# Group interactions of dissipative solitons in a laser cavity: the case of 2+1

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**Abstract:** What can be the outcome of the interaction between a dissipative soliton pair and a soliton singlet? We report an experimental observation of "elastic" collisions as well as "inelastic" formation of triplet soliton states in a fiber laser setup. These observations are supported with the numerical simulations based on the dispersion (parameter) managed cubic-quintic Ginzburg-Landau equation model.

**OCIS codes:** (060.5530) Pulse propagation and solitons; (140.3510) Fiber lasers.

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## References and links

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## 1. Introduction

In integrable systems, such as the one modeled by the nonlinear Schrödinger equation, two bright temporal solitons having different amplitudes and velocities can undergo elastic collision, but they cannot form a stable bound state of two solitons. In contrast, in the case of dissipative solitons, it was found theoretically that stable asymmetric soliton pairs can be formed, when we model the system by the complex cubic-quintic Ginzburg-Landau equation (CCQGLE) [1]. These stable soliton pairs were later observed experimentally in a fiber ring laser [2]. The key feature of asymmetric soliton pairs (with their phases in quadrature) is their group velocity that is different from the group velocity of a single soliton. Hence, when the soliton pair (doublet) and a soliton singlet exist simultaneously in the cavity, they must collide at some stage. In the present work, we studied such interactions in a dissipative system such as a fiber laser. In Section 2, we report the experimental observation of endlessly repeated collisions between the pair and a single soliton. By tuning cavity losses, we were also able to observe the formation of a stable triplet soliton state. Section 3 gives the description of our numerical simulations, which use the CCQGLE, as well as the concept of dispersion (parameter) management. These reproduce the behavior observed in the experiment. After discussing the simulations, we finally conclude in Section 4.

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## 2. Dissipative solitons in interaction: experimental observation

The mode-locked laser (see Fig. 1), which emits around 1.5-µm, is a dispersion-managed fiber ring laser [3, 4]. The gain is provided by a 1.9-m long, 1400-ppm erbium-doped fiber (EDF) that is pumped at 980 nm and features normal chromatic dispersion [$D$ = -40 (ps/nm)/km]. The path-averaged cavity dispersion is adjusted with the use of an appropriate length of a SMF-28 fiber that has anomalous dispersion [$D$ = +16.5 (ps/nm)/km]. A 50-cm long open air section is used to insert polarization components. Due to the nonlinear polarization evolution that takes place along with propagation in the fibers, the transmission through the polarizer P1 is intensity dependent, and an appropriate adjustment of the preceding wave plates triggers the mode-locked laser operation. A second polarizer P2, preceded by a half-wave plate, provides a convenient variable output coupler. In the present work, the output coupling is set to around 40%. The laser also features a 10%-output fiber coupler, where a small length of dispersion compensating fiber (DCF) is spliced to the SMF-28 fiber output in order to compress the chirped pulses. Pulsed laser operation is analyzed by a homemade optical autocorrelator and by an optical spectrum analyzer (MS 9710B). Output intensity is also monitored on a 500-MHz digital phosphor oscilloscope.

In the present work, the path-averaged dispersion is in the normal regime and close to zero [$D \approx -2$ (ps/nm)/km]. In this regime, dissipative solitons acquire much larger energy and frequency chirping than in the anomalous regime [3]. In most cases, this allows observing fewer solitons at a relatively large pumping power. Soliton pairs and triplets are rather easily formed while adjusting the wave plates in the open-air section [4]. Pumping power is around 350 mW, and a single soliton has intracavity energy of around 0.5 nJ after the EDF (note that the soliton energy is not precisely quantized with respect to pumping power).

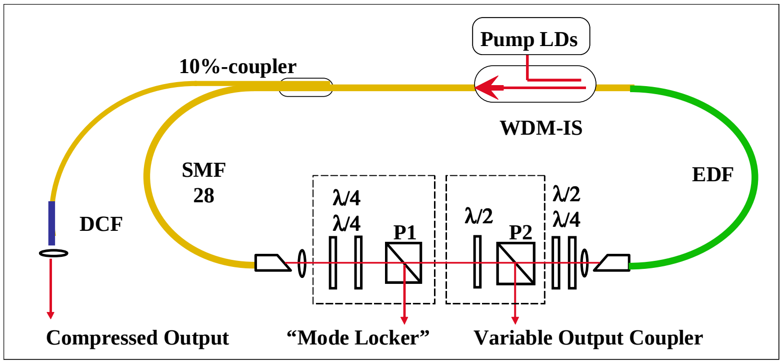

Fig. 1. Fiber ring laser experimental setup. WDM-IS: polarization-insensitive coupler isolator.

Operating with three dissipative solitons in the cavity, we adjust (≈±5°) the orientation of the half-wave plate preceding P2, which is equivalent to changing the amount of linear losses in the cavity (≈±0.8 dB), and observe the following. For a small range of orientations, the oscilloscope traces feature two peaks per cavity round trip. The larger one is twice the other one in amplitude and appears stably on the screen, since its level is used for trace synchronization. The second one moves endlessly with a constant relative velocity (at this scale of observation). This is illustrated by Fig. 2(a) and the video recording which is linked to it. The largest peak, as we shall see more clearly below, is identified as a doublet or soliton pair, whereas the second peak is identified as a singlet soliton. As the singlet moves as in Video 1, one round trip difference with the doublet, which represents 5.3 meters of fiber length, is traveled in 0.64 seconds, which means the group velocity difference between the doublet and singlet is 8.3 m/s. This difference is less than $10^{-7}$ of each soliton group velocity! We should also stress that the relative motion can be stable for hours once obtained. Although the existence of relative motions between pulses in fiber lasers was already mentioned [5], we believe the present letter is the first to report and analyze the case of a simple repeated motion.

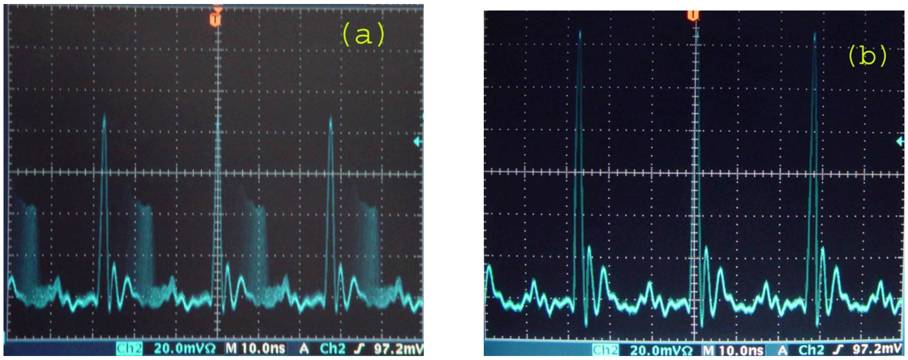

Fig. 2. (a) “Collisions”: the QuickTime video file (1475 Kb) presents live recording of the oscilloscope trace featuring a doublet and a moving singlet soliton. (b) “Collisions and triplets”: the QuickTime video file (1497 Kb) presents formation and decomposition of stable triplet soliton, when cavity losses are slightly changed back and forth. Note that the small sub peaks of amplitude around or less than one division are electronic artifacts due to imperfect impedance matching.

Varying slightly the output coupling, a stable triplet soliton state can be formed as a result of the interaction between the doublet and singlet states. This triplet corresponds to the oscilloscope trace in Fig. 2(b). In the video which is link to Fig. 2(b), we adjust back and forth the output coupling, to show how we can form or decompose repeatedly the triplet state. Although the triplet state is highly stable once obtained, varying back the output coupling results in the decomposition of the triplet into doublet and singlet solitons and, quite remarkably, the same cycle can be repeated again. The transition 2+1->3, although with some hysteresis, is reversible, as illustrated in Video 2.

The appearance of singlet, doublet and triplet soliton states in the cavity finds more justification when considering optical autocorrelation traces and optical spectra. Figure 3(a) represents the autocorrelation traces recorded at the variable output coupler. The blue curve is taken when the soliton pair and the soliton singlet are moving with different group velocities, as in Fig. 2(a). We can see that the amplitude of the central peak is three times larger than the amplitude of side peaks. This is compatible with the fact that the soliton pair or doublet is made of two equal pulses that have the same amplitude as the third moving pulse. The red curve is taken when the triplet is formed, as in Fig. 2(b). This is the autocorrelation function of three identical, equally separated pulses. The same analysis is confirmed by recording the autocorrelation function at a different location, namely after the erbium doped fiber, at the 10%-output of the fiber coupler. It is displayed on Fig. 3(b). Dispersion compensation allows clearer identification of pulses that form either a doublet or a triplet state. The separation between the bound pulses is 1.2 ps, and does not practically change during the round trip, whereas individual pulse width varies significantly due to the important frequency chirping effect.

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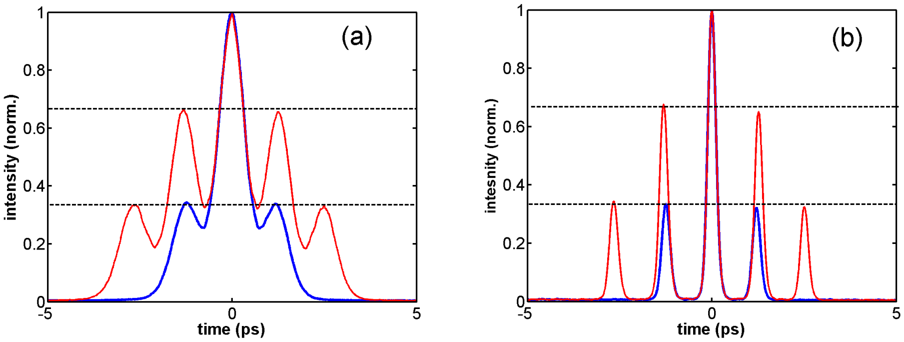

Fig. 3. Autocorrelation traces taken from (a) the variable output coupler and (b) the 10% coupler. In blue: doublet and singlet in endlessly relative motion. In red: when the triplet soliton state is formed.

The spectrum in Fig. 4 is recorded at the variable output coupler, in the regime, when the soliton doublet and singlet states move endlessly relative to each other. We can see clearly that this spectrum consists of two contributions. The first one is the soliton pair spectrum, with an interfringe separation of 6.5 nm that corresponds exactly to the soliton pair separation of 1.2 ps. The second contribution is the baseline of the spectral fringes, represented in dotted line. The latter corresponds to the spectrum of a singlet. Namely, the integral of the recorded spectrum (in blue line) is three times the integral below the dotted line with high accuracy.

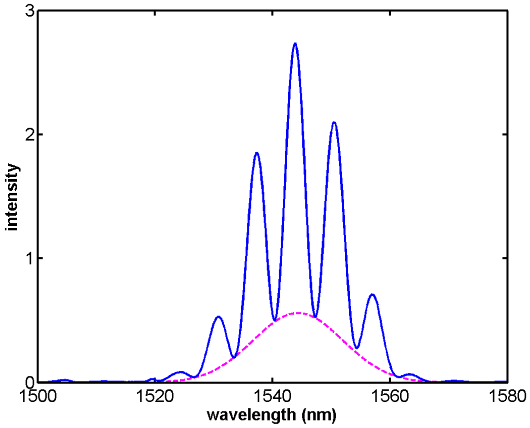

Fig. 4. Recorded optical spectrum (in blue line) and its baseline (in magenta dotted line), for doublet and singlet solitons in endlessly relative motion.

We can clearly see from Fig. 4 that the contribution corresponding to the spectrum of the soliton pair is asymmetric. The presence of this asymmetry provides an explanation for the group velocity difference between the pair and the singlet solitons state.

## 3. Numerical simulations

Our numerical simulations are based on the CCQGLE model:

$$i\psi_z+\frac{D}{2}\psi_{tt}+|\psi|^2\psi+\nu|\psi|^4\psi=i\delta\psi+i\varepsilon|\psi|^2\psi+i\beta\psi_{tt}+i\mu|\psi|^4\psi, \tag{1}$$

where the parameters $D,\delta,\varepsilon,\beta,\nu,\mu$ represent chromatic dispersion, linear losses, nonlinear gain, spectral filtering, saturation of nonlinear index and saturation of nonlinear gain,

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respectively. All the parameters are piece-wise periodic functions of z. The model and the variation of parameters are sketched in Fig. 5. The model does not copy exactly the laser cavity but has the elements that are essential for the existence of the observed phenomenon. Dispersion has opposite signs in the two sections of the map as it does in the laser cavity. Average normal dispersion that is close to zero is one of the requirements for modeling the collisions.

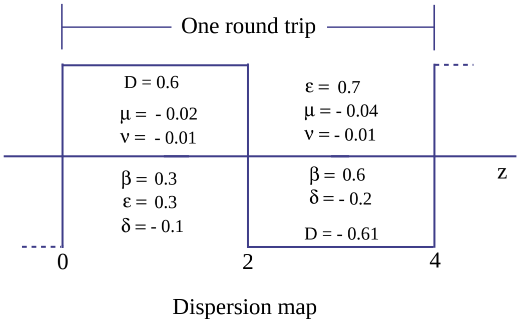

Fig. 5. Dispersion map and values of parameters used in the simulation

Simulations show that two solitons can form stable asymmetric soliton pairs similar to those in the continuous model of Eq. (1) [1]. Due to the periodic width variations solitons interact at larger separations than in the case of the continuous model. Figure 6 shows that the distance R between the soliton centers in the pair is around 6 dimensionless units and the phase difference is ±π/2. Due to their asymmetry, the soliton pairs have group velocity that differs from the group velocity of singlet solitons. The plus or minus sign of the phase difference defines the direction of motion of the pair relative to the single pulse. The two fixed points in the interaction plane are stable foci and thus the soliton pairs are stable.

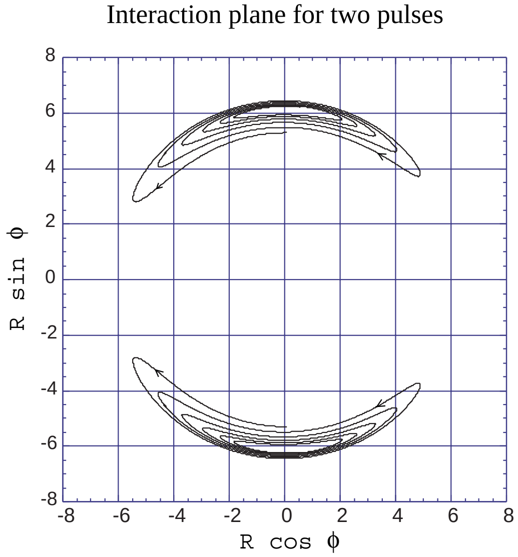

Fig. 6. Formation of stable soliton pairs in the interaction plane. R represents the separation between the two solitons, whereas ϕ is their relative phase.

Two nearby solitons will merge into a coupled state and propagate indefinitely until the pair is disturbed by a collision with a single pulse. The collision is a strong perturbation that destroys the coupling but a new bound state is formed after the interaction. The collision

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between the soliton pair and the soliton singlet is shown in Fig. 7. This scenario of collision might seem to correlate with the laws of classical mechanics when two particles of the same mass moving in one dimension collide with the third one resulting in the motion of the third particle along with one of the incident ones. However, the dissipative system is ruled by different laws. Firstly, the binding energy of solitons is nonzero. Secondly, the difference in velocities of the pair and the singlet is fixed and must be the same before and after collision. These two reasons result in the "elastic collisions" of dissipative solitons shown in Fig. 7. As in the experiment, each collision happens in the scale of hundreds of roundtrips. When periodic boundary conditions are used, the same type of collision is repeated many times indefinitely in time.

Soliton triplets exist on the same basis as pairs. They move with the same velocity provided the phase difference between the outermost solitons is twice $\pi/2$, i.e., $\pi$. When the phase difference between the outermost solitons is zero, i.e., $\pi/2-\pi/2$, the soliton triplet has zero velocity. The change of some parameters in the dispersion map can result in either of these states being excited after the collision.

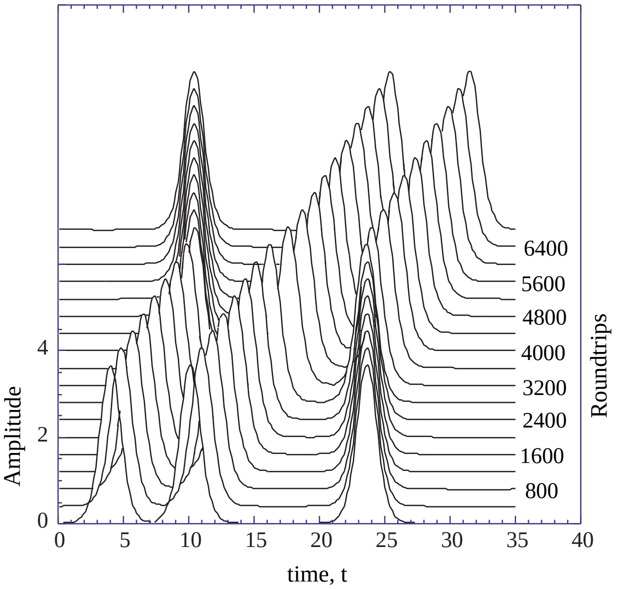

Fig. 7. "Elastic" collision of a pair of coupled dissipative solitons with a soliton singlet.

## 4. Conclusion

We have studied both experimentally and numerically group collisions of dissipative solitons. We have found that two solitons can be coupled into a stable pair that has group velocity different from the velocity of a single soliton. Collision of the pair with a single soliton destroys the bound state but another pair is formed that moves away with the same velocity leaving one of the solitons of the previously moving pair in rest. Depending on the number of solitons in the cavity and cavity parameters, more complicated interactions are also possible.

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