” versions of those for the original Battle of the Sexes game.

| | | |
|---|-------|-------|
| | 1 | 0 |
| 1 | (4,4) | (5,6) |
| 0 | (6,5) | (3,3) |

Figure 1: Competition at Point of Sale

Being away from the other player is beneficial, because it reduces the number of others who strike a deal with the buyer at any one time. There are two pure strategy Nash equilibria, (0,1) and (1,0), and a mixed strategy Nash equilibrium $p^* = \frac{1}{2}$. Pure strategy equilibria have the seller's joint payoff increase from 8 to 11 by avoiding (1,1) in the example, and the buyer's payoff is reduced accordingly. Our main contribution is to show that with discounting and competition, an increase in the number of players in such a game can lead to an increase in the equilibrium probability of delay, and hence an increased cost to the buyer.

Our model might appear at first to be analogous to the literature on the public goods contribution problem (Bergstrom, Blume and Varian (1986)), the problem of the commons (Cornes and Sandler (1982)), or indeed the corporate takeover problem (Grossman and Hart (1980)). While the inefficiency result for large n has a similar flavor, our analysis is quite different to those problems. Figure 2 illustrates a simple 'public goods contribution', or 'problem of the commons' game. Choice 1 represents a high contribution, and choice 0 a lower contribution. The dominant strategy equilibrium (0,0) is jointly inefficient as players do not account for the negative externality they impose on each-other through their choice of 0. In contrast, the equilibria in the inter-temporal coordination game of Figure 1 yield sellers the highest possible joint payoff. Instead, it is the buyer who suffers a reduction in utility, because the buyer must pay more in aggregate and suffers delay. Indeed, we derive a model in

| | | |
|---|-------|-------|
| | 1 | 0 |
| 1 | (5,5) | (2,6) |
| 0 | (6,2) | (3,3) |

Figure 2: Public Goods Contribution Game

which delay increases with the number of sellers –to the detriment of the buyer who has a preference for consumption today. The efficient coordination between sellers is in stark contrast to the inefficiencies generated between players who donate funds towards public goods, or between farmers who share a common.

The paper by Segal (1999) provides an excellent benchmark against which to distinguish our contribution from the literature on free riding in the public goods problem. Among other things, Segal examines a very general setting that nests a large set of papers in this area. He examines a situation in which a principal makes bilateral contracts with N agents. The agent’s payoffs are affected by other agents trade with the principal – regardless of whether or not they contract with the principal. For example, the agents might own land that is to be sold to a principal who wishes to build a shopping centre. If an agent does not contract with the principal, then her utility might be increased or decreased due to her proximity to the shopping centre, depending on her tastes. In Segal’s framework, the buyer in our model is the principal, and the agents are the sellers. However, we examine a situation in which the principal is not able to make an initial contract offer to the sellers. That is, the sellers are able to avoid dealing with the buyer in order to gain a strategic advantage in later contracting. Our contracting environment is harsher than Segal’s. Our buyer cannot write contingent contracts, nor can our buyer make initial contract offers. All the buyer can do is operate on a spot market should the sellers decide to enter

the sales process. Thus, the focus of our paper is pre-contractual strategic behavior, rather than behavior within a contracting framework.

In their seminal paper, Mailath and Postlewaite (1990) consider a public goods contribution game with private values and show that as the number of players increases it becomes asymptotically impossible to implement the first best.² The inefficiency is derived from the private information nature of the problem. In our model, however, decreased efficiency due to delay arises from the greater difficulty that sellers face in “dis-coordinating” their actions when the number of sellers increases.

Although the holdout problem that we analyze is related to the literature on coordination games, the questions we examine are different. The coordination literature³ examines games that exhibit multiple Nash equilibria which are Pareto-rankable (see for example, Schelling (1960), Harsanyi and Selten (1988), Katz and Shapiro (1985), Kohlberg and Mertens (1986)). The goal of this literature has been to examine how players will select among the equilibria. In our model, simple competition at the bargaining table is sufficient for the existence of a unique symmetric mixed-strategy equilibrium in the n -player (dis)coordination game (see Proposition 4). This result is very useful, since it avoids the multiplicity problem of the coordination literature, and allows us to focus instead on comparative statics as a point of departure from this literature.

²The reason is as follows. For provision, it is necessary that some agents contribute more than their per-capita cost of provision. An agent, however, will not contribute truthfully according to her valuation unless this agent is pivotal. In a large economy, the probability of an individual agent being pivotal is very small and therefore, all agents net utilities must be nearly constant, which implies the probability of provision goes to zero.

³The term was coined by Thomas Schelling (1960, ch.4) to refer to games with multiple Nash equilibria yielding identical payoffs. A standard example includes two automobiles approaching on a road. Each driver must select a side on which to drive. Equilibrium obtains when they select opposite sides. There are two such equilibria with identical payoffs.

2 Model

There are $n + 1$ players. Player 0, the buyer, is interested in purchasing n objects, and realizing value from the entire set. The objects could be, for example, blocks of land, patents, other firms, labor contracts or property rights over pollution –among other things. The objects are owned by n players (or owners) $i = 1, 2, \dots, n$. Ideally, the buyer would like to engage each of the sellers together, make a take-or-leave-it offer, and realize the full value of the combination. However, an owner may find it in her interests to avoid selling to player 0, and perhaps delaying sale for strategic advantage. Thus, each player $i = 1, 2, \dots, n$ simultaneously (and independently) chooses her probability $p_i \in [0, 1]$ of selling at date A . With probability $1 - p_i$, player i delays sale until date B . We assume that the buyer is passive in this process, and cannot influence the probability that a player sells at date A . In other words, the buyer is not strategic in this model; we leave such considerations for future work. In addition, each player ends up selling either at date A or date B for certain. This captures the assumption that it is not credible for player 0 to refuse to buy an item that delivers positive value.

We make a key simplifying assumption throughout the paper: The buyer is not able to write contracts that are contingent on the sale of any of the objects. We do not make this assumption because we believe that it always holds. Rather, we make it because there is an important class of circumstances in which there is at least a subset of objects where such contracts will not be feasible.⁴

Let $t_i \in \{0, 1\}$ denote whether player i sells her object at date A . The value $t_i = 1$ indicates that the player sells at date A , and 0 indicates that the player sells at date B . The notation is chosen so that 1 indicates presence at date A , and 0 indicates

⁴For example, suppose the buyer offers to pay an owner $\$P$ for object X, if the owner of object Y accepts $\$Q$. The actual transfer of funds between the buyer and each seller cannot be observed, so the contract on transfer price might not be enforceable. Similarly, suppose the buyer offers to purchase only in the event that both all owners agree to sell their objects. This may not be credible where the objects are complex and difficult to describe; for example, such difficulties could be encountered with labor contracts, complicated pieces of machinery, or complex combinations of assets like firms.

absence at date A , and therefore presence at date B . Thus, t_i is the outcome of i 's choice p_i of the probability of selling at date A . Since we assume that players must sell at either of the two focal dates A or B , a player could end up selling at the same time as a multitude of other sellers. To capture this, the payoff to player i from sale when the outcome is $t \equiv (t_1, t_2, \dots, t_n)$ is $s_i : \{0, 1\}^n \mapsto \mathbb{R}$. Thus, for example, $s_i(1, 1, \dots, 1)$ is the payoff when all players present their objects for sale at date A , and $s_i(0, 0, \dots, 0)$ is the payoff when all players sell their objects at date B .

Let $\pi_i : \{0, 1\}^n \times [0, 1]^n \mapsto \mathbb{R}$ denote player i 's expected payoff. Therefore

$$\begin{aligned} \pi_i = \sum_{t \in \{0, 1\}^n} & [(1 - t_1)(1 - p_1) + t_1 p_1] \cdot [(1 - t_2)(1 - p_2) + t_2 p_2] \\ & \cdots [(1 - t_n)(1 - p_n) + t_n p_n] \cdot s_i(t_1, t_2, \dots, t_n). \end{aligned}$$

The expressions $[(1 - t_j)(1 - p_j) + t_j p_j]$, $j = 1, \dots, n$ are the probabilities of outcomes t_1, t_2, \dots, t_n respectively, and $s_i(t_1, \dots, t_n)$ is the corresponding payoff. For example with probability $p_1 \cdot p_2 \cdots p_n$, outcome $(1, 1, \dots, 1)$ occurs, and player i receives $s_i(1, 1, \dots, 1)$. Similarly, outcome $(t_1, \dots, t_n) = (1, 0, 1, \dots, 1)$ delivers payoff $s_i(1, 0, 1, \dots, 1)$ with probability $p_1 \cdot (1 - p_2) \cdot p_3 \cdots p_n$ etc.

A Nash equilibrium of the game is a vector of probabilities $(p_1^*, p_2^*, \dots, p_n^*)$ that satisfies $p_i \in \arg \max \pi_i(p_i, p_{-i}^*)$ for all i . Assuming an identical payoff function s for each i , the set of symmetric Nash equilibria are given by consideration of the zeros and corner solutions of the derivative of expected profit with respect an agent's choice of probability:

$$\begin{aligned} \frac{\partial \pi_1}{\partial p_1} = \sum_{t_{-1} \in \{0, 1\}^{n-1}} & [(1 - t_2)(1 - p) + t_2 p] \cdot [(1 - t_2)(1 - p) + t_2 p] \\ & \cdots [(1 - t_n)(1 - p) + t_n p] \cdot [s(1, t_2, \dots, t_n) - s(0, t_2, \dots, t_n)]. \end{aligned} \quad (1)$$

Note that this expression is the difference in expected payoff to the player from being present at date A for certain as opposed to date B for certain (i.e., it is the

probability-weighted sum of payoff differences over all states). Clearly if the expression is positive, then $p = 1$; if it is negative $p = 0$, and the roots of (1) yield interior solutions. In the appendix we demonstrate the existence of a symmetric Nash equilibrium to this general problem. This paper examines the determinants of the symmetric Nash equilibrium p^* . In particular, we wish to solve the problem of how p^* changes as the number of players increases. The highly stylized specific examples immediately below help us understand the structure of this problem.

2.1 Examples

The following two examples (Extreme Co-operation and Extreme Competition) are very stylized, and are for illustrative purposes only. The third case (Aggregate Count Payoffs) is more general, and is the model we adopt to explore the question of holdout more generally.

2.1.1 Extreme Cooperation

Suppose that whenever all of the players are present at sale, at any one time, each individual player can extract the value V^A of her block of land at date A , and V^B at date B . This captures – in reduced form – the idea that sellers are somehow able to collude on the sale price. When less than the full number of players are present, the payoff is $v < \max(V^A, V^B)$ per player. From equation (1) $p^* = 1$, and $p^* = 0$ could be pure strategy Nash equilibria.⁵ This is straightforward, since

$$s(1, t_2, \dots, t_n) - s(0, t_2, \dots, t_n) = \begin{cases} V^A - v & \text{for } t_j = 1, j > 1 \\ v - V^B & \text{for } t_j = 0, j > 1 \\ 0 & \text{otherwise} \end{cases},$$

which yields $\frac{\partial \pi_1}{\partial p_1} > 0$ for $p^* = 1$, and $\frac{\partial \pi_1}{\partial p_1} < 0$ for $p^* = 0$. We might expect players coordinate on the Pareto dominant equilibrium. For example, if $V^A > V^B$ due to

⁵There are other Nash equilibria, where players have no strict incentive to deviate. For example this is true, if half the players sell at date A , and the other half sell at date B , since deviation yields the same payoff v .

discounting, then this would be $p^* = 1$. With this kind of extreme cooperation at point of sale, the model predicts that there will be no holdout problem.

2.1.2 Extreme Competition

An extreme form of competition is captured by the assumption that the presence of at least one other player at point of sale limits a player's return to zero. It is extreme in the sense that payoffs could come from a reduced form in which players compete vigorously. Again note that this example is for illustrative purposes only; we do not claim that such severe competition is necessarily applicable in practice.

Suppose that if a seller is alone with the buyer at the date A sale, then she gets payoff $S^A > 0$ at date A , and payoff $S^B > 0$ at date B . Substitution of these payoffs into equation (1) yields

$$(1 - p)^{n-1} S^A - p^{n-1} S^B \equiv 0. \quad (2)$$

We get the following result:

Proposition 1 *With extreme competition, the symmetric equilibrium probability of holdout is*

$$h \equiv 1 - p = [(S^A/S^B)^{\frac{1}{n-1}} + 1]^{-1}.$$

Thus holdout occurs with positive probability for all n . Moreover,

- (i) *if $S^A > S^B$, then h is increasing in n ,*
- (ii) *if $S^A < S^B$, then h is decreasing in n , and*
- (iii) *h converges to $\frac{1}{2}$ as $n \rightarrow \infty$.*

In part (i) $S^A > S^B$ – which is probably the leading case if players discount the payoff from sale at the later date B compared to date A . Here, holdout gets worse with n , despite the favorable payoff from date A . The intuition for this result comes from the following thought-experiment. Suppose all other players but player 1 choose

a fixed probability p . Now add another player, who also chooses this level. The payoff to player 1 from sale at date A falls to $(1 - p)^n S^A$ from $(1 - p)^{n-1} S^A$, and the payoff to player 1 from sale at date B falls to $p^n S^B$ from $p^{n-1} S^B$. However, $S^A > S^B$ implies that $p > \frac{1}{2}$. Therefore the expected payoff from date A falls proportionately more than its counter-part for date B . As a result, player 1 finds it in her interests to deviate from p and choose a lower value. In equilibrium all players will therefore reduce p , leading to an increase in holdout. Result (ii) is analogous. Part (iii) follows from a related thought experiment. Fix the choice of p by the other players. As n increases, the probabilities $(1 - p)^{n-1}$ and p^{n-1} that a player is alone at point of sale both shrink towards zero, and this dominates any difference in the relative payoff at each date.

The implication of (iii) is that the value-per-object to the buyer falls. That is, there is an increased transactions cost with the number of sellers. To see why, suppose that the buyer gets value W per object that is combined, and only realizes the total value nW after all objects are sold. Therefore, the buyer's expected payoff per-object with discount factor δ is $W \cdot [p^n + (1 - p^n) \delta]$. For case (i), p falls with n . This leads to a reduction in the buyer's payoff per object – an increase in the transactions costs due to increased delay as the number of sellers increases. Note that the more general case where the buyer gets some value from objects purchased at date A , but must discount the benefits gives similar results: As long as there is delay, the buyer's payoff is lower than otherwise.

One objection to the analysis of this case, is that there are pure strategy Nash equilibria of the game that yield a higher joint surplus to the sellers. For example, if player i chooses $p_i = 1$, and all other players j choose $p_j = 0$, this Nash equilibrium gives S^A as joint surplus of the sellers. Under the mixed strategy, the joint surplus of the sellers is

$$n \cdot [p(1 - p)^n S^A + (1 - p) p^n S^B]$$

which is less than S^A .⁶ However, note that such pure strategy equilibria do not Pareto dominate the mixed strategy symmetric equilibrium of proposition 1, nor would any non-symmetric mixed strategy equilibrium. Each player would prefer to be the one earning the highest expected payoff, which strengthens the prediction that the mixed strategy symmetric equilibrium will be the outcome of such a game.

2.2 Aggregate Count Payoffs

Here, we consider a more realistic simplification of the model. We assume that the payoff to player i from bargaining when the outcome is (t_1, t_2, \dots, t_n) depends only on the count of the number of players present with player i , and the date of i 's sale. Thus, $s(k)$ is the payoff from sale when only k other players are present at date A , and $\delta s(k)$ is the present value payoff in this situation from date B , where $\delta \in [0, 1]$ is the discount factor. With this notation, equation (1) becomes

$$\frac{\partial \pi_1}{\partial p_1} = \sum_{k=0}^{n-1} \binom{n-1}{k} p^k (1-p)^{n-1-k} \Delta_k \quad (3)$$

where

$$\Delta_k \equiv s(k) - \delta s(n-k-1)$$

is the gain that the player makes from being present at date A as compared to date B , taking the participation outcomes (k present at date A , and $n-1-k$ present at date B) of other players as given.

Throughout the paper, we assume that there is discounting:

Discounting The buyer and sellers discount date B payoffs by $\delta \in (0, 1)$.

Of course, discounting means that players have a preference for income today. This assumption ensures that any equilibrium delay will generate inefficiency. Since our goal is to examine how seller's payoffs at point of sale affects sellers behavior, we examine two alternatives. The first is competition:

⁶To see why, note that $\arg \max_p \{n(1-p)^n p\} = \frac{1}{n}$, that the value at this maximum is $\frac{(1-n)^{n-1}}{n^{n-1}} < 1$, and that $S^B < S^A$.

Competition $s(k)$ is decreasing in k

Competition is the assumption that an increase in the number of players selling at any one time, leads to a decrease in each player's payoff. Implicit in this assumption is that there is at least a degree of substitutability between the items being sold. The opposite assumption is collusion:

Collusion $s(k)$ is increasing in k

When there are more players who collude, they are able to collectively extract a larger per-owner surplus from player 0.

Finally, we admit any profile of our (non-strategic) buyer's benefit from sale consistent with discounting. A simple case mentioned in the extreme competition example had the buyer realize no value until all units were purchased. However we could assume, for example, that the buyer receives W^A per object combined at date A , and $W^B < W^A$ per object combined at date B ; delay would cost $W^A - W^B$ per object (due to discounting).

Note that any form of discounting implies that the first-best outcome must involve sale of all the objects at date A . The following result is immediate:

Proposition 2 $p = 1$ is the Pareto Dominant Nash equilibrium under Collusion, and yields the first-best outcome.

Proof. This is easily proved by substitution of $p = 1$ into equation (3), and noting that expected surplus per seller falls with p as p falls below unity. There is no delay in this equilibrium. ■

As in the illustrative example of extreme collusion considered above, a more moderate collusion assumption leads to efficiency. The results below show that the situation is quite different when there is competition at point of sale.

Understanding of the behavior of the zeros of equation (3), is crucial for analyzing our problem. First note that if competition holds, then the coefficients Δ_k of

this expression are monotonic decreasing: By definition $\Delta_k = s(k) - \delta s(n - k - 1)$ and $\Delta_{k+1} = s(k + 1) - \delta s(n - k - 2)$. Competition implies $s(k) > s(k + 1)$ and $s(n - k - 2) > s(n - k - 1)$. Hence $\Delta_k > \Delta_{k+1}$. The next result demonstrates that relatively weak assumptions are needed to ensure that there is costly delay in equilibrium:

Proposition 3 *Assume discounting, competition and $\Delta_{n-1} < 0$. Then neither $p = 0$ nor $p = 1$ can be a symmetric Nash equilibrium, and the first-best is not attained.*

Proof. Substitute $p = 1$ into (3). This yields $\frac{\partial \pi_1}{\partial p_1} = \Delta_{n-1} < 0$, so that player 1 will deviate from $p_1 = 1$. Substitution of $p = 0$ into (3) gives $\frac{\partial \pi_1}{\partial p_1} = \Delta_0$ which is positive by competition, and player 1 will deviate from $p_1 = 0$. Since $p = 1$ is not an equilibrium, the discounting assumption means the first best is not attained. ■

The assumption $\Delta_{n-1} < 0$ has a straightforward economic interpretation as a weak additional competition assumption. The term Δ_{n-1} equals $s(n - 1) - \delta s(0)$; if it is negative⁷, then players get a higher return selling alone at date B , than selling in the presence of all other players at date A . This rules out $p = 1$ as an equilibrium. Competition ensures that the payoff from selling today when every other player sells tomorrow is positive. This rules out $p = 0$. Since $p \in (0, 1)$, and there is discounting, the joint surplus of buyer and sellers is lowered.

While competition-like assumptions and discounting ensure that symmetric equilibria are inefficient, the goal of our paper is to examine not only the phenomenon of costly delay, but to examine comparative statics, such as behavior as the number of players increases, or behavior due to changes in the discount factor. Such analysis is problematic if there are multiple symmetric equilibria. However, as the following result shows, existing assumptions are sufficient for a unique symmetric Nash equilibrium.

⁷Note that the assumption is an implicit restriction on the nature of competition and the discount factor: $\frac{s(n-1)}{s(0)} < \delta$.

Theorem 4 *If $\Delta_{n-1} < 0$, and competition holds, then there exists a unique positive and real symmetric Nash equilibrium probability.*

Proof. Since both $p = 0$, and $p = 1$ are ruled out as symmetric Nash equilibria (SNE), we can divide equation (3) by $(1 - p)^{n-1}$. Defining $\rho = \frac{p}{1-p}$ this implies that SNE are found from

$$\sum_{k=0}^{n-1} \binom{n-1}{k} \rho^k \Delta_k = 0 \quad (4)$$

If the sequence $\Delta_{n-1}, \Delta_{n-2}, \dots, \Delta_0$ changes sign only once, then by Descartes's rule of signs, there is a unique positive real root of the polynomial in (4) (Kostrikin (1982), p. 310-331). New York: NY.). Since $\Delta_{n-1} < 0$, this sequence begins with a negative sign, and since $\Delta_0 > 0$ by competition, the sequence changes sign at least once. However, competition also implies that $\Delta_{n-1}, \Delta_{n-2}, \dots, \Delta_0$ is strictly increasing. Therefore, the sequence only changes sign once and there is a unique positive real SNE. ■

The assumptions of competition and $\Delta_{n-1} < 0$ are fairly weak, and yet they lead to the very strong conclusion of a unique symmetric Nash equilibrium. As mentioned, the advantage of this result, is that it makes comparative statics predictions much stronger than when there are multiple equilibria. In addition, for $\delta = 1$ (i.e., no discounting) we can show that the equilibrium probability is $\frac{1}{2}$, and that when $\delta < 1$, the equilibrium probability exceeds $\frac{1}{2}$. These two results are important in the construction of our results concerning the effect on p of an increase in n :

Proposition 5 *When $\delta = 1$, the unique SNE is given by $p^* = \frac{1}{2}$.*

Proof. Define

$$\frac{\partial \pi_1}{\partial p_1} = \sum_{k=0}^{n-1} \binom{n-1}{k} p^k (1-p)^{n-1-k} [s(k) - \delta s(n-k-1)] \equiv f(p, \delta, n) \quad (5)$$

It suffices to show that $p^* = \frac{1}{2}$ is a SNE, i.e. $f(\frac{1}{2}, 1, n) = 0$. Substitution of $p^* = \frac{1}{2}$

gives

$$f\left(\frac{1}{2}, \delta, n\right) = \left(\frac{1}{2}\right)^{n-1} \sum_{k=0}^{n-1} \binom{n-1}{k} [s(k) - s(n-k-1)] = 0.$$

Since

$$\binom{n-1}{k} = \binom{n-1}{n-1-k},$$

telescoping guarantees that indeed $\sum_{k=0}^{n-1} \binom{n-1}{k} [s(k) - s(n-k-1)] = 0$. ■

Below, we show that when $p = \frac{1}{2}$ and $\delta < 1$ the derivative $\frac{\partial \pi_1}{\partial p_1}$ is greater than zero. That is, the next corollary shows that for a fixed n , the equilibrium probability of immediate sale is strictly greater than $\frac{1}{2}$.

Proposition 6 *When $0 < \delta < 1$, the unique SNE is given by $p^* > \frac{1}{2}$ for any finite n .*

Proof. First we demonstrate that $f_p < 0$. Differentiating (5) with respect to p yields

$$\begin{aligned} f_p &= -(n-1)(1-p)^{n-2} \Delta_0 + \binom{n-1}{1} (1-p)^{n-2} \Delta_1 \\ &\quad - \binom{n-1}{1} (n-2)p(1-p)^{n-3} \Delta_1 + \binom{n-1}{2} 2p(1-p)^{n-3} \Delta_2 \\ &\quad - \binom{n-1}{2} (n-3)p^2(1-p)^{n-4} \Delta_2 + \binom{n-1}{3} 3p^2(1-p)^{n-4} \Delta_3 \\ &\quad + \dots \\ &\quad - \binom{n-1}{n-3} p^{n-3} 4(1-p)^3 \Delta_{n-3} + \binom{n-1}{n-2} (n-2)p^{n-3}(1-p) \Delta_{n-2} \\ &\quad - \binom{n-1}{n-2} p^{n-2} \Delta_{n-2} + (n-1)p^{n-2} \Delta_{n-1}. \end{aligned}$$

The first two terms of f_p reduce to $(n-1)(1-p)^{n-2}(\Delta_1 - \Delta_0)$ which is negative, since $\Delta_0 > \Delta_1$ by competition. In a similar fashion, the second two terms reduce to

$$(n-1)(n-2)p(1-p)^{n-3}[\Delta_2 - \Delta_1]$$

which is negative. All such pairs of terms are negative, for example consider the next to last pair of terms, which reduces to $(n-1)(n-2)p^{n-3}(1-p)[\Delta_{n-2} - \Delta_{n-3}] < 0$. Thus, $f_p < 0$.

Now consider the case of n even. It can be shown that

$$f(p, \delta, n) = \sum_{k=0}^{\frac{n}{2}-1} \binom{n-1}{k} \{s(k)[p^k(1-p)^{n-1-k} - \delta p^{n-1-k}(1-p)^k] \\ + s(n-1-k)[p^{n-1-k}(1-p)^k - \delta p^k(1-p)^{n-1-k}]\}$$

Therefore we have

$$f\left(\frac{1}{2}, n, \delta\right) = \left(\frac{1}{2}\right)^{n-1} (1-\delta) \sum_{k=0}^{\frac{n}{2}-1} \binom{n-1}{k} \{s(k) + s(n-1-k)\} > 0.$$

Since $f_p < 0$, $f(\frac{1}{2}, \delta, n) > 0$ ensures that $p = \frac{1}{2}$ lies below the solution to $f(p, \delta, n) = 0$.

The argument for n odd is similar. Thus, $p^* > \frac{1}{2}$. ■

These set of results above demonstrate that delay is an important feature of equilibrium under competition. Delay occurs because of sellers' desire to dis-coordinate, that is, to minimize the impact competition has on their surplus. Under our assumption of discounting, these results demonstrate that equilibrium delay is a source of endogenous transactions cost that the buyer must face. Below, we demonstrate that such costs are increase with the discount rate δ , as might be expected.

Proposition 7 *A rise in the discount factor leads to an increase in holdout.*

Proof. Recall the definition of $f(p, \delta, n)$ in equation (5) and note that for all equilibrium values of $p = p(\delta)$, $f(p(\delta), \delta, n) \equiv 0$. Differentiating this identity with respect to δ yields

$$p'(\delta) = -f_\delta / f_p. \quad (6)$$

Thus we seek to find the sign of f_δ from (3), since $f_p < 0$ from the proof of Proposition (6). Differentiating f with respect to δ yields

$$f_\delta = - \sum_{k=0}^{n-1} \binom{n-1}{k} p^k (1-p)^{n-1-k} \cdot s(n-k-1) < 0.$$

Substitution in the expression for f_p yields $p'(\delta) < 0$. ■

The direct effect of a rise in δ is to reduce the marginal profitability of p_1 ($f_\delta < 0$). Competition ensures that π_1 is concave in p_1 (i.e. $f_p < 0$ as shown in the proof of Proposition 6. Therefore, in equilibrium, p must fall. Intuitively, when δ rises, each player discounts the future payoff from delay by less: If no other players were to change their strategy, an individual player would wish to delay sale. However, every player is aware that this incentive exists and that there is a higher likelihood of competition from delaying sale. This diminishes the marginal profitability of holdout, although it is still positive. Thus $(1-p)$ must still rise.

The main contribution of this paper is to analyze what can happen to the holdout probability as n increases. A-priori theorizing does not suggest a strong case either way. One might argue that the increased competition following from a rise in the total number of players will diminish the gains from delay. If there is discounting and the number of layers increases, it may be less likely that an individual will be alone at point of sale today, than if that individual delays. This would suggest a fall in the probability of holdout. However, some reflection (based on the extreme competition example above) suggests that as the number of players increases, it will only be the payoffs in the extreme ends of the distribution that motivate players. The result below shows that weak conditions lead to an increase in holdout.

Theorem 8 *An increase in the number of owners increases $h \equiv 1 - p$, and h approaches $\frac{1}{2}$ from above as the number of owners approaches infinity, if all of the following conditions hold:*

(i) *Competition (i.e. $s(k)$ is decreasing);*

(ii) $s(\cdot)$ and $\delta \in (0, 1)$ satisfy, $\Delta_{n-1} \equiv s(n-1) - \delta s(0) < 0$; and

(iii) *Strong Competition*: $s(k)$ diminishes sufficiently rapidly after $n = M$ (specifically, $\lim_{n \rightarrow \infty} \sum_{k=M}^{n-1} s(k) \leq \varepsilon$, some small $\varepsilon > 0$)

Proof. From competition and Proposition (3), for a fixed n , the equilibrium price belongs to the interval $(\frac{1}{2}, 1)$. Thus, we need only consider $p \in (\frac{1}{2}, 1)$.

Noting that $\binom{n-1}{k} = \binom{n-1}{n-1-k}$, f from equation (3) can be re-written as the sum of two terms by matching arguments in $s(\cdot)$ starting from $k = 0$ upwards, and from $k = n-1$ downwards. Specifically, $s(0), s(1), \dots, s(M-1)$ can be derived from $s(k)$ for $k = 0, \dots, M$, and from $s(n-1-k)$ for $k = n-1, n-2, \dots, n-M$. The sequence $s(M), s(M+1), \dots, s(n-1)$ is derived similarly. Thus,

$$\begin{aligned} f(p, \delta, n) = & \\ & \sum_{k=0}^{M-1} \binom{n-1}{k} s(k) \left[p^k (1-p)^{n-1-k} - \delta p^{n-1-k} (1-p)^k \right] \\ & + \\ & \sum_{k=M}^{n-1} \binom{n-1}{k} s(k) \left[p^k (1-p)^{n-1-k} - \delta p^{n-1-k} (1-p)^k \right]. \end{aligned}$$

Consider the second of these sums. By strong competition for $n \geq M$, $\lim_{n \rightarrow \infty} \sum_{k=M}^{n-1} s(k)$

$\leq \varepsilon$. (This is satisfied, for example, if $s(k) = \lambda r^k$ with $0 < r < 1$, since the sum converges to λ times a finite positive number.) Note that $\binom{n-1}{k} p^k (1-p)^{n-1-k}$

and $\binom{n-1}{k} p^{n-1-k} (1-p)^k$ are each weights of a binomial probability distribution.

The sum of each of these terms from M to $n-1$ is therefore less than unity, and the weights themselves are between zero and unity. Therefore, by choosing ε sufficiently small, the second sum can be made arbitrarily small. Thus for sufficiently large n , we can rewrite f as

$$f(p, n) \simeq \sum_{k=0}^{M-1} \binom{n-1}{k} s(k) \left[p^k (1-p)^{n-1-k} - \delta p^{n-1-k} (1-p)^k \right] \quad (7)$$

Now define

$$g^k(p) \equiv p^k (1-p)^{n-1-k} - \delta p^{n-1-k} (1-p)^k,$$

and note that for a fixed k ,

$$g^k(p) < 0 \Leftrightarrow p^k (1-p)^{n-1-k} < \delta p^{n-1-k} (1-p)^k$$

That is,

$$p > \frac{1}{1 + \delta \frac{1}{n-1-2k}} \equiv \bar{p}_k.$$

Moreover, $\frac{\partial \bar{p}_k}{\partial k} > 0$, $g^k(\bar{p}_k + \varepsilon) < 0$ and $g^j(\bar{p}_k + \varepsilon) < 0 \forall k \geq j$. Therefore we can conclude that

$$g^j(\bar{p}_{M-1}) < 0 \forall j \leq M-1. \quad (8)$$

Replacing (8) into (7) we obtain

$$f(\bar{p}_{M-1}, n, M) < 0.$$

Since $f_p < 0$ from proposition 7, this indicates that the equilibrium price belongs to the interval $(\frac{1}{2}, \bar{p}_{M-1})$. However,

$$\bar{p}_{M-1} = \frac{1}{1 + \delta \frac{1}{n-M}}$$

so that as n gets larger, the interval $(\frac{1}{2}, \bar{p}_{M-1})$ must shrink. In particular, as $n \rightarrow \infty$, $\bar{p}_{M-1} \rightarrow \frac{1}{2}$. ■

This is the main result of the paper. The assumption of strong competition ensures that the states of nature in which there are many other players present have a negligible weight in a player's choice of p . Only states in which there are less than $M-1$ other players present determine players' decisions. Following the thought experiment from the extreme competition example in section 2.1.2, fix the choice of p by all other players, and add an additional identical player. Consider the term

$$p^k (1-p)^{n-1-k} - \delta p^{n-1-k} (1-p)^k \quad (9)$$

in equation (7) for fixed k . With $\delta < 1, p > \frac{1}{2}$. Therefore the term $p^k (1 - p)^{n-1-k}$ falls by a proportionately smaller amount than the term $\delta p^{n-1-k} (1 - p)^k$ for all k . In other words, in the comparison between the expected payoff from sale now and sale later, the latter falls by proportionately less. The best response of the player is therefore to choose a lower level of p ; an increase in holdout. Moreover, the probability of holdout approaches $\frac{1}{2}$ because both terms in (9) converge to zero, and hence approach each other as n increases. If the buyer's payoff suffers because of delay, then our result can be interpreted directly as endogenous transactions costs faced by such a player as the number of sellers increases.

3 Concluding Comments

We have modeled a situation in which a buyer wishes to purchase n objects from n independent owners. Each owner is able to delay selling her object. If the presence of other owners at point of sale improves the sale price, then there is no delay in equilibrium, and hence the buyer does not face any transactions costs. This case can be thought of as collusion at point of sale. However, if the presence of other owners leads to a reduced sale price – competition at point of sale – then there is delay in equilibrium. Thus, competition at point of sale imposes transactions costs on the buyer; with discounting, the symmetric equilibrium probability of immediate sale is $p \in (\frac{1}{2}, 1)$.

The model developed is very tractable. Existence of a unique symmetric equilibrium is guaranteed by the relatively weak competition assumptions; that sale price declines in the number of players, and that the sale price obtained being alone but at a later date dominates the immediate sale price with all other players present. The uniqueness result makes the model very amenable to comparative-statics analysis. An increase in discounting reduces the probability of delay. The main result of the paper is to show that with sufficiently strong competition and a sufficiently large number of players, a rise in the number of players leads to an increase in the

symmetric equilibrium probability of delay. This provides a novel explanation for the transactions costs that a buyer can face when purchasing from multiple owners. The owners end up dis-coordinating in equilibrium – that is, to engage in a divide and conquer strategy – so as to reduce the competition they generate at the two periods in which sale takes place. But this behavior leads to an increase in equilibrium delay: As n increases, the probability that a given player has relatively few fellow sellers present immediately, falls more rapidly than the counter-part probability for sale in the later period, leading each player to increase her probability of holdout.

The transactions costs identified in our paper are quite different to those present in current literature. Our model has symmetric information, so that private information is not a source of transactions costs. The public goods contribution problem leads to increased costs as the number of players increases, because of a worsening negative externality. In contrast our problem has players taking actions that are beneficial to their fellow sellers. We do not claim that our source of transactions costs is more or less relevant than these other sources. Rather, the aim of this paper is to develop a formal model that explores equilibrium delay as a source of such costs.

Several areas of future research suggest themselves. The buyer in our model is non-strategic. In practice (particularly with smaller numbers of sellers) the buyer could take actions to increase the likelihood that an owner is present early on in the sale process. Contractual solutions may partially solve the problem. For example, in some circumstances the buyer may be able to make contingent-sale contracts. This raises the more general comment that the buyer’s action of purchasing early could affect the surplus that sellers earn in the later period.

4 Appendix

We now show that a symmetric Nash equilibrium for the general problem stated in section 1 always exists. Note that the first order condition given by (1) is clearly a continuous function of p . Moreover

$$\frac{\partial \pi_1}{\partial p_1} \Big|_{p=0} =$$

$$\sum_{(t_2, \dots, t_n) \in \{0,1\}^{n-1}} [(1-t_2)(1-t_3) \cdots (1-t_n)] \cdot [s(1, t_2, \dots, t_n) - s(0, t_2, \dots, t_n)]$$

This expression is either zero or equal to $[s(1, 0, \dots, 0) - s(0, 0, \dots, 0)] > 0$. In addition,

we can evaluate the first-order condition at $p = 1$:

$$\frac{\partial \pi_1}{\partial p_1} \Big|_{p=1} =$$

$$\sum_{(t_2, \dots, t_n) \in \{0,1\}^{n-1}} [t_2 t_3 \cdots t_n] \cdot [s(1, t_2, \dots, t_n) - s(0, t_2, \dots, t_n)]$$

This expression is either zero or equal to $[s(1, 1, \dots, 1) - s(0, 1, \dots, 1)] < 0$. Therefore,

there exists a solution by the Intermediate Value Theorem.

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