# Excitation Energies of Superdeformed States in $^{196}$Pb: Towards a Systematic Study of the Second Well in Pb Isotopes

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The excitation energy of the lowest-energy superdeformed band in $^{196}$Pb is established using the techniques of time-correlated $\gamma$-ray spectroscopy. Together with previous measurements on $^{192}$Pb and $^{194}$Pb, this result allows superdeformed excitation energies, binding energies, and two-proton and two-neutron separation energies to be studied systematically, providing stringent tests for current nuclear models. The results are examined for evidence of a "superdeformed shell gap."

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Atomic nuclei constitute a unique testing ground for the understanding of finite Fermionic systems in which individual quantum states can be observed and characterized. As in metallic clusters, shell effects play a major role in determining both the static (shape) and dynamic (transition) properties of heavy nuclei. The observation of *superdeformed* nuclei (excited states adopting ellipsoidal shapes with an axis ratio around 2:1) associated with new shell gaps, predicted in even simple models but previously unconfirmed by experiment, has been one of the most exciting discoveries of contemporary nuclear structure studies. This Letter focuses on the opportunity presented by these exotic states to test established nuclear models, which have been developed and refined using data obtained at normal nuclear shapes.

The superdeformed (SD) shape is thought to be associated with a distinct minimum in the nuclear potential energy, separated from the primary minimum by a high barrier. The lowest-energy states in this second well can thus be thought of as quasivacuum states. The properties of these "SD ground states" (in particular the effective binding energy systematics), as well as the size and location of the SD shell gaps, should provide excellent benchmarks for any model of the nucleus.

Over the past two decades, rotational bands associated with SD shapes have been observed in several regions of the nuclear chart [1], with 85 SD bands observed in nuclei with $79 \leq Z \leq 84$ (the $A \approx 190$ region) alone. Unfortunately, measurement of the fundamental properties of these states—their excitation energy, spin, and parity—has only rarely been possible. The difficulty lies with observing the very weak discrete transitions which link SD levels with levels of normal deformation (ND levels). To date, linking transitions have been identified in the decay of only five SD bands in the $A \approx 190$ region: two bands in $^{194}$Hg [2,3], and one band in each of $^{194}$Pb [4,5], $^{192}$Pb [6], and $^{191}$Hg [7]. Less precise measurements have been achieved in $^{192}$Hg [8] and $^{195}$Pb [9] following analyses of the quasicontinuum component of the decay.

The present Letter reports on the measurement of the excitation energy of the yrast (lowest energy for a given spin) SD band in $^{196}$Pb. Together with earlier measurements of the excitation energies of SD states in $^{194}$Pb [4,5] and $^{192}$Pb [6], the results allow a systematic study of the energy of the SD well in a single isotope chain. Comparisons with both macroscopic and microscopic calculations and an analysis of the data aimed at identifying the SD neutron shell closure are presented.

The data were obtained in an experiment carried out using the Euroball IV array [10] at the Institut de Recherches Subatomiques, Strasbourg. High-spin states were populated in the reaction $^{170}$Er($^{30}$Si, $4n$)$^{196}$Pb. The target consisted of a 1.65 mg/cm$^2$ layer of $^{170}$Er on a Au backing of thickness 6.6 mg/cm$^2$. The beam (which was not pulsed) was provided by the Vivitron tandem accelerator at an energy of 144 MeV. An "inner ball" of 210 BGO detectors was used to measure the sum energy and multiplicity of the $\gamma$ rays emitted in each reaction. Events were recorded when signals were obtained from at least five of the inner ball elements and at least four escape-suppressed Ge detectors.

It was recently demonstrated that selection of decay paths to isomeric states in the normal minimum can simplify the study of the decay out of a SD minimum [6]. The yrast levels of spin and parity $I^{\pi} = 5^-, 9^-, 11^-$, and $12^+$ in $^{196}$Pb are known to be isomeric [11]; the lifetimes of these states (between 50 and 250 ns) are such that, if any are fed in the decay from a SD state towards the ground state, the techniques of time-correlated $\gamma$-ray spectroscopy should be applicable.

This approach is straightforward when the data are obtained with a pulsed beam, but may be more complicated when a continuous beam is used. In the present case, the time of each $\gamma$ ray was recorded relative to a reference time $t_0$ defined by the "master trigger"—that is, relative to the last of the nine detector signals required for an event to be accepted. Because several strongly populated isomers can be fed during the decay from the highest spin states in $^{196}$Pb to the ground state, a $t_0$ derived in this way is not fixed relative to the reaction time. To circumvent this problem, when creating spectra gated on the SD band we have used the average time of the gating transitions (rather than the master trigger) to define $t_0$. This resulted not only in a more reliable time reference, but also in improved time resolution, and hence a better signal-to-background ratio. Energy-dependent time gates were then applied to determine whether a particular $\gamma$ ray was prompt (within $\approx$ 30 ns) or delayed (by 30–300 ns) with respect to the SD band.

Figure 1 shows a spectrum obtained by double-gating on the 388 and 429 keV transitions in the yrast SD band in $^{196}$Pb and requiring that all $\gamma$ rays are in prompt coincidence. The in-band transitions are marked with their energies. The peak at 1049 keV corresponds to the $2^+ \rightarrow 0^+$ ground-state transition in $^{196}$Pb. A comparison of this transition with the 260 keV transition (which carries 100% of the in-band SD flux) gives an indication of the fraction of the SD intensity decaying through isomeric states. By studying this and similar spectra we find that only 23(3)% of the SD band intensity is in prompt coincidence with the $2^+ \rightarrow 0^+$ transition, indicating that most of the decay occurs via paths which feed isomers.

Figure 2 presents evidence for two transitions linking states in the SD band to known ND levels. The candidate peaks are marked by dotted lines at 3698 and 4062 keV. All four panels show triple-gated, background-subtracted spectra, created under various energy and time conditions. The energy region shown is that in which single-step transitions between SD and ND states might be expected. The intensities of these two $\gamma$ rays ( $\leq 1\%$ of the SD intensity) are too low for energy coincidence relations alone to unambiguously determine the deexcitation paths. However, the combination of both energy and time correlations allows firm assignments to be made.

Figure 2(a) shows that both transitions are in prompt coincidence with the SD band, although the 3698 keV transition is barely resolvable. Both are also visible in Fig. 2(b), which was created requiring correlation with a delayed (by $\approx$ 30–300 ns) 689 keV $4^+ \rightarrow 2^+$ or 1049 keV $2^+ \rightarrow 0^+$ $\gamma$ ray. Figure 2(c) shows that only the 3698 keV transition is in prompt coincidence with transitions above the $5^-$ isomer, and Fig. 2(d) shows that neither is correlated with delayed decays between the $9^-$ and $5^-$ isomers. Although the lower energy transition is weak, it is at greater than the $3\sigma$ level above background in Figs. 2(b) and 2(c).

This information, combined with energy sums for the two paths, leads us to place the 4062 keV transition as

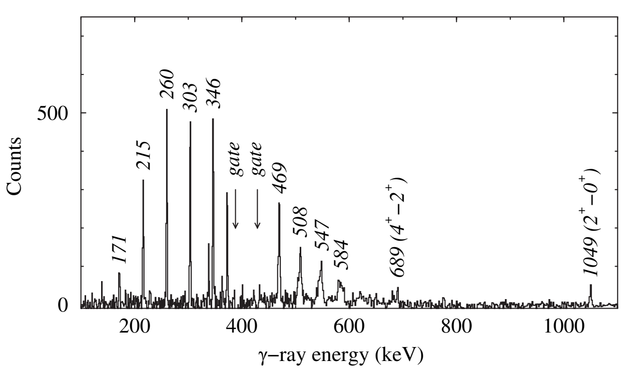


FIG. 1. Gamma rays in coincidence with both the 388 and 429 keV SD transitions. Members of the SD band are marked with the transition energies.

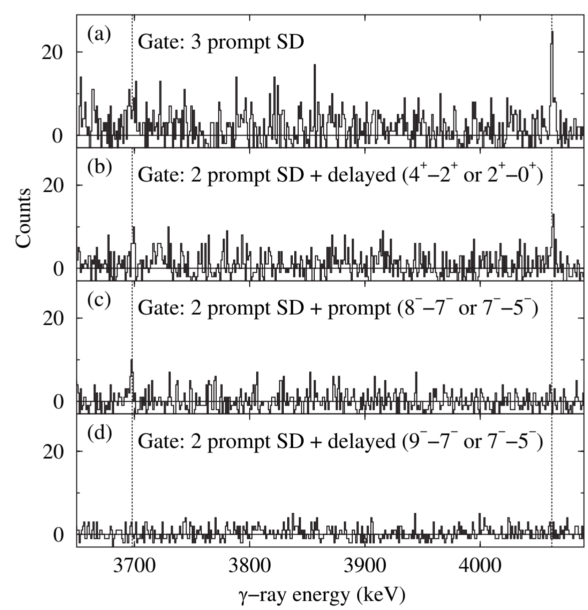


FIG. 2. Spectra showing the candidate SD-ND linking transitions. The gating conditions are indicated in the figure.

linking the $6^+$ SD level to the (isomeric) yrast $5^-$ level, and the 3698 keV transition as linking the $8^+$ SD level to the yrast $8^-$ level (the SD spin assignments are discussed below). The resulting decay scheme is shown in Fig. 3. The lowest observed SD level is placed at excitation energy $E_{\mathrm{SD}}(6^+) = 5.859(2)$ MeV.

Assuming that the band is based on a $K^\pi = 0^+$ configuration (as expected for a band built on a quasivacuum state in an even-even nucleus), the in-band levels will have even spin and positive parity. The proposed assignment implies $E1$ multipolarity for both observed single-step linking transitions; raising or lowering the spins by $2\hbar$ gives $M2$ and $E3$ assignments to the linking transitions, which would make competition with the highly collective in-band $E2$ transitions extremely unlikely. The spins in Fig. 3 are consistent with previous work [12] and a fit to the moment of inertia of the band [13].

Extrapolation of this fit to low spin indicates a bandhead energy $E_{\mathrm{SD}}(0^+) = 5.630(5)$ MeV, significantly higher than the values measured for $^{194}$Pb (4.643 MeV) [4] and $^{192}$Pb (4.011 MeV) [6]. This can be compared to existing predictions of SD bandhead energies, concentrating on those which have been applied to all three even-even Pb isotopes. A macroscopic approach has been taken using the Strutinsky method with a Woods-Saxon potential [14]. The microscopic Hartree-Fock-Bogoliubov (HFB) method has been employed by several authors using different meanfield parameterizations, including density-dependent Skyrme [15], SLy4 and SkP [16], and Gogny [17] interactions. Most recently, a cluster model has been applied to the problem [18]. The results of these approaches are compared to the extrapolations from experimental data in Fig. 4.

While most of the calculations predict the gross trend of decreasing energy with decreasing neutron number, neither the absolute energies nor their differences are consistently reproduced by *any* model. The macroscopic, Strutinsky-method calculations [14] (stars) reproduce the energy differences along the Pb chain to within 200 keV, but underestimate the absolute values by $\approx$1 MeV. The meanfield calculations using the SLy4 parameterization [16] (downward-pointing triangles), which is the most sophisticated of the Skyrme interactions, result in Pb excitation energies which are too high by $\approx$1 MeV and energy differences which are too large by $\approx$0.5 MeV. The Gogny force [17] calculations perform best overall for the Pb nuclei, but these too overestimate the energy differences by $\approx$0.5 MeV. Importantly, neither the Strutinsky-method nor the meanfield approaches reproduces the difference between Pb and Hg isotones. The cluster model [18] does well for all nuclei except $^{192}$Pb.

In fact, the decrease in $E_{\mathrm{SD}}(0^+)$ with decreasing neutron number $N$ is predominantly due to the increasingly unfavored ND ground states as we move further from stability, rather than of intrinsic properties of the SD states. Figure 5(a) shows the binding energy per nucleon of the ND ground states and SD bandheads obtained using the tabulated ND binding energies [19] and the experimental SD energies. The SD binding energies decrease at a slower rate, approaching the ND values with decreasing $N$. Accurate calculation of $E_{\mathrm{SD}}(0^+)$ depends strongly on the ground-state energy, and hence Heenen *et al.* [16] caution that this is not a reliable test of the HFB approach. More significant probes are the differences in the binding energies (the two-neutron and two-proton separation energies, $S_{2n}$ and $S_{2p}$) in the second well. These are shown in Fig. 5(b), along with the predictions of the HFB approach using the SLy4 and SkP forces [16].

For $^{196}$Pb, the SLy4 interaction (long dashed lines) does slightly better than the SkP interaction (short dashed lines), reproducing $S_{2n}$ and $S_{2p}$ to within 300 keV. However, both versions underestimate $S_{2p}$ for $^{194}$Pb by $\approx$1 MeV and SLy4 overestimates $S_{2n}$ for $^{194}$Pb by $\approx$0.8 MeV. Thus even the more sophisticated of these two Skyrme parameterizations fails to reproduce not only the absolute excitation energies, but also the differences in binding energy.

The presence of SD shell closures should also be reflected in the separation energies. (A related analysis aimed

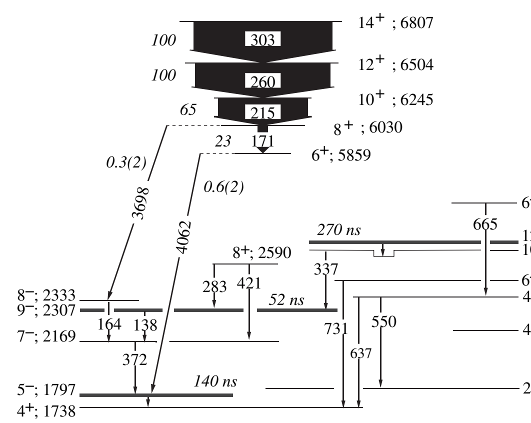


FIG. 3. Partial level scheme indicating the decay from the lowest observed SD states to the yrast normal-deformed levels.

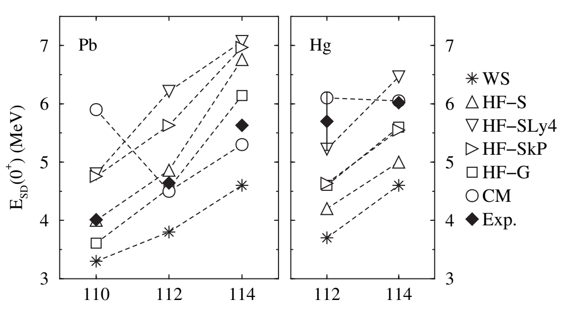


FIG. 4. Experimental and theoretical SD bandhead energies. WS: Strutinsky method using a Woods-Saxon potential [14]; HF-S, HF-SLy4, HF-SkP, and HF-G: HFB using density-dependent Skyrme [15], SLy4 [16], SkP [16], and Gogny [17] interactions; CM: cluster model [18].

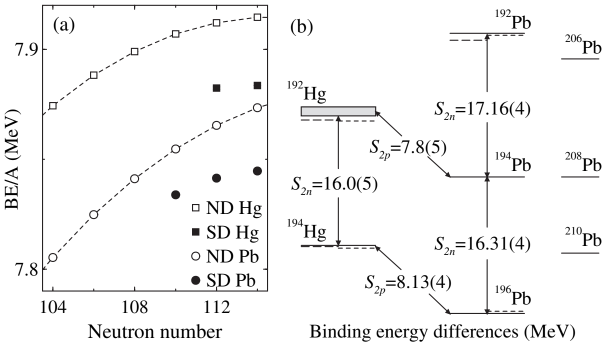


FIG. 5. (a) Binding energy per nucleon for ND and SD states at $I = 0\hbar$. (b) Differences in SD binding energies. Long/short dashed lines indicate SLy4/SkP HFB calculations [16]. ND data around $^{208}$Pb are shown on the right.

at examining pairing gaps has been performed for the Nd isotopes [20].) At normal deformations in heavy nuclei, the two-neutron separation energy changes such that $\Delta S_{2n,\mathrm{ND}} = S_{2n,\mathrm{ND}}(N) - S_{2n,\mathrm{ND}}(N+2)$ is typically around 0.5 MeV. Immediately above the ND shell gaps, $S_{2n,\mathrm{ND}}$ decreases markedly, resulting in $\Delta S_{2n,\mathrm{ND}} \approx 4$–5 MeV. This pattern, which is evident in the significantly smaller value of $S_{2n,\mathrm{ND}}$ below the $N = 126$ shell closure at $^{208}$Pb shown on the right hand side of Fig. 5(b), is one of the experimental signs of a shell gap. The SD separation energies $S_{2n}$ are significantly larger than the typical ND value of 0.5 MeV, possibly reflecting a lower average level density at superdeformations. However, no indication of the characteristic sharp drop in the binding energy is seen above $N = 112$ (where a shell closure is predicted). This could indicate either no significant shell gap, or gaps of approximately equal size at both $N = 112$ and $N = 114$.

Table I compares $\Delta S_{2n}$ predicted by the SLy4 and SkP calculations [16] for SD Pb and Hg isotopes and the value obtained from the data for $N = 112$. The SLy4 calculations suggest a shell closure at $N = 114$ rather than $N = 112$ for the Pb isotopes, but almost no difference between $N = 112$ and $N = 114$ for the Hg isotopes. The SkP calculations suggest very little difference between the gaps at $N = 112$ and $N = 114$ for either Pb or Hg. The experimental result strongly disagrees with the SLy4 calculations, and is in fact much closer to the value obtained with the SkP interaction.

TABLE I. Differences between two-neutron separation energies in the second minimum.

| | $\Delta S_{2n,\mathrm{SD}}$ Pb (MeV) | | $\Delta S_{2n,\mathrm{SD}}$ Hg (MeV) | |
|---|---|---|---|---|
| | $N = 112$ | $N = 114$ | $N = 112$ | $N = 114$ |
| SLy4 | −0.206 | 1.490 | 0.940 | 0.912 |
| SkP | 0.871 | 0.787 | 0.778 | 0.816 |
| Experiment | 0.85(8) | | | |

In conclusion, we have observed two high energy transitions in the decay of the yrast SD band of $^{196}$Pb which directly feed yrast states in the normal well. The data establish the excitation energy of the SD states, and confirm the predicted trend of decreasing excitation energy with decreasing neutron number. A detailed comparison with the available model predictions shows that none correctly reproduces the excitation energies. Where theoretical binding energy differences are available, it is found that these too are not well reproduced. The binding energy systematics suggest similar "gaps" at $N = 112$ and $N = 114$, perhaps indicating that both neutron numbers are semimagic at superdeformation. It remains a challenge for current models of the nucleus to reproduce these properties of superdeformed nuclei.

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