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Abstract

When projects are evaluated using a conventional Harberger (1971) cost-benefit analysis the welfare effects are separated with lump-sum transfers. But this does not appear possible when governments raise revenue with distorting taxes. Evidence to support this view can be found in Mayshar (1990) and Wildasin (1984) who derive a marginal social cost of public funds (MCF) that depends on how the government spends the extra revenue raised. Ballard and Fullerton (1992) use this MCF in place of the conventional Harberger (1964) measure to amend the revised Samuelson condition obtained by Pigou (1947). We show that a conventional cost-benefit analysis is possible in this setting by decomposing their revised condition into conventional Harberger terms. The welfare effects of marginally increasing the public good are isolated by *hypothetical* lump-sum transfers that are offset separately with a distorting tax. We also demonstrate that when the marginal costs and benefits of providing the public good are measured by changes in utility (denominated in units of a chosen numeraire), the income effects are irrelevant because they impact equally on each dollar of cost and benefit. Consequently, projects can be evaluated correctly using uncompensated welfare changes.

1. Introduction

When projects are evaluated using a conventional cost-benefit analysis there are lump-sum transfers between the public and private sectors of the economy that separate the welfare effects of each input and output. Harberger (1971) exploited this property to compute shadow prices for individual goods. And while this is analytically convenient, it is also important for the way governments operate because it allows them to assign the task of measuring welfare changes to separate agencies. For example, treasury and finance can estimate the social cost of raising revenue for a variety of taxes without knowing how the funds will be spent by departments like social security and defence. The same also applies when spending departments evaluate the benefits from their outputs; they do not need to know how the revenue is raised.[1]

In practice, however, governments rarely transfer revenue in a lump-sum manner. Instead, they use distorting taxes, and this appears to rule out a conventional cost-benefit analysis. For this reason, Ballard and Fullerton (1992) argue the Pigou-Harberger-Browning approach that "*measures the efficiency effects of taxes, given the level of government spending*"......."*seems poorly suited to the cost-benefit problem of whether the level of government spending should increase, given that the spending must be financed with additional distortionary taxes*".[2] They demonstrate this for the revised Samuelson condition obtained by Pigou (1947), where:

$$MRS = MCF \cdot MRT,$$

with MRS being the summed marginal consumption benefits from additional output of a public good G, MRT its marginal production cost, and MCF the conventional Harberger (1964) measure of the marginal social cost of public funds. Ballard and Fullerton argue this conventional cost of funds should be replaced by a measure that "*ultimately depends not just on the tax, but also on the nature of the government expenditure under consideration*".[3]

Accordingly, they use a MCF that includes the "revenue effect" from the public good when it impacts on the demands for other taxed goods, and obtain a revised Samuelson condition of:[4]

$$MRS = MCF^{*} \cdot MRT,$$

where MCF^{*} is isolated by raising a distorting tax and returning the revenue to consumers as additional output of the public good. It differs from the conventional measure which isolates the tax inefficiency by returning the revenue to consumers as a lump-sum transfer. But this creates a problem because MCF^{*} cannot be computed without knowing how the government spends the revenue.[5] And this means the welfare effects of the project are not separated in a conventional Harberger manner.

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[1] Clearly, this separation applies for incremental project evaluation.

[2] Page 119 in Ballard and Fullerton.

[3] Page 125 in Ballard and Fullerton.

[4] Diamond and Mirrlees (1971) and Stiglitz and Dasgupta (1971) isolate this change in tax revenue and observe that it is not captured in the revised Samuelson obtained by Pigou. Atkinson and Stern (1974) refer to it as the "revenue effect".

[5] Mayshar (1990) and Wildasin (1979, 1984) also obtain measures of the MCF that incorporate this "revenue effect".

This paper shows how a conventional cost-benefit analysis proceeds when revenue is transferred with distorting taxes. The welfare effects are decomposed into conventional Harberger terms using *hypothetical* lump-sum transfers. And we demonstrate this by decomposing the revised Samuelson condition obtained Ballard and Fullerton. When the public good is marginally increased utility rises by the summed consumption benefits (MRS) less the cost of balancing the government budget; it is driven into deficit by the production costs (MRT) minus the "revenue effect" ($\partial R/\partial G$). When this deficit is financed by lump-sum transfers, the Samuelson condition will be:

$$MRS = MRT - \frac{\partial R}{\partial G}.$$

Notice how the "revenue effect" arises, not because distorting taxes change, but because they are present. If, however, the government raises revenue with a distorting tax then the cost of financing the deficit is increased by any tax inefficiency. It is the excess burden when the extra revenue is returned to taxpayers as a (hypothetical) lump-sum transfer. By summing this tax inefficiency on each dollar of revenue raised to unity we obtain a conventional MCF, where this provides a revised Samuelson condition, of:

$$MRS = MCF\left(MRT - \frac{\partial R}{\partial G}\right).$$

When MCF > 1, as it will be for Ramsey optimal taxes, the public good is more costly to supply. And since the distorting tax changes to offset the hypothetical lump-sum transfers that would fund the budget deficit, this revised condition leads to the same optimal provision of the public good obtained by Ballard and Fullerton. It differs only by the way the costs and benefits are decomposed. Using this decomposition we can see that their MCF, is:

$$MCF^{*} = \left(1 - \frac{1}{MRT}\frac{\partial R}{\partial G}\right),$$

which makes clear the linkage between the tax and the government expenditure.

We also address the important issue about which welfare measure to use in project evaluation. The measures examined to this point are obtained in full equilibrium models where the government balances its budget and private utility changes. Unfortunately they are normally path dependent and lack economic meaning unless we know how the marginal utility of income changes with relative prices.[6] Compensated welfare measures, being path independent, are therefore preferred in project evaluation.

But this creates a dilemma because compensated welfare measures rely on hypothetical outside funding to balance the government budget.[7] And when no foreign aid is actually received or paid, the government must transfer revenue from the private economy, thereby causing utility to change. This dilemma over whether to work with hypothetical or actual welfare measures is apparent in Ballard, Shoven and Whalley (1985) and Stuart (1984) who

[6] There are also distributional effects in models with heterogeneous consumers. Sandmo (1998), Slemrod and Yitzhaki (2001) and Wilson (1991) examine this issue in detail. Kaplow (1996) shows they do not impact on the Samuelson rule for the optimal provision of the public good when preferences are separable into goods and leisure, and the government uses a non-linear income tax to optimally redistribute income.

[7] Ballard and Fullerton neatly avoid this issue by choosing preferences that are linear in the public good. By doing so the compensated and uncompensated welfare measures coincide. We examine why this happens in section 4 below.

compute the MCF by combining them. They deduct actual (uncompensated) changes in tax revenue from the hypothetical (compensated) changes in consumer surplus.

By formalising the relationship between the two welfare measures we prove they each provide the same optimal level of government spending. In other words, the choice between the actual and hypothetical welfare measures is irrelevant to project evaluation. This quite surprising result is demonstrated for the project that supplies a unit of the public good when revenue is raised with tax D. In summary, the compensated welfare gain from marginally increasing the public good is the rise in the budget surplus when tax D changes to hold utility constant ($\hat{\pi}_D$); it is the foreign aid the economy could pay at no cost to utility. When this surplus is transferred to consumers (by lowering tax D), each unit raises utility by the shadow value of foreign aid (ϕ_D), where the actual gain in welfare from the project, is:

$$\pi_D = \phi_D \hat{\pi}_D.$$ [8]

ϕ_D converts the budget surplus into utility because it measures the gain from endowing a unit of foreign aid on the government who balances the budget with tax D. It isolates any income effects, and being independent of the public good, is irrelevant to project evaluation. This means we can find the optimal provision of the public good (with $\hat{\pi}_D = 0$) by equating actual costs and benefits (with $\pi_D = 0$).[9] And this is particularly useful in computational work because analysts can evaluate projects in full equilibrium models by reporting actual, rather than hypothetical, changes in activity. They do not have to separately compute compensated welfare changes to find the optimal level of government spending.

We commence the formal analysis in section 2 of the paper by adopting the general equilibrium (GE) model in Ballard and Fullerton to derive a conventional welfare equation. This equation will isolate the welfare effects of government policy as conventional Harberger terms and is used in section 3 to obtain a revised Samuelson condition. It is compared to the revised Samuelson condition obtained by Wildasin, and Ballard and Fullerton, which allows us to reconcile their MCF to the conventional Harberger measure. The relationship between the uncompensated and compensated welfare changes is obtained in section 4 using the shadow value of foreign aid, and it is confirmed in section 5 using structural form decompositions to separate the income and substitution effects. Basically, we do this by extending to the GE setting the standard practice of using a Slutsky decomposition on partial equilibrium (PE) welfare measures.[10] The paper concludes in section 6 with a summary of the main results.

2. A Conventional Compensated Welfare Equation

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[8] Using the previous notation for the Samuelson condition, the project changes utility by:

$$\pi_D = MRS - MCF\left(MRT - \frac{\partial R}{\partial G}\right).$$

$MCF = 1$ when tax D is non-distorting.

[9] Furthermore, with ϕ_D positive the two welfare changes always have the same sign away from the optimum supply of the public good. And ϕ_D is positive whenever transferring a unit of revenue to taxpayers with tax D raises utility.

[10] For examples of the way the Slutsky decomposition is used see Auerbach and Hines (2001) and Diamond and Mirrlees.

Consider an economy with a single consumer who chooses a private good X and leisure H, taking output of the non-marketed (non-excludable) pure public good G as given, to:[11]

maximise $u(X, H, G)$

s.t.	$X = \eta + (w - \tau)N + L$	- consumer budget constraint;
	$\eta = F(N) - wN$	- private profits;
	$R \equiv \tau N = G + L$	- government budget constraint; and,
	$T = N + H$	- time constraint.

The utility function u(X, H, G) is assumed to be well-behaved, and expenditure on good X is financed from profits (η), after-tax labour income ((w-τ)N) and lump-sum transfers from the government budget (L). All private markets are competitive, and producers of good X use a strictly concave technology F(N). Good X is chosen as numeraire, so the wage rate per unit labour supply (w), and the specific wage tax (τ), are both measured in units of good X. The government budget constraint equates wage tax revenue (R) to expenditure on the public good (G) and lump-sum transfers (L). Only the government supplies the public good, and it does so using a simple linear technology that converts a unit of tax revenue into a unit of the public good (where MRT=1). Finally, the hours devoted to leisure (H) and labour supply (N) are constrained by a fixed endowment of time (T).

To simplify the analysis we use the profit equation to write the consumer budget constraint as:

(1) $$X = F(N) - \tau N + L.$$

The wage tax and the lump-sum transfers constrain government spending, and this is captured in the government budget surplus, which is defined to be:

(2) $$Z = \tau N - G - L.$$

There are four possible equilibrium outcomes in the economy that are determined by the way the government raises revenue, using either lump-sum transfers or the wage tax, and whether it balances its budget (in full equilibrium) or holds utility constant (in a compensated equilibrium). The following table summarises the exogenous variables in each of these four equilibrium closures of the model.

The Exogenous Variables in General Equilibrium

	Full Equilibrium	Compensated Equilibrium
With Lump-Sum Transfers	G, τ, Z	G, τ, u
Without Lump-Sum Transfers	G, L, Z	G, L, u

When the government raises revenue with lump-sum transfers it has two independent policy variables, G and τ, but when it raises revenue with the wage tax it can choose G and L. The budget surplus (Z) is exogenous in the full equilibrium because the government balances its

[11] This is equivalent to having many identical consumers. We set aside equity considerations in this paper.

budget, while utility (u) is exogenous in the compensated equilibrium. The welfare analysis commences in the next section using a full equilibrium model (where utility changes) because that is the setting used to obtain the Samuelson condition in the papers cited above.

The welfare effects of government policy are obtained by totally differentiating the constrained optimisation problem for the consumer, where after applying the first order conditions for price-taking consumers and firms, we have:[12]

(3) $$\frac{du}{u_X} = q\,dG - N\,d\tau + dL.$$

$q \equiv u_G/u_x$ is the summed marginal consumption benefits from the public good (MRS) with $u_X = \partial u/\partial X$. The terms in (3) measure the changes in utility (in units of good X) for increases in the public good, the wage tax and the lump-sum transfers, respectively. The lump-sum transfers are obtained from the government budget constraint in (2), where:

(4) $$dL = dR - dG - dZ,$$

with $dR = \tau\,dN + N\,d\tau$ being the change in wage tax revenue.

A ***conventional welfare equation,*** which uses lump-sum transfers to separate the welfare effects as Harberger terms, is obtained by substituting (4) into (3), where:

(5) $$\frac{du}{u_X} = (q-1)\,dG + \tau\,dN - dZ.$$

Welfare rises by the consumption benefits from the public good ($q\,dG$) and the extra wage tax revenue ($\tau\,dN$), and falls by additional production costs (dG) and exogenous foreign aid payments (dZ).

3. The Revised Samuelson Condition

To find the optimal provision of the public good in full equilibrium (with $dZ = 0$) we write the conventional welfare equation in (5), as:

(6) $$\frac{du(G,\tau,Z)}{u_X} = (q-1)\,dG + \tau\,dN(G,\tau,Z).$$

The lump-sum transfers that separate the terms in (6) are isolated by (4), where the first two terms are the increase in tax revenue, and the last two the increase in expenditure on producing the public good and making foreign aid payments, respectively. When the government uses the wage tax to balance its budget the lump-sum transfers that separate the welfare effects in (6) will be hypothetical, and we solve the change in the tax needed to offset them using (4).

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[12] After totally differentiating the utility function, and using the first order conditions to the consumer problem, with $u_H/u_X = w - \tau$, this yields:

$$du/u_X = dX - (w-\tau)\,dN - q\,dG.$$

Using the budget constraint in (1), the change in private consumption of good X is:

$$dX = F'dN + \tau\,dN + N\,d\tau + dL.$$

When this is substituted into the change in utility above, together with the first order condition for private firms with $F' = w$, we obtain (3) above.

3.1 With Lump-Sum Transfers

We begin the analysis in a conventional setting where the government balances its budget using lump-sum transfers. This is the setting in which the original Samuelson (1954) condition was derived.

Proposition 1: The change in utility from a marginal increase in the supply of the public good when it is financed by lump-sum transfers, is:

$$\pi_L = MRS - MRT + \frac{\partial R(G,\tau,Z)}{\partial G}. \tag{7}$$

[13]

Proof: From (6), we have:

$$\pi_L = \frac{du(G,\tau,Z)}{dG}\frac{1}{u_X} = q - 1 + \tau\frac{\partial N(G,\tau,Z)}{\partial G}, \tag{8}$$

where (7) is obtained using $MRS \equiv q$ and $MRT = 1$.‖

The Samuelson condition is obtained by equating (7) to zero, and is:

$$MRS = MRT - \frac{\partial R(G,\tau,Z)}{\partial G}. \tag{9}$$

This differs from the original Samuelson condition because it includes the "revenue effect" identified by Diamond and Mirrlees, and Stiglitz and Dasgupta when distorting taxes are present in the economy. It collapses to the original Samuelson condition in a partial equilibrium analysis, or when there are no cross effects between the public good and tax distorted markets in general equilibrium.

The cost of funding the project with the wage tax can also be isolated in a conventional manner using lump-sum transfers. It is the Harberger measure of the MCF, which is the cost to consumer surplus of transferring a unit of revenue to the government budget. It is equal to one plus the marginal excess burden (meb) of the tax, where:

$$meb(G,\tau,Z) = -\left(\frac{du(G,\tau,Z)}{d\tau}\frac{1}{u_X}\Big/\frac{dL(G,\tau,Z)}{d\tau}\right). \tag{10}$$

Using (6) and (4), we have:

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[13] Slemrod and Yitzhaki define the marginal benefit from the public good G, as:

$$MBP_G = \frac{MRS}{MRT - \partial R/\partial G}.$$

It is the increase in the social welfare per dollar spent by the government on a unit of the public good. Using this notation we can rewrite (8), as:

$$MBP_G = 1 + \frac{\pi_L}{MRT - \partial R/\partial G},$$

which collapses to $MBP_G = 1$ when the good is optimally supplied.

(11) $$MCF(G,\tau,Z) = 1 + meb(G,\tau,Z) = N \Big/ \frac{\partial R(G,\tau,Z)}{\partial \tau}.$$

Clearly, this MCF is independent of the way the government spends the revenue it raises because it is transferred as a lump sum to the consumer.

3.2 With Distorting Transfers

A revised Samuelson condition is obtained by using this MCF. The welfare change for the project can be decomposed in a conventional manner by using hypothetical lump-sum transfers that are offset with the wage tax. This allows us to separate the welfare effects of the public good and the wage tax.

Proposition 2: When the supply of the public good is increased marginally and financed with the wage tax, utility rises by:

(12) $$\pi_\tau = MRS - MCF(G,\tau,Z)\left(MRT - \frac{\partial R(G,\tau,Z)}{\partial G}\right).$$

Proof: Using the conventional welfare equation in (6), the change in utility is:

(13) $$\pi_\tau = \frac{du(G,L,Z)}{dG}\frac{1}{u_X} = q - 1 + \frac{\partial R(G,\tau,Z)}{\partial G} + \tau\frac{\partial N(G,\tau,Z)}{\partial \tau}\frac{d\tau(G,L,Z)}{dG}.$$ [14]

The wage tax rises to offset the *hypothetical* lump-sum transfers that result from the public good, where from (4), the tax change solves:

$$\frac{dL(G,L,Z)}{dG} = \frac{\partial R(G,\tau,Z)}{\partial G} - q + \frac{\partial R(G,\tau,Z)}{\partial \tau}\frac{d\tau(G,L,Z)}{dG} = 0,$$

with:

(14) $$\frac{d\tau(G,L,Z)}{dG} = \frac{\dfrac{\partial R(G,\tau,Z)}{\partial G} - q}{\dfrac{\partial R(G,\tau,Z)}{\partial \tau}}.$$

When (14) is substituted into (13) we obtain (12).‖

The revised Samuelson condition is obtained from (12), and is:

(15) $$MRS = MCF(G,\tau,Z)\left(MRT - \frac{\partial R(G,\tau,Z)}{\partial G}\right).$$ [15]

[14] The change in utility in (12) can be solved as π_L plus the tax inefficiency that arises from balancing the government budget using the wage tax, where:

$$\pi_\tau = \pi_L + meb(G,\tau,Z)\frac{dL(G,\tau,Z)}{dG}.$$

[15] Slemrod and Yitzhaki would obtain (14) by equating their marginal benefit from the public good G to the conventional MCF in (11), where:

$$MBP_G = \frac{MRS}{MRT - \partial R/\partial G} = MCF.$$

They include the "revenue effect" with the benefits from the extra output of the public good in a

(continued...)

Following Pigou, the conventional MCF is used here to gross up the cost of funding the project. The only difference, however, is the inclusion of the "revenue effect" to determine the change in the budget deficit.

Ballard and Fullerton choose to write this revised Samuelson condition, as:

$$MRS = MCF^{*} \cdot MRT, \tag{16}$$

where MCF^{*} differs from the Harberger measure in (11) above. It is decomposed using (15), as:

$$MCF^{*} = MCF(G,\tau,Z)\left(1 - \frac{1}{MRT}\frac{\partial R(G,\tau,Z)}{\partial G}\right). \tag{17}$$

By capturing the "revenue effect" in this way there are as many measures of their MCF for each tax as there are goods on which the revenue can be spent.[16] It is clear from (17) how Ballard and Fullerton find the MCF can be less than or equal to unity when a positive "revenue effect" offsets the conventionally measured tax inefficiency. And this cannot happen to the conventional MCF for a single distorting tax, or with many taxes when they are Ramsey optimal.

These welfare changes are illustrated in diagrams below by assuming the combined effects of the project leave labour supply unchanged. In other words, the public good raises the supply of labour, while the higher wage tax reduces it back to its initial level. This is the outcome examined in Ballard and Fullerton where the public good is weakly separable in

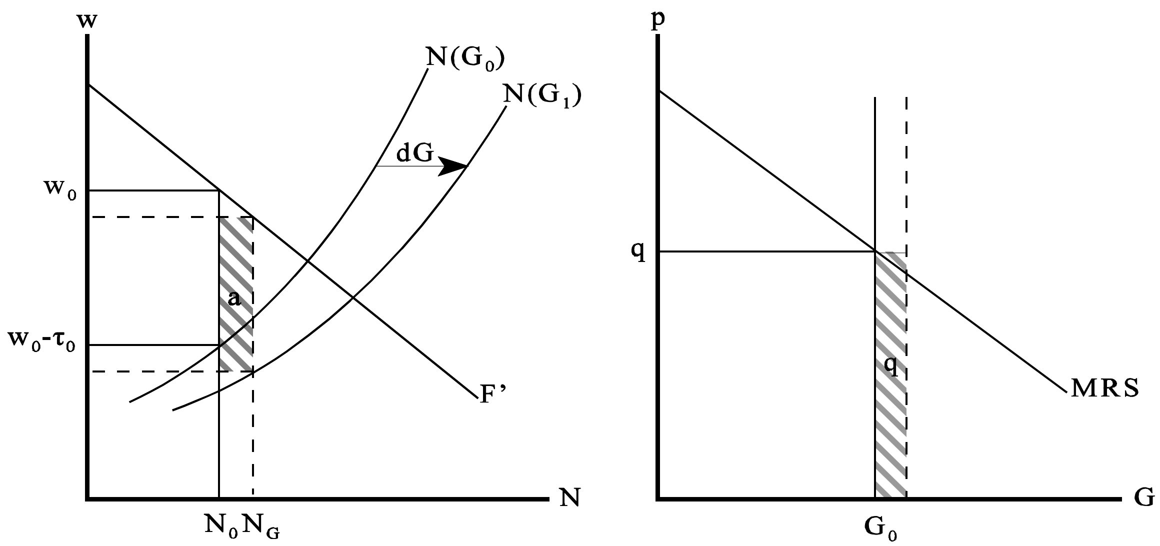

Figure 1: The Welfare Effects with Lump-sum Transfers

[15](...continued)
conventional manner.

[16] Wildasin and Mayshar also define measures of the MCF in this way.

utility. First, the welfare effects are illustrated in Figure 1 when the extra output of the public good is funded from lump-sum transfers. There are additional consumption benefits in q and extra tax revenue in a. And since the cost of producing the public good (MRT = 1) is funded from lump-sum transfers, utility increases by $\pi_L = q - (1 - a)$. This provides the Samuelson condition in (9), where q = 1- a in Figure 1. The budget deficit of 1- a is the amount of revenue transferred as a lump-sum from the private economy.

When these funds are transferred by the wage tax the welfare effects of the project are summarised in Figure 2 where the higher tax reduces labour supply back to N_0. This tax change lowers private surplus by the lined areas b, and raises government revenue by b - c. Once this extra revenue is transferred as a lump-sum back to the consumer, utility falls by the conventional tax inefficiency in area c, where the marginal excess burden for the tax is:

$$meb(G,\tau,Z) = \frac{c}{b-c}.$$

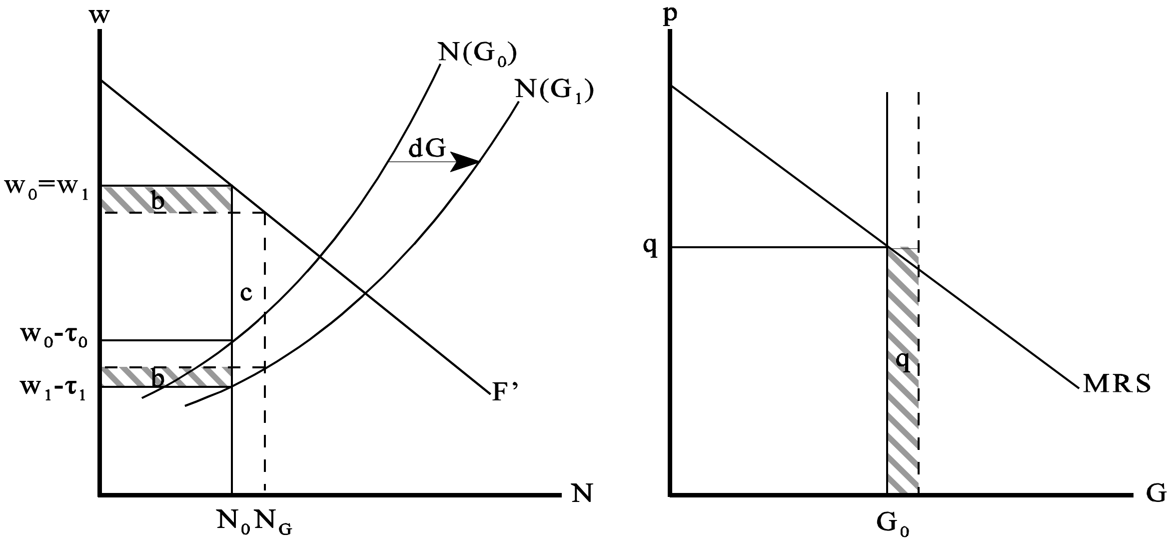

Figure 2: The Welfare Effects with Distorting Transfers

This makes the conventional MCF in (11):

$$MCF(G,\tau,Z) = \frac{b}{b-c}.$$

When the budget deficit (1 - a) is multiplied by this MCF, the final change in utility will be:

$$\pi_\tau = q - \frac{b}{b-c}(1-a),$$

where the revised Samuelson condition is:

$$q = \frac{b}{b-c}(1-a).$$

For the special case illustrated in Figure 2 the “revenue effect” offsets the tax inefficiency, with c = a, so the Samuelson condition collapses to q = b = 1.

The MCF used by Ballard and Fullerton, is:

$$MCF^* = \frac{b}{b-c}(1-a) = \frac{1-a}{1-c}.$$

It is unity in the special case illustrated. If the “revenue effect” is positive and larger than the tax inefficiency (with a > c because labour supply doesn't fall back to N_0) MCF^* is less than unity. But we cannot conclude from this that using a conventional MCF as Pigou does will lead to under-provision of the public good. This only happens when the "revenue effect" is ignored, which is not the case in a conventional cost-benefit analysis because it is included in the welfare effects arising from the provision of the public good (rather than from the change in the wage tax).

4. Compensated Welfare Measures

It is well known that actual (or uncompensated) welfare measures are in general unreliable because they are path dependent. For non-homothetic and non-quasi-linear preferences, the marginal utility of income changes with relative prices, so that dollar changes in surplus under uncompensated demand schedules do not map into utility at the same rate. The recognised way of avoiding this problem is to use, path independent, compensated welfare changes.[17] But in most circumstances they rely on payments or receipts of foreign aid to offset the effects of policy changes on consumer utility. In practice, however, this aid is never actually paid or received, so they are hypothetical welfare measures that isolate potential welfare gains or losses. For example, the inefficiency from raising a distorting tax is the amount of aid the economy would have to receive from foreign donors to hold utility constant. When this aid is not received, private utility falls. And, as noted earlier, this creates a dilemma for the policy analyst wanting to balance the government budget.

In some applications the actual and hypothetical welfare measures are combined. For example, Ballard, Shoven and Whalley estimate the MCF for a range of taxes in the US using a general equilibrium model. The marginal excess burdens for each tax are obtained by deducting *actual* changes in revenue from the cost to surplus measured by equivalent variations. Similarly, Stuart computes the MCF for labour income taxes in the US by deducting *actual* changes in tax revenue from compensating variations, while Browning (1987) estimates their marginal excess burdens as the compensated tax inefficiency per dollar change in *actual* tax revenue.[18] None of these measures coincide with the compensated measures in Auerbach (1985) and Diamond and McFadden (1974) who use compensated (rather than actual) changes in tax revenue.[19]

Mayshar (1990), who examines these and other variants of the MCF, argues the *"conceptual problems that underlie the existing excess burden measures may be the source of the dearth of applications"*.[20] Kay (1980) observes there are frequently problems from

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[17] In economies with heterogeneous consumers there are no distributional effects either because the compensating transfers are personalised to hold constant the utility of each individual. This is not a problem for the single consumer economy being examined in this paper.

[18] Stuart measures the tax inefficiency as the difference between the "compensating surplus" and the change in actual tax revenue. Hicks (1954) distinguishes between "compensating surplus" and "compensating variation". The former holds utility constant by transferring a numeraire good to consumers, while the latter holds utility constant with lump-sum transfers of revenue. The "revenue effect" is also included in the MCF. Ballard, Shoven and Whalley, and Stuart include the "revenue effect" in the MCF. Browning does not.

[19] Fullerton (1991) provides an excellent summary of these different measures of the MCF.

[20] Page 263 in Mayshar. Håkonsen (1998) also examines the different measures of the
(continued...)

confounding hypothetical with actual reimbursement of tax revenue.[21] We now provide a way of resolving this conflict by formalising the relationship between the actual and hypothetical welfare changes. As a first step we derive the compensated welfare change for the project.

4.1 The Revised Samuelson Condition

In a compensated equilibrium the government makes transfers to hold utility constant, so the project impacts on the government budget surplus. This makes the conventional welfare equation in (5), equal to:

(18) $$dZ(G,\tau,u) = (q-1)dG + \tau dN(G,\tau,u).$$

The compensating lump-sum transfers that separate the terms in (18) are obtained using (3) (with $du/u_X = 0$), where:

(19) $$dL = N d\tau - q\,dG.$$

$N d\tau$ is the compensating variation (CV) that offsets the effects of the tax change, while $-q\,dG$ is the CV that offsets the additional consumption benefits from the extra public good.

4.1.1 With Lump-Sum Transfers

Proposition 3: When utility is held constant with lump-sum transfers a marginal increase in the public good raises the government budget surplus by:

(20) $$\hat{\pi}_L = MRS - MRT + \frac{\partial R(G,\tau,u)}{\partial G}.$$

Proof: From (18), we have:

(21) $$\hat{\pi}_L = \frac{dZ(G,\tau,u)}{dG} = q - 1 + \tau\frac{\partial N(G,\tau,u)}{\partial G}.\|$$

This provides a compensated measure of the Samuelson condition, of:

(22) $$MRS = MRT - \frac{\partial R(G,\tau,u)}{\partial G},^{22}$$

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[20](...continued)
MCF and argues they arise from differences in terminology and in the choice of numeraire.

[21] There is still a further choice between the two compensated welfare measures i.e., whether to use equivalent or compensating variations. Most computational studies choose the compensating variation because it holds utility constant at its initial level. In any case, both measures provide unique welfare measures, and are therefore preferable to uncompensated welfare measures.

[22] If we follow the approach used by Slemrod and Yatzhaki, the compensated marginal benefit from the public good G will be:

(continued...)

where the "revenue effect" is the compensated change in tax revenue.

The welfare change in (20) is the hypothetical budget surplus the project generates, and is therefore the potential welfare gain. It is the foreign aid the economy would need to receive to replicate the effects of the project.

Proposition 4: The relationship between the hypothetical welfare change in (20) and the actual change in utility in (7), is:

(23) $$\pi_L = \phi_L \hat{\pi}_L.$$ [23]

where: $\phi_L = -\frac{du(G,\tau,Z)}{dZ}\frac{1}{u_X}$ is the shadow value of foreign aid; it is the rise in utility when the government receives a unit of foreign aid and uses lump-sum transfers to balances its budget.[24]

Proof: Write the reduced form expression for utility as u(G, τ, Z), where Z represents exogenous foreign aid payments (in units of good X). After it is totally differentiated, we have:

(24) $$du(G,\tau,Z) = \frac{\partial u(G,\tau,Z)}{\partial G}dG + \frac{\partial u(G,\tau,Z)}{\partial \tau}d\tau + \frac{\partial u(G,\tau,Z)}{\partial Z}dZ.$$

With lump-sum transfers ($d\tau = 0$) the compensated welfare change from marginally increasing G, solves:

(25) $$\frac{du(G,\tau,Z)}{dG}\frac{1}{u_X} = \frac{\partial u(G,\tau,Z)}{\partial G}\frac{1}{u_X} + \frac{\partial u(G,\tau,Z)}{\partial Z}\frac{1}{u_X}\frac{dZ(G,\tau,Z)}{dG} = 0.$$

The relationship in (23) is obtained using the welfare changes in (8) and (21), where:

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[22](...continued)

$$M\hat{B}P_G = \frac{\dfrac{\partial L(G,\tau,u)}{\partial G}}{\dfrac{\partial Z(G,\tau,u)}{\partial G} + \dfrac{\partial L(G,\tau,u)}{\partial G}} = \frac{MRS}{MRT - \dfrac{\partial R(G,\tau,u)}{\partial G}}.$$

The Samuelson condition in (22) is obtained by setting $M\hat{B}P_G = 1$.

[23] This relationship is verified in section 5.1 where the welfare changes are separated into income and substitution effects using structural form decompositions. Sieper (1981) proves this relationship for the shadow prices of goods in a more general setting.

[24] Using (5), we have: $\phi_L = -\frac{du(G,\tau,Z)}{dZ}\frac{1}{u_X} = 1 - \frac{\partial R(G,\tau,Z)}{\partial Z}$. When a unit of budget surplus is transferred to consumers it has income effects on their demands for taxed goods. Any extra tax revenue must also be transferred to consumers to balance the government budget. Thus, utility changes by more than the initial unit of foreign aid. In economies with heterogenous consumers there is a separate π_L for each of them because personalised transfers are required to hold constant the utility of each individual.

$$\frac{du(G,\tau,Z)}{dG}\frac{1}{u_X} = \pi_L - \phi_L\hat{\pi}_L = 0 . \|$$

There is a straightforward intuition for Proposition 4. When the hypothetical gain, which is the compensated budget surplus in (20), is transferred as a lump-sum to the private economy each unit of surplus raises utility by ϕ_L, where the final change in utility from the project is measured in (7). The shadow value of foreign aid isolates the income effects, and since they impact equally on each dollar of cost and benefit, are irrelevant to project evaluation.[25] This is an important result because it means the optimal provision of the good can be determined by using actual welfare changes obtained in full equilibrium models. There is no need to choose between the compensated and uncompensated welfare measures in project evaluation.[26]

4.1.2 With Distorting Transfers

This same relationship between the welfare changes also applies when revenue is transferred by the wage tax. Before it can be demonstrated we need to isolate the compensated welfare gain for the project.

Proposition 5: When utility is held constant with the wage tax, a marginal increase in the public good raises the government budget surplus by:

(26) $$\hat{\pi}_\tau = \frac{MRS}{MCF(G,\tau,u)} - \left(MRT - \frac{\partial R(G,\tau,u)}{\partial G}\right),$$

where:

(27) $MCF(G,\tau,u) = N \Big/ \frac{\partial R(G,\tau,u)}{\partial \tau}$ is the compensated MCF; it is the direct cost to consumer surplus (i.e., the CV) when a unit of revenue is transferred with the wage tax from the private to the public sector at constant utility.[27]

Proof: This welfare change is isolated as conventional Harberger terms using the welfare

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[25] With heterogeneous consumers the income effects do not play a role when non-linear income taxes are set to equate the marginal utility of income across consumers.

[26] In fact, Sieper points out that this is precisely why Little and Mirrlees (1969) obtain their result that the ratio of the (uncompensated) shadow prices for any two fully traded goods is equal to the ratio of their border prices. The income effects cancel leaving the ratio of the compensated shadow prices, where the compensated shadow price of a fully traded good in a small open economy is just its border price.

[27] The compensated MCF solves:

$$MCF(G,\tau,u) = \frac{dL(G,\tau,u)}{d\tau} \Big/ \left\{\frac{dZ(G,\tau,u)}{d\tau} + \frac{dL(G,\tau,u)}{d\tau}\right\}.$$

It can also be written as: $MCF(G,\tau,u) = \frac{1}{1 - meb(G,\tau,u)}$, where:

$meb = -\frac{dZ(G,\tau,u)}{d\tau} \Big/ \frac{dL(G,\tau,u)}{d\tau}$ is the tax inefficiency per dollar of compensating transfer.

equation in (18), where the budget surplus rises by:

$$\hat{\pi}_\tau = \frac{dZ(G,L,u)}{dG} = q - 1 + \frac{\partial R(G,\tau,u)}{\partial G} + \tau \frac{\partial N(G,\tau,u)}{\partial \tau} \frac{d\tau(G,L,u)}{dG}. \tag{28}$$

The wage tax makes the *hypothetical* lump-sum transfers that hold utility constant, where from (19), the tax change solves:

$$\frac{dL(G,L,u)}{dG} = N \frac{d\tau(G,L,u)}{dG} - q = 0,$$

with:

$$\frac{d\tau(G,L,u)}{dG} = \frac{q}{N}. \tag{29}$$

After substituting (29) into (28) we obtain (26).‖

A compensated measure of the revised Samuelson condition is obtained from (26), and is:

$$\frac{MRS}{MCF(G,\tau,u)} = MRT - \frac{\partial R(G,\tau,u)}{\partial G}. \tag{30}$$ [28]

Notice how the consumption benefits are discounted by the MCF. MRS is the amount that must be transferred from the consumer to hold utility constant, but because there is tax inefficiency, the budget surplus rises by less than MRS. No transfers are required for the production cost or the "revenue effect" because they do not impact on consumer utility. Instead, they impact directly on the government budget.

Once again, this revised Samuelson condition can be illustrated in Figure 2 for the special case where labour supply is unchanged. The marginal increase in the public good confers consumption benefits q on the consumer. To hold utility constant, the wage tax is raised to reduce private surplus in the labour market by the same amount i.e., by b = q. Since the supply of labour remains at N_0 the "revenue effect" exactly offsets the tax inefficiency, so the project has no impact on the government budget surplus with b = q = 1. Thus, there is no change in utility or the government budget surplus in this special case.

In general, however, there are income effects that make the two welfare measures different when the public good is not optimally supplied. We now isolate these income effects.

Proposition 6: The relationship between the hypothetical welfare change in (26) and the actual change in utility in (12), is:

[28] Following the approach adopted by Slemrod and Yitzhaki, the welfare change in (30) is obtained by equating the compensated marginal benefits from the public good G (derived in footnote 21 above) to the MCF in (27), where:

$$M\hat{B}P_G = \frac{MRS}{MRT - \dfrac{\partial R(G,\tau,u)}{\partial G}} = MCF(G,\tau,u)$$

(31) $$\pi_\tau = \phi_\tau \hat{\pi}_\tau,$$[29]

where:

(32) $\phi_\tau = -\frac{du(G,L,Z)}{dZ}\frac{1}{u_X}$ is the shadow value of foreign aid; it is the rise in utility when the government receives a unit of foreign aid and uses the wage tax to balance its budget.[30]

Proof: In the absence of lump-sum transfers we write the reduced form expression for utility as u(G, L, Z). After it is totally differentiated (with dL=0), we have:

(33) $$\frac{du(G,L,Z)}{dG}\frac{1}{u_X} = \frac{\partial u(G,L,Z)}{\partial G}\frac{1}{u_X} + \frac{\partial u(G,L,Z)}{\partial Z}\frac{1}{u_X}\frac{dZ(G,L,u)}{dG} = 0.$$

The relationship in (31) is obtained from the welfare changes defined in (13) and (26), where:

$$\frac{du(G,L,Z)}{dG}\frac{1}{u_X} = \pi_\tau - \phi_\tau \hat{\pi}_\tau = 0. \|$$

Once again, the income effects on each unit of cost and benefit are identical, so they are irrelevant to project evaluation.

5. Decomposing the Welfare Changes into Income and Substitution Effects

A Slutsky decomposition is frequently used to remove the income effects from uncompensated welfare changes obtained in a PE setting. For example, Diamond and Mirrlees (1971) and Auerbach and Hines (2001) do this to find Ramsey optimal taxes that are determined by substitution effects alone. We now extend this practice to a GE setting and prove the relationships in Propositions 4 and 6 above.

To this point the uncompensated welfare changes are derived as reduced form expressions of the exogenous policy variables G, τ and Z. For example, labour supply is the function $N(G,\tau,Z)$. A Slutsky decomposition cannot be applied directly to these reduced form expressions because it separates income and substitution effects when consumers and firms take the wage and income as exogenous. In GE these variables are endogenously determined, so we need to derive the welfare changes as functions of the structural form parameters w,τ,I and G, and then solve the endogenous changes in the wage and income separately. On this basis, labour supply is the function $N(w,\tau,I,G)$, with $w = w(G,\tau,Z)$ and $I = I(G,\tau,Z)$. To simplify the notation we define reduced form expressions as functions of $(G,\tau,Z) = (\cdot\cdot)$, and structural form expressions as functions of $(w,\tau,I,G) = (\cdot)$.

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[29] This relationship is proved in section 5.2 where the welfare changes are separated into income and substitution effects using structural form decompositions. Jones (2000) proves this relationship using the more general model of Sieper.

[30] Using (5), we can show that in our model: $\phi_\tau = -\frac{du(G,L,Z)}{dZ}\frac{1}{u_X} = MCF(G,\tau,Z)\cdot\phi_L$.

In a more general model there may be additional terms. For an example see Jones 2000.

5.1 With Lump-Sum Transfers

First we decompose the change in utility in (7) by deriving a structural form expression for the "revenue effect", which solves:

$$\frac{\partial R(\cdot\cdot)}{\partial G} = \frac{\partial R(\cdot)}{\partial w}\frac{\partial w(\cdot\cdot)}{\partial G} + \frac{\partial R(\cdot)}{\partial I}\frac{\partial I(\cdot\cdot)}{\partial G} + \frac{\partial R(\cdot)}{\partial G}. \tag{34}$$

The change in income is obtained from the consumer budget constraint, where income is equal to:

$$\mathrm{I} \equiv F(N) - MRT{\cdot}G + (w - \tau)H - Z.$$ [31]

After it is totally differentiated, we have:

$$dI = H(dw - d\tau) - \tau dH - MRT dG - dZ, \tag{35}$$

which yields:

$$\frac{\partial I(\cdot\cdot)}{\partial G} = H\frac{\partial w(\cdot\cdot)}{\partial G} + \frac{\partial R(\cdot\cdot)}{\partial G} - MRT. \tag{36}$$

The wage rate changes to equate the supply and demand for labour, where:

$$\frac{\partial N(\cdot)}{\partial w}\frac{\partial w(\cdot\cdot)}{\partial G} + \frac{\partial N(\cdot)}{\partial I}\frac{\partial I(\cdot\cdot)}{\partial G} + \frac{\partial N(\cdot)}{\partial G} = \frac{\partial N_D(w)}{\partial w}\frac{\partial w(\cdot\cdot)}{\partial G},$$

with $N_D(w)$ being the demand for labour.[32] Using the income effect in (36), and applying a Slutsky decomposition, we have:[33]

$$\frac{\partial w(\cdot\cdot)}{\partial G} = \alpha\frac{\partial R(\cdot)}{\partial I}\left(\frac{\partial R(\cdot)}{\partial G} - MRT\right) + \alpha\frac{\partial R(\cdot)}{\partial G}, \tag{37}$$

with: $\alpha = \dfrac{-1}{\tau\left.\dfrac{\partial N(\cdot)}{\partial w}\right|_{\bar{u}} - \tau\left.\dfrac{\partial N_D(w)}{\partial w}\right|_{\bar{u}}}.$

[31] From the budget constraint we have:

$$I = X + (w - \tau)H \equiv \text{Expenditure}.$$

[32] Firms produce good X using labour as an input, and they make this choice by taking the relative wage as given.

[33] From a Slutsky decomposition:

$$\left.\frac{\partial R(\cdot)}{\partial w}\right|_{\bar{u}} = \frac{\partial R(\cdot)}{\partial w} + \frac{\partial R(\cdot)}{\partial I}\frac{\partial I(\cdot)}{\partial w},$$

where: $\partial I(\cdot)/\partial w = H$. This is the income effect when all other variables are held constant.

When (37) and (36) are substituted into (34), the “revenue effect” becomes:

(38)
$$\frac{\partial R(\cdot\cdot)}{\partial G} = \frac{\beta MRS - \beta MRT + \left.\frac{\partial R(\cdot\cdot)}{\partial G}\right|_{\bar{u}}}{1 - \beta},^{34}$$

with: (i) $\beta = \frac{\partial R(\cdot)}{\partial I}\left(\alpha \left.\frac{\partial R(\cdot)}{\partial w}\right|_{\bar{u}} + 1\right)$;[35] and,

(ii) $\left.\frac{\partial R(\cdot\cdot)}{\partial G}\right|_{\bar{u}} = \left.\frac{\partial R(\cdot)}{\partial G}\right|_{\bar{u}}\left(\alpha \left.\frac{\partial R(\cdot\cdot)}{\partial G}\right|_{\bar{u}} + 1\right)$.[36]

This allows us to write the uncompensated change in utility in (7), as:

(39)
$$\pi_L = \frac{MRS - MRT + \left.\frac{\partial R(\cdot\cdot)}{\partial G}\right|_{\bar{u}}}{1 - \beta} = \frac{\hat{\pi}_L}{1 - \beta},$$

where $\hat{\pi}_L$ is the compensated welfare change in (20).

The Shadow Value of Foreign Aid: The relationship between the welfare changes in Proposition 4 is confirmed by deriving a structural form expression for the shadow price of foreign aid.[37] The reduced form expression for this shadow price, is:

(40)
$$\phi_L = -\frac{du(\cdot\cdot)}{dZ}\frac{1}{u_X} = 1 - \frac{\partial R(\cdot\cdot)}{\partial Z}.$$

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[34] In deriving (38) we use the Slutsky decomposition:

$$\frac{\partial R(\cdot)}{\partial G} = \left.\frac{\partial R(\cdot)}{\partial G}\right|_{\bar{u}} - \frac{\partial R(\cdot)}{\partial I}\frac{\partial I(\cdot)}{\partial G},$$

where: $\partial I(\cdot)/\partial G = -MRS$. Also, the compensated change in tax revenue, $\left.\frac{\partial R(\cdot\cdot)}{\partial G}\right|_{\bar{u}}$, is not solved as a function of the structural form parameters because there are no income effects to isolate.

[35] β measures the impact on the budget surplus of a one unit rise in real private spending.

[36] We obtain the structural form expression for compensated “revenue effect” from:

$$\left.\frac{\partial R(\cdot\cdot)}{\partial G}\right|_{\bar{u}} = \left.\frac{\partial R(\cdot)}{\partial w}\right|_{\bar{u}}\left.\frac{\partial w(\cdot\cdot)}{\partial G}\right|_{\bar{u}} + \frac{\partial R(\cdot)}{\partial G},$$

where: $\left.\frac{\partial w(\cdot\cdot)}{\partial G}\right|_{\bar{u}} = \alpha \left.\frac{\partial R(\cdot)}{\partial G}\right|_{\bar{u}}$.

[37] We can see from (39) that we must have: $\phi_L = 1/(1 - \beta)$.

Using structural parameters the change in tax revenue will be:

$$\frac{\partial R(\cdot\cdot)}{\partial Z} = \frac{\partial R(\cdot)}{\partial w}\frac{\partial w(\cdot\cdot)}{\partial Z} + \frac{\partial R(\cdot)}{\partial I}\frac{\partial I(\cdot\cdot)}{\partial Z},$$

where from (35), the income effect is:

$$\frac{\partial I(\cdot\cdot)}{\partial Z} = H\frac{\partial w(\cdot\cdot)}{\partial Z} + \frac{\partial R(\cdot\cdot)}{\partial Z} - 1,$$

and the change in the wage rate:

$$\frac{\partial w(\cdot\cdot)}{\partial Z} = \alpha\frac{\partial R(\cdot)}{\partial I}\left(\frac{\partial R(\cdot\cdot)}{\partial Z} - 1\right).$$

After substitution the change in tax revenue will be:

$$\frac{\partial R(\cdot\cdot)}{\partial Z} = \frac{-\beta}{1-\beta},$$

where the structural form shadow price of foreign aid in (40) is:

(41) $$\phi_L = \frac{1}{1-\beta}.$$ [38]

Sieper provides the economic intuition for this structural decomposition. When a unit of budget surplus is transferred as a lump-sum to the private sector it has income effects which initially increase wage tax revenue by β. This extra revenue is also transferred to the private sector to balance the government budget, and it raises tax revenue again by β^2, and so on. The sum of this sequence of changes to real private spending means:

$$1 + \beta + \beta^2 + \beta^3 + \cdots = \phi_L.$$

Finally, we use this shadow price in (41) and the change in utility in (39) to confirm the relationship between the welfare changes in Proposition 4. It is clear from (41) that the income effects from the project are independent of the policy variable G.

5.2 With Distorting Transfers

To decompose the change in utility in (12) we also need a structural form expression for the change in revenue from the higher wage tax, which solves:

(42) $$\frac{\partial R(\cdot\cdot)}{\partial \tau} = N + \tau\frac{\partial N(\cdot)}{\partial w}\frac{\partial w(\cdot\cdot)}{\partial \tau} + \tau\frac{\partial N(\cdot)}{\partial \tau} + \tau\frac{\partial N(\cdot)}{\partial I}\frac{\partial I(\cdot\cdot)}{\partial \tau}.$$

The GE income effect is obtained using (35), where:

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[38] This is proposition 8 in Sieper.

$$\frac{\partial I(\cdot\cdot)}{\partial \tau} = H\frac{\partial w(\cdot\cdot)}{\partial \tau} + \frac{\partial R(\cdot\cdot)}{\partial \tau} - H,$$

while the change in the wage rate is:

$$\frac{\partial w(\cdot\cdot)}{\partial \tau} = \alpha\frac{\partial R(\cdot)}{\partial I}\left(\tau\frac{\partial N(\cdot\cdot)}{\partial \tau} - H\right) + \alpha\tau\frac{\partial N(\cdot)}{\partial \tau}.$$[39]

After substituting this and the income effect into (42), and using a Slutsky decomposition, we obtain the structural form decomposition:

$$\frac{\partial R(\cdot\cdot)}{\partial \tau} = \frac{\left.\frac{\partial R(\cdot\cdot)}{\partial \tau}\right|_{\bar{u}} - \beta N}{1 - \beta}. \tag{43}$$

Once (43) and (38) are substituted into (12), we have:

$$\pi_\tau = \frac{N}{\left.\frac{\partial R(\cdot\cdot)}{\partial \tau}\right|_{\bar{u}} - \beta N}\left\{\frac{MRS}{MCF(\cdot\cdot)|_{\bar{u}}} - MRT + \left.\frac{\partial R(\cdot\cdot)}{\partial G}\right|_{\bar{u}}\right\}, \tag{44}$$

where: $\mathrm{MCF}(\cdot\cdot)|_{\bar{u}} = \frac{N}{\left.\frac{\partial R(\cdot\cdot)}{\partial \tau}\right|_{\bar{u}}}$ is the compensated MCF in (27).

The terms inside the brackets in (44) are the compensated welfare change from the project in (26). All that remains to be done is to prove the terms in front of the brackets are the shadow value of foreign aid.

The Shadow Value of Foreign Aid: When the government transfers a unit of foreign aid to consumers using the wage tax, utility rises by:

$$\phi_\tau = -\frac{du(G,L,Z)}{dZ}\frac{1}{u_X} = -\frac{du(\cdot\cdot)}{dZ}\frac{1}{u_X} - \frac{du(\cdot\cdot)}{d\tau}\frac{1}{u_X}\frac{d\tau(G,L,Z)}{dZ}.$$

The tax change offsets any hypothetical lump-sum transfers, and solves:

$$\frac{dL(G,L,Z)}{dZ} = \frac{\partial R(\cdot\cdot)}{\partial \tau}\frac{d\tau(G,L,Z)}{dZ} - \left(1 - \frac{\partial R(\cdot\cdot)}{\partial Z}\right) = 0.$$

After solving the tax change the shadow value of foreign aid becomes:

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[39] This is obtained from the market clearing condition in the labour market and using the Slutsky decomposition:

$$\frac{\partial N(\cdot)}{\partial \tau} = \left.\frac{\partial N(\cdot)}{\partial \tau}\right|_{\bar{u}} + \frac{\partial N(\cdot)}{\partial I}\frac{\partial I(\cdot)}{\partial \tau},$$

with $\frac{\partial I(\cdot)}{\partial \tau} = -H.$

$$\phi_\tau = \phi_L \left\{ 1 - \frac{\frac{du(\cdot\cdot)}{d\tau}\frac{1}{u_X}}{\frac{\partial R(\cdot\cdot)}{\partial \tau}} \right\} = \phi_L MCF(\cdot\cdot).$$

Using the structural form expressions for the change in revenue in (43), we have:

$$MCF(\cdot\cdot) = \frac{N(1-\beta)}{\left.\frac{\partial R(\cdot\cdot)}{\partial \tau}\right|_{\bar{u}} - \beta N},$$

where this allows us to write shadow value of foreign aid, using (41), as:

$$\phi_\tau = \frac{N}{\left.\frac{\partial R(\cdot\cdot)}{\partial \tau}\right|_{\bar{u}} - \beta N}. \tag{45}$$

On this basis the change in utility in (44) becomes:

$$\pi_\tau = \phi_\tau \left\{ \frac{MRS}{MCF(\cdot\cdot)|_{\bar{u}}} - MRT + \left.\frac{\partial R(\cdot\cdot)}{\partial G}\right|_{\bar{u}} \right\} = \phi_\tau \hat{\pi}_\tau,$$

which confirms Proposition 6.

6. Conclusion

This paper obtains a revised Samuelson condition using a ***conventional*** Harberger cost-benefit analysis. The welfare effects are separated by *hypothetical* lump-sum transfers that are offset separately with a distorting tax. This allowed us to reconcile the ***conventional*** MCF with the measure used by Ballard and Fullerton, and Wildasin. By including the "revenue effect" in their MCF it cannot be measured without knowing how the government spends the extra revenue. In contrast, the conventional measure allows us to separate the welfare effects of the project; a property that is crucial in allowing separate agencies to evaluate projects in practice.

Finally, the change in utility from the project was decomposed into its income and substitution effects. And because the income effects are independent of the public good they are irrelevant to project evaluation. This means projects can be evaluated correctly using uncompensated measures of the marginal costs and benefits i.e., there is no need to compute the compensated costs and benefits to find the social optimum.

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